Chaotic Modeling and Simulation (CMSIM) **4: 185-189, 2019**

Rössler multiattractor

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Abstract: The stochastic dynamic system which movement occurs on the 2-dimensional composite multiattractor consisting of attractors of Rössler is considered. The formation of a multiattractor is provided by the introduction into the Rössler system of a replicating operator containing an additional operation of mutual displacement of adjacent phase cells containing copies of the Rössler attractor. Displacement is used to combine the areas of attraction of chaotic attractors at the boundaries between adjacent cells. The presence inherent in such multiattractors of the phenomenon of transformation of random transitions of the phase point between adjacent attractors in a directed rotational displacement of the active region of attraction of phase trajectories.
Keywords: Dynamic chaos, chaotic oscillator, composite chaotic multiattractor, replicating operator, phase cell, dynamics of active regions of attraction of phase trajectories.

The Rössler attractor [1] has an asymmetrical shape. Therefore, to build a dynamic system with a multiattractor consisting of Rössler attractors, it is necessary to use a replication operation with an offset. The displacement is necessary to combine the regions of attraction of neighboring attractors, which allows the phase trajectories to carry out transitions between adjacent chaotic attractors in both directions [2].

Using this approach, it is possible to build different versions of dynamic systems that implement all kinds of configurations of multi-attractors, combining attractors Rössler.

For example, consider the following system:

$$\begin{cases} \dfrac{dx_1}{dt} = -H_2(x_2) - H_3(x_3)\,; \\ \dfrac{dx_2}{dt} = D(x_1, x_3) + AH_2(x_2)\,; \\ \dfrac{dx_3}{dt} = B + [D(x_1, x_3) - C]H_3(x_3)\,, \end{cases} \qquad (1)$$

Received: 20 July 2019 / Accepted: 17 October 2019
 ISSN 2241-0503

where

$$H_k(x_k)=\begin{cases} x_k, & x_k \le h2_k\ ; \\ x_k - h1_k - h2_k, & x_k > h2_k\ , \end{cases}$$

$$D(x_k, x_p)=\begin{cases} x_k, & x_p \le h2_p\ ; \\ x_k - \Delta, & x_p > h2_p\ , \end{cases}$$

$h1_\xi$ and $h2_\xi$ –coordinates of the lower and upper boundaries of the phase cell, containing the replicated attractor [3], by coordinate x_ξ., Δ –the amount of offset copies of the original Rössler attractor by coordinate x_1.

The system (1) has a simple two-dimensional multiattractor (this refers to the composite dimension of the multiattractor, equal to the number of replication variables replaced by replicating functions [4]) obtained by replication of the Rössler attractor at x2 and x3 coordinates, using the $H_2(x_2)$ and $H_3(x_3)$ replicating operators and the displacement of copies of the original Rössler attractor by the x_1 coordinate using the $D(x_1,x_3)$ operator.

Figures 1,2 show the projections of the multiattractor of this system on the plane (x_2,x_3) and (x_1,x_3), respectively, when $A=0.2$, $B=0.2$, $C=5$, $h2_1=-8.9$, $h2_2=6.63$, $h3_1=0.0157$, $h3_2=15.1$, $\Delta=10$.

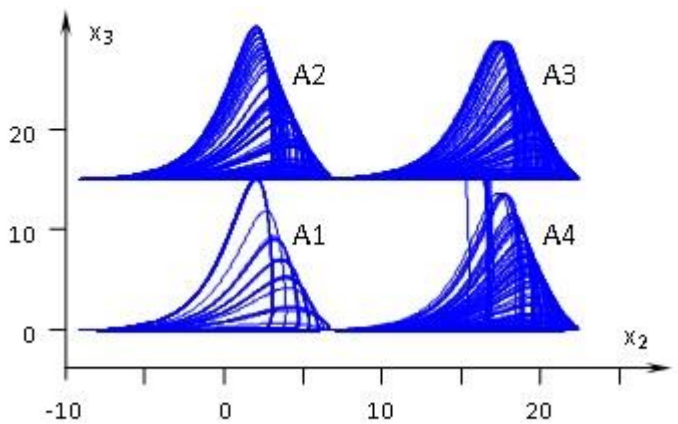

Fig.1. Projection of the multiattractor of the system (1) on the plane (x_2,x_3).

Feature composite multiattractor consisting of asymmetric local attractors, is the possibility of inequality of probabilities of phase transitions points between adjacent local attractors [5]. In the system (1), on those time intervals when the motion is concentrated, for example, only on attractors A1 and A4 or only on attractors A2 and A3, the probability of finding a phase point on the attractor A1(respectively, A2) is approximately 0.65, and on the attractor A4(A3) – approximately 0.35. A similar probability inequality for pairs of local attractors A2-A1 and A3-A4 is much less pronounced; the appropriate probability for them is equal to approximately 0.502 and 0.498. Consequently, in the composite multiattractors, consisting of asymmetric local attractors, as well as in inhomogeneous multiattractors, there is, as a rule, an uneven probability distribution for the local attractors. In system (1) this distribution has the following form: p(A1)≈0.42, p(A2)≈0.1, p(A3)≈0.14, p(A4)≈0.34. That is, at large intervals of observation, the movement spends about 42% of the time on the local attractor A1, 10% on the attractor A2, 14% on the attractor A3 and 34% on the attractor A4.

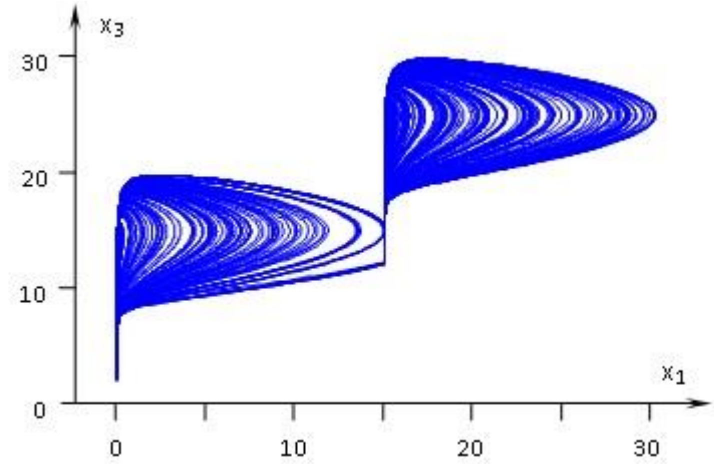

Fig.2. Projection of the multiattractor of the system (1) on the plane (x_1,x_3).

In addition, the considered dynamic system is characterized by a noticeable rotational instability of the position of the active region of attraction of phase trajectories (the attractor, on which at the current time there is a movement). As a result, this area is systematically shifted in the direction of rotation clockwise.

As a numerical characteristic of this motion element, it is convenient to use a scalar value that increases by one when moving from the attractor to the attractor in the direction of rotation of the arrow, and decreases by one when moving in the opposite directions (the phase index). The character of the change in the phase index of the system (1) over time is shown in Fig.3. The average growth rate of the phase index corresponding to this figure is $1.2 \cdot 10^{-3}$ with a standard deviation of $0.2 \cdot 10^{-3}$.

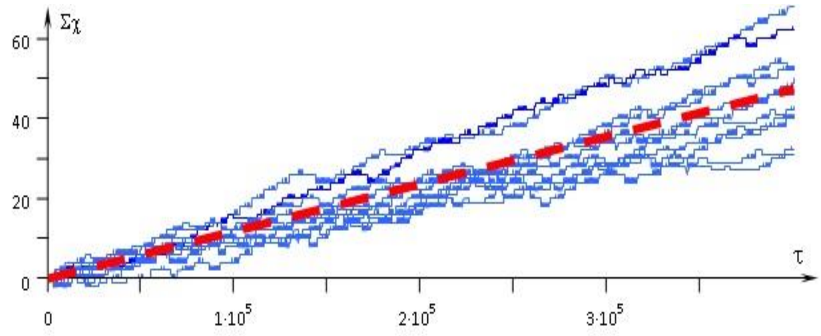

Fig.3. 11 realizations of a random process $\Sigma\chi(\tau)$ (thin broken lines) and the change of the phase index with the average for these implementations speed (a straight line).

It should be noted that the systematic shift of the active local attractor in the multiattractor of the system (1) is not related to the distribution of motion probabilities by local attractors. This is due to the fact that the inequalities in the probability of transitions of phase trajectories between neighboring local attractors are mutually compensated in pairs (A1-A4, A2-A3) and (A1-A2, A3-A4). The reason for this shift, as well as in homogeneous multiattractors consisting of symmetric local attractors, is a different configuration of closed displacement circuits in opposite directions. This causes a difference in the corresponding probabilities of systematic displacement of the active local attractor.

Thus, it is possible to construct composite multiattractors consisting of the Rössler attractor by supplementing the operation of the attractor replication of the original dynamic system with the operation of displacement of the obtained copies of this attractor.

Since the displacement of the active region of attraction of phase trajectories is nothing but the evolution of the state of the dynamic system, it can be stated that the movement on the composite multi-attractor Rössler is characterized by the presence of such evolution, generated by random interactions between the elements of the multiattractor. A distinctive feature of the Rössler multiattractor is that in the process of such evolution the probability of motion localization on the active local attractor changes in accordance with the probability distribution on local attractors.

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