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Transition from Chaos to Order in a Classical Yang Mills Higgs System

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Abstract. Yang Mills theory is a gauge theory having a central role in our current understanding of fundamental interactions. Today most field theories of weak, electromagnetic and strong interactions are well described by gauge theories. Yang Mills theories are special example of gauge theory with a nonabelian symmetry group. Chaos to order transitions in a classical Yang Mills Higgs system is analyzed with the aid of Lyapunov exponents and Poincare sections. Addition of oscillatory term to the Hamiltonian of the corresponding system shows that there is a transition from chaotic behavior to regular motion as its intensity is increased. The maximal Lyapunov exponents of the system in a specific range of parameters are calculated and the region where the chaos-order transition occurs identified with an eye on transition back to order.
Keywords: Dynamical systems, Lyapunov Exponents, Yang Mills Higgs, Chaos Theory, Poincare Sections

Introduction

An inspiration for taking a attempt for an alternate view in the established acknowledgement of the Yang-Mills or Yang Mills expanded by the Higgs mechanism mathematical statements is the significance of this framework in the starting instability or stability of the universe, since in the beginning stages all associations were of the same quality and depended on non-abelian gauge hypotheses, of which the SU(2) Yang Mills is a first example. In this study we consider the accompanying two degree of freedom viable Hamiltonian recommended by Biro, Matinyan and Müller [1].

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$$H = \frac{P_x^2 + P_y^2}{2} + \frac{1}{2}x^2y^2 - \frac{1}{2}y^2 + \frac{1}{8}x^4 + \frac{1}{4}py^4 \qquad (1)$$

In gauge speculations, the presence of diverse stages, especially a limiting stage connected with a disordered field setup and the Higgs stage portrayed by an all-around requested field condensate, is a focal issue. The qualification between established dynamical frameworks and the non-existence of a complete arrangement of nontrivial integrals of motion is of essential significance. In this part, we study a coupled Yang-Mill-Higgs framework which displays an all-inclusive requested stage because of unconstrained symmetry breaking.

If we perform appropriate gauge field configurations and scale transformations, we obtain the following simple dynamical system;

$$\begin{aligned} \ddot{x} &= -xy^2 - \frac{1}{2}x^3 \\ \ddot{y} &= -x^2y + y - py^3 \end{aligned} \qquad (2)$$

It can be shown that the system possesses chaotic behavior for a large range of p values.

In this work, we analyze the suppression of chaotic behavior with an isotropic oscillator term. The Hamiltonian of the system under consideration is

$$H = \frac{P_x^2 + P_y^2}{2} + \frac{1}{2}x^2y^2 + \frac{1}{2}g^2(x^2 + y^2) - \frac{1}{2}y^2 + \frac{1}{8}x^4 + \frac{1}{4}py^4 \qquad (3)$$

This Hamiltonian resembles to the 4th order truncation of three particle Toda Lattice Hamiltonian [2] except the 3rd order terms which are responsible for the chaos in the Henon Heiles system [3].
The equation of motions for the system is

$$\begin{aligned} \ddot{x} &= -xy^2 - \frac{1}{2}x^3 - g^2x \\ \ddot{y} &= -x^2y + y - py^3 - g^2y \end{aligned} \qquad (4)$$

Lyapunov Exponents for Indication of Chaos

One of the most effective methods for the determination of chaos in dynamical systems is calculation of Lyapunov exponents. In strict mathematical terms, chaotic motion is defined in terms of the long-term exponential divergence of neighboring trajectories with very small initial separations in phase space. On the other hand, neighboring orbits of integrable systems, either perform stable oscillations around each other or diverge at most as a finite power of time.

Let's consider the system of first order differential equations

$$\dot{x} = F(x_1, \dots x_N) \qquad i = 1, \dots, N \tag{5}$$

For a given solution $x_i(t)$, we can linearize the equations of motion around this reference orbit and obtain a set of linear differential equations for the deviations $\delta x_i(t) = x_i(t) - \tilde{x}_i(t)$:

$$\frac{d}{dt}(\delta x_i) = \sum_{k=1}^{N} \delta\, x_k(t) \left(\frac{\partial F_i}{\partial x_k}\right)_{x=x_k(t)} \tag{6}$$

The length of the vector δx_i is,

$$d(t) = \sqrt{\sum_{i=1}^{N} \delta x_i(t)^2} \tag{7}$$

which gives a measure of the divergence of the two neighboring trajectories $x_i(t)$ and $\tilde{x}_i(t)$. The maximal Lyapunov exponent λ_1 is defined as the long time average of its logarithmic growth rate,

$$\lambda_1 = \lim_{t\to\infty} \lim_{d(0)\to 0} \frac{1}{t} \ln\left(\frac{d(t)}{d(0)}\right) \tag{8}$$

Where the $\lim_{d(0)\to 0}$ ensures the validity of the linear approximation at any time. Regions of phase space for which $\lambda_1 > 0$ exhibit sensitive dependence of the orbits on the initial conditions. An infinitesimal change in initial data results in macroscopic deviations after sufficiently long time, $d(t) = d(0)e^{\lambda_1 t}$ for large t . Hence, dynamical systems exhibiting a positive Lyapunov Exponent are truly unpredictable over the long term and has a predictability horizon related to the reciprocal of the Lyapunov Exponent. Hence if the Lyapunov Exponent decreases in magnitude, the system behaves more predictably.

For $N-$ dimensional systems there exist a set of N characteristic exponents λ_i, corresponding to the long time limit of the eigenvalues of the associated tangent space. They can be ordered by size,

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \tag{9}$$

One of the λ_i always vanishes, corresponding to the displacement along the reference trajectory. For Hamiltonian systems with n degrees of freedom, $N = 2n$ and the $2n$ Lyapunov Exponents come in pairs of opposite sign,

$$\lambda_i = -\lambda_{2n-i+1} \tag{10}$$

In other words, for every direction in phase space that expands exponentially there is one that contracts at an equal rate, thus ensuring the validity of Liouville's theorem about the constancy of the phase space volume.

Calculations and Results

The results below were obtained numerically using a Fortran code [4] that implement the Wolf algorithm [5] to determine trajectories and Lyapunov Exponents of dynamical system, and a reduce code [6] which calculates the variational equations needed for the Wolf algorithm

The Yang Mills Higgs system that we consider possess chaotic behavior in a certain range of parameter p, but we show that the addition of oscillator term to the Yang Mills Higgs system shifts the system towards periodic or quasi periodic behavior. There is an obvious transition from chaotic behavior to ordered behavior when the oscillator parameter increased.

The maximal Lyapunov Exponents of the system are analyzed at a specific region of parameters. In some regions the exponents gets larger where the chaoticity of the system is dominant, then suddenly exponents gets smaller showing that chaos order transition take place. But after a certain value of parameters the exponents gets smaller smoothly and approaches to zero.

With the aid of MINUIT[7] program, we analyzed Maximal Lyapunov Exponents when one of the parameters is fixed and the other is changing, and we have seen that there is a sudden decrease in Exponents after certain value of parameter showing a possible bifurcation scheme.

Constantinescu and Ionescu[8] have indicated the possibility of stabilization mechanisms for the Yang Mills system, including the Higgs mechanism itself and mass terms[7]. In this work, we wish to illustrate that both mechanisms are needed together in the Matinyan Model.

Some of the Poincare Sections and Lyapunov Exponents corresponding to chaotic and periodic behavior of the system is given below.

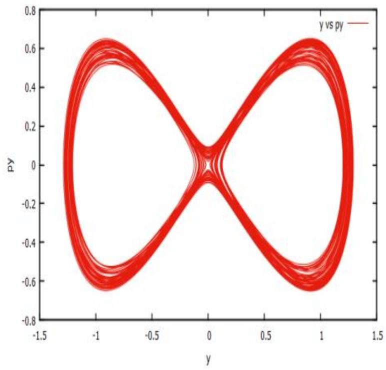

Fig. 1. y versus py (p=1.2, g=0)

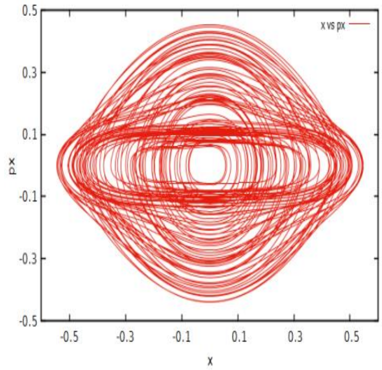

Fig. 2. x versus px (p=1.2, g=0)

Chaotic behavior for Yang-Mills-Higgs system in the absence of additional oscillator.

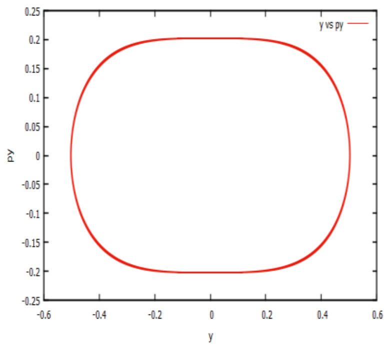

Fig. 3. y versus py (p=1.2, g=1)

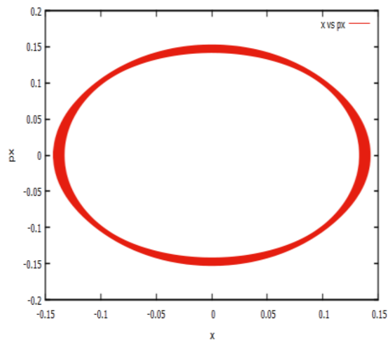

Fig. 4. x versus px (p=1.2, g=1)

Ordered phase of the system showing limit cycles for an additional oscillator term.

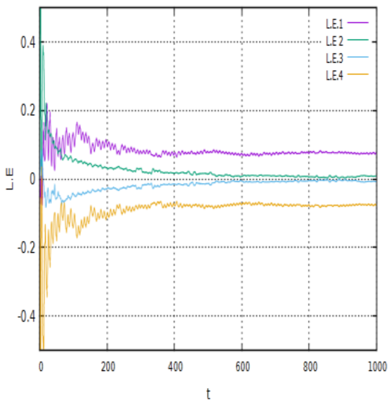

Fig. 5.a. t versus L.E (p=1.2, g=0)

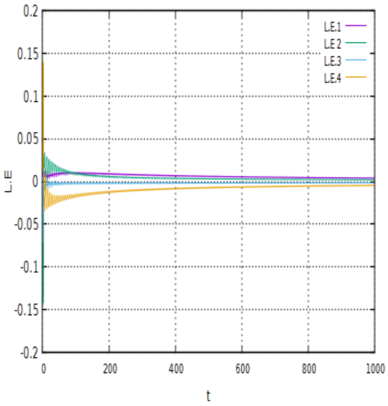

Fig. 5.b. t versus L.E (p=1.2, g=1)

Lyapunov Exponents without oscillator (Fig .5.a) ,with oscillator (Fig.5.b)

Lyapunov Exponents and Poincare Sections above shows the stabilizing effect of additional oscillator for the system.

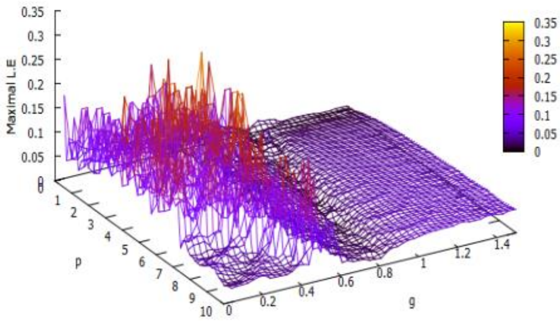

Fig. 6. Maximal Lyapunov Exponents for interval 0<g<1.5, 0<p<10

The Lyapunov Exponents are maximum in the region where oscillator parameter g is smaller than 0.6. After g=0.6 they approaches 0.

The MINUIT program is used to minimize the equation of motions and an operating point is found. Then one of the parameters held fixed when the other parameter is varied. The results are given below.

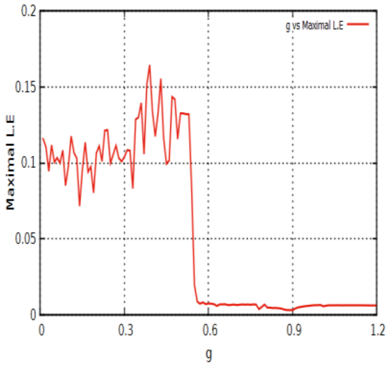

Fig. 7.a. g versus Maximal L.E (p=4.91)

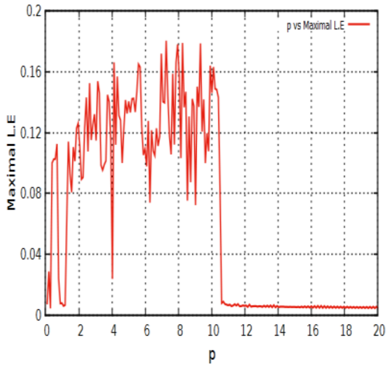

Fig. 7.b. p versus Maximal L.E (g=0.50)

Lyapunov Exponents for one parameter change indicating chaos order transitions for a critical value. For Fig.7.b. there are sudden changes for some critical values of p parameter.

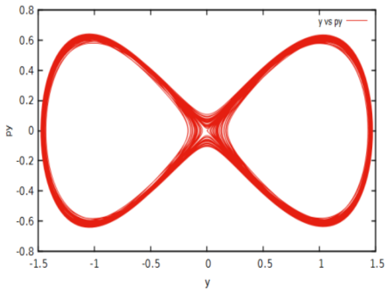

Fig. 8.a. y versus py (p=0.68,g=0.50)

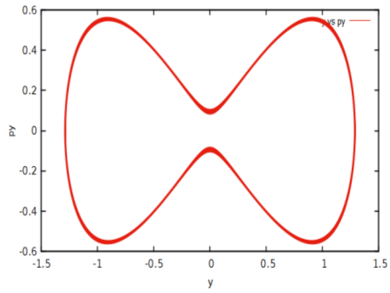

Fig. 8.b. y versus py (p=0.89,g=0.50)

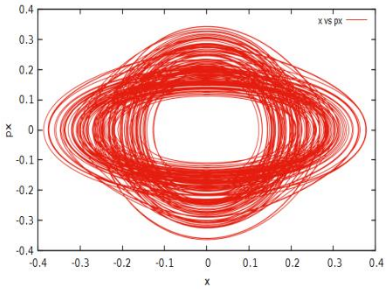

Fig. 9.a. x versus px (p=0.68,g=0.50)

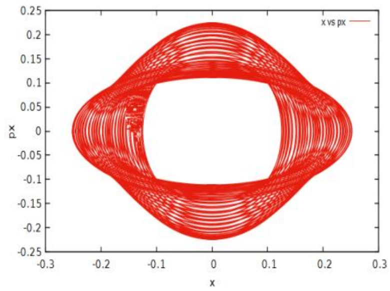

Fig. 9.b. x versus px (p=0.89,g=0.50)

From the Figures above it can be seen that while p=0.68 the chaotic motion is dominant, for p=0.89 transition to quasiperiodic motion begins. Then again chaotic and quasiperiodic motions take place until p=10.55, after this value the motion becomes completely periodic.

Conclusions

It is clearly seen that there is a well-defined range of parameters for which a chaotic regime is observed in the Yang Mills Higgs system, corroborating the results of Matinyan and coworkers. However, increasing the intensity of the oscillator term indicates a transition from chaos to quasi periodicity and introduces stability. The oscillator term, as well as majority of the terms in the Matinyan et. al model are related to the truncations of the cyclic three body next neighbor interaction model [2].

We have seen that chaotic behavior frequently occurs in models inspired by classical gauge field theories. Gauge field theories are usually formulated in phase spaces of larger dimensions, then special solutions with known homogeneity properties are used to convert systems such as Yang Mills or Yang Mills plus Higgs into dynamical systems. In this way, the system can be reduced to an evolution equation of a dynamical system in time. Stability of the universe, mechanism of the onset of the instability that led to the big bang becomes a relevant dynamical system problem.
Stability of the universe, mechanism of the onset of the instability that led to the big bang becomes an important dynamical system problem. In that light, the recent statements that gravity may have prevented the universe from collapsing 13.8 billion years ago and saved early universe. It was thought the Higgs particle should have made the cosmos unstable. However, gravity interacted with the Higgs boson to provide stability. If a system has a stable equilibrium point, motion near it is simple harmonic and our observation that as the intensity of the harmonic oscillator term increases, we transition from chaos to multi periodicity and this is of relevance in this context.

References:

[1] T S Biró, S G Matinyan, B Müller, "Chaos and Gauge Field Theory" World Scientific Publishing Co. Singaport (1994).

[2] G.Contopoulos, C Polymilis, " Approximations of the 3-particle Toda lattice" Physica D: Nonlinear Phenomena Volume 24, Issues 1–3, Pages 328-342 (1987)

[3] Hénon, M.; Heiles, C. "The applicability of the third integral of motion: Some numerical experiments". The Astronomical Journal 69: 73–79.doi:10.1086/109234 (1964).

[4] Michael Metcalf, "Fortran 90/95 Explained" 2nd Edition, Oxford University Press, Oxford, UK (1999).

[5] Alan Wolf, Jack B. Swift, Harry L. Swinney and John A. Vastano. "Determining Lyapunov Exponents From a Time Series" Physica D Volume 16, p.285-317 (1985)

[6] AnthonyC.Hearn "REDUCE ™ REDUCE User'sManual Version3.8" SantaMonica,CA,USA February2004

[7] F. James, "MINUIT Function Minimization and Error Analysis Reference Manual", CERN Program Library Long Writeup D506, Geneva (1994).

[8] Radu Constantinescu, Carmen Ionescu, "Chaos and Stabilizing Mechanisms for Yang-Mills Mechanical Models", Physics AUC, Vol 21 Sp. Issue, 207-215 (2011).