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Anti-Synchronization of Chaotic Systems with Adaptive Neuro-Fuzzy Inference System

Uğur Erkin Kocamaz [1], and Yılmaz Uyaroğlu [2]

[1] Department of Computer Technologies, Vocational School of Karacabey, Uludağ University, 16700 Karacabey, Bursa, Turkey
E-mail: ugurkocamaz@gmail.com
[2] Department of Electrical & Electronics Engineering, Faculty of Engineering, Sakarya University, 54187 Serdivan, Sakarya, Turkey
E-mail: uyaroglu@sakarya.edu.tr

Abstract. In this study, Adaptive Neuro-Fuzzy Inference System (ANFIS), which is a combination of fuzzy inference system and artificial neural network learning algorithms, is proposed for the anti-synchronization of chaotic systems. Based on an adaptive model reference control technique, two identical chaotic systems that have different initial conditions are trained by backpropagating the anti-synchronization errors. In the simulations, well-known Lorenz chaotic system is used. Simulation results show that the proposed approach is very effective for the anti-synchronization of chaos.
Keywords: Anti-synchronization, Chaos, Adaptive Neuro-Fuzzy Inference System, Neuro-Fuzzy, ANFIS.

1 Introduction

The aim of synchronization is to use a master system's output to induce a slave system so that the slave system's output could follow the master system's output asymptotically. Anti-synchronization means that the synchronized slave system's output has the same absolute values but opposite signs. Since the synchronization of chaotic systems was first proposed by Pecora and Carroll in 1990 [1], chaos synchronization has become one of the most interesting research subjects and many control techniques have been proposed for the synchronization and anti-synchronization of the chaotic systems. Active control method was used for the synchronization of chaotic Lorenz [2], Rössler [3], Chen [3], Chua [4], between Lorenz and Rössler [5], and many other identical and non-identical systems. The synchronization and anti-synchronization were applied with active control for chaotic Colpitts [6], extended Bonhöffer–van der Pol [7], and hyperchaotic Chen [8] systems. Active controllers were also constructed for chaos anti-synchronization between chaotic Lü and Rössler [9], and between hyperchaotic Lorenz and Liu [10] systems. Anti-synchronization between hyperchaotic Lorenz and Liu [10], hyperchaotic Lorenz and Chen [11], between two different hyperchaotic four-scroll [12], and a modified three-

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dimensional chaotic finance [13] systems were realized by means of adaptive control technique. Feedback controllers were proposed to anti-synchronize Rössler [14], between Rössler and Chen [14], Chua [15], and Liénard [16] chaotic systems. The anti-synchronization was achieved Lorenz [17], Tigan [17], between Tigan and Lorenz [17], between Genesio and Rössler [18], hyperchaotic Chen [19], hyperchaotic Lü [19], and between hyperchaotic Chen and Lü [19] systems on the basis of nonlinear control scheme. Sliding mode control was applied to Rikitake [20], hyperchaotic Lorenz [21], hyperchaotic Lü [22], and hyperchaotic Qi [23] systems. Anti-synchronization of chaotic systems were also presented with passive control [24], $H\infty$ control [25], and backstepping design [26] techniques.

Furthermore, the synchronization and anti-synchronization of chaotic systems implemented with artificial intelligence approaches. Artificial Neural Networks (ANNs) were used for the synchronization of chaotic Lorenz [27], Rössler [27, 28], unified [29], Genesio-Tesi [30], Duffing-Holmes [31] systems. The synchronization of chaotic Lorenz [32, 33], Rössler [32], Chen [32], Duffing-Holmes [33], Chua [34], and Rikitake [35], between Chen and Lü [36], between Chen and hyperchaotic Lorenz [36] systems were applied with fuzzy logic. Anti-synchronization between hyperchaotic Wu and hyperchaotic Lorenz systems [36], chaotic Lorenz [37], and hyperchaotic Lorenz [37] were achieved owing to the fuzzy logic controllers in recent years. ANFIS was used for the chaos synchronization only in a few papers [38, 39].

According to the literature review, the anti-synchronization of chaos has not been investigated with ANFIS based controllers. In this paper, the anti-synchronization of two identical Lorenz chaotic systems is applied by using an adaptive model reference ANFIS control technique.

The rest of this paper is organized as follows: In Section 2, the Lorenz chaotic system, and ANFIS are described briefly. Then, the proposed ANFIS model is constructed for chaos anti-synchronization in Section 3. Afterwards, ANFIS controllers assigned to Lorenz chaotic system and the simulation results are presented graphically to verify the anti-synchronization in Section 4. Finally, the paper is concluded in Section 5.

2 Materials and Methods

2.1 Lorenz Chaotic System

The Lorenz model is used for fluid conviction that describes some feature of the atmospheric dynamic. The differential equations of the Lorenz chaotic system is described by

$$\dot{x} = \sigma(y - x),$$
$$\dot{y} = -xz + rx - y, \quad (1)$$
$$\dot{z} = xy - \beta z,$$

where x, y, z are state variables that represent measures of fluid velocity, horizontal and vertical temperature variations, and σ, r, β are positive real constant parameters that represent the Prandtl number, Rayleigh number and geometric factor, respectively [40]. The Lorenz system is a chaotic attractor according to the parameters $\sigma = 10$, $r = 28$, and $\beta = 8 / 3$ [40]. The time series of the Lorenz chaotic system with the initial conditions $(x(0), y(0), z(0)) = (9, 15, 17)$ are shown in Fig. 1, the 2D phase portraits are shown in Fig. 2, and the 3D phase plane is shown in Fig. 3.

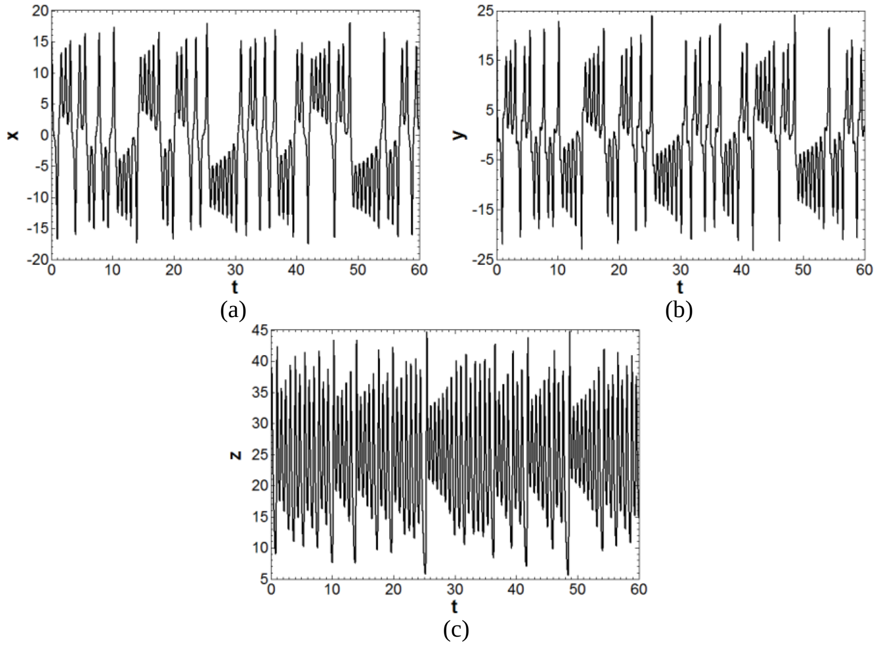

Fig. 1. Time series of Lorenz chaotic system for (a) x, (b) y, and (c) z signals.

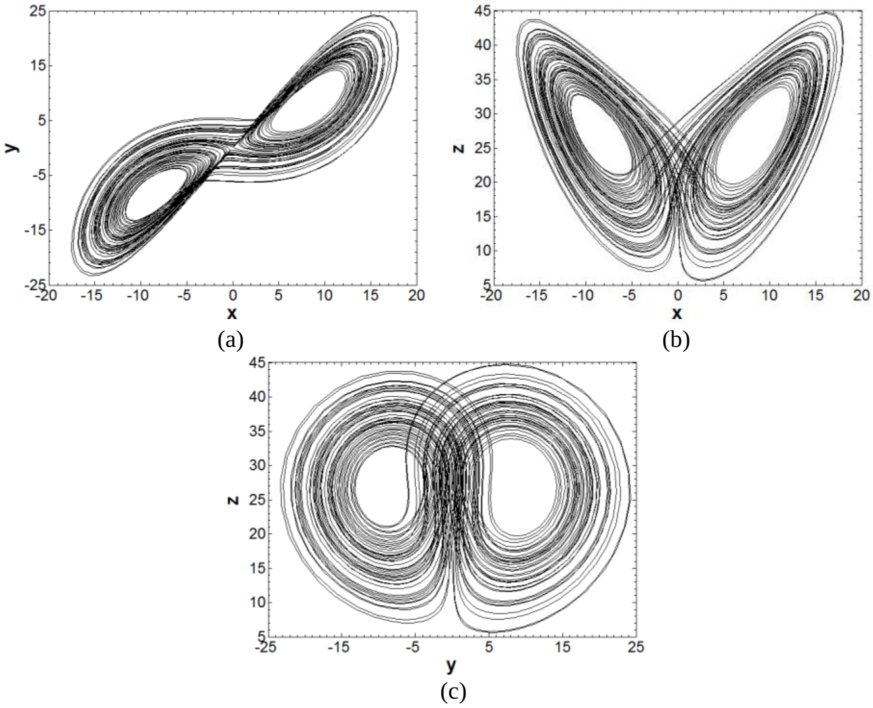

Fig. 2. Phase portraits of Lorenz chaotic system in (a) *x*–*y*, (b) *x*–*z*, and (c) *y*–*z* phase plot.

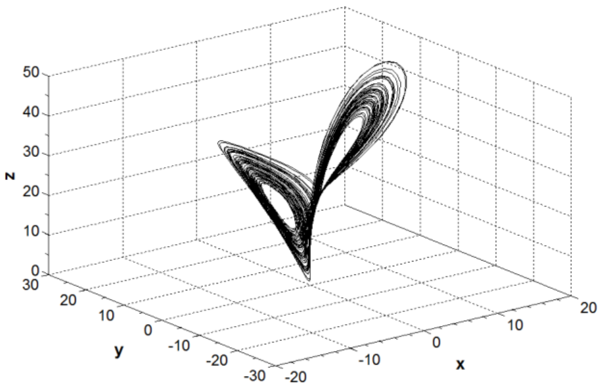

Fig. 3. *x*–*y*–*z* phase plane of Lorenz chaotic system.

2.2 Adaptive Neuro-Fuzzy Inference System (ANFIS)

Introduced by Jang in 1992 [41], ANFIS is a Sugeno fuzzy model where the final Fuzzy Inference System (FIS) is optimized with an ANN training. It is a universal intelligent computing methodology and it is capable of approximating any real continuous function on a compact set to any degree of accuracy [42].

ANNs, which are inspired from biological neural networks, have the ability of learning functional relations with limited amounts of training data. There are mainly two approaches for FISs, namely Mamdani [43] and Sugeno [44]. The differences between them arise from the outcome part where fuzzy membership functions are used in Mamdani's approach, while linear or constant functions are used in Sugeno's approach. Since ANFIS is based on the Sugeno type fuzzy model, it should be always interpretable in terms of function based fuzzy If-Then rules. Then, its parameter values are determined by a learning algorithm of ANN. Either a backpropagation method or a hybrid method which is a combination of least squares estimation with backpropagation can be utilized.

For better understanding of ANFIS, an example with two inputs x, y and one output f is given briefly. In an ANFIS model, the output of each rule is a linear combination of input variables by adding a constant term. For a first-order Sugeno fuzzy model, when it is assumed that each input has two membership functions, the fuzzy If-Then rules can be written as

$$
\begin{aligned}
&\text{Rule 1: If } x \text{ is } A_1 \text{ and } y \text{ is } B_1, \text{ then } f_1 = p_1x + q_1y + r_1,\\
&\text{Rule 2: If } x \text{ is } A_1 \text{ and } y \text{ is } B_2, \text{ then } f_2 = p_2x + q_2y + r_2,\\
&\text{Rule 3: If } x \text{ is } A_2 \text{ and } y \text{ is } B_1, \text{ then } f_3 = p_3x + q_3y + r_3,\\
&\text{Rule 4: If } x \text{ is } A_2 \text{ and } y \text{ is } B_2, \text{ then } f_4 = p_4x + q_4y + r_4,
\end{aligned}
\tag{2}
$$

where A_i and B_i are the membership functions for inputs x and y, respectively, and p_i, q_i, r_i are the parameters of output function with i = 1, 2, 3, … , n corresponding to Rule 1, Rule 2, Rule 3, … , Rule n. In ANFIS, the final output f is computed by the weighted average of each rule output as:

$$f = \sum_i \overline{w}_i (p_i x + q_i y + r_i) \tag{3}$$

where

$$\overline{w}_i = \frac{w_i}{\sum_i w_i}. \tag{4}$$

Fig. 4 shows the composed layers of an ANFIS structure: input fuzzification, fuzzy rules, normalization, defuzzification, and total output. In layer 1, the fuzzy membership functions are represented. Layer 3 calculates the firing strength of the signals received from layer 2 and forwards it to layer 4, which calculates an adaptive output for giving them as input to the layer 5, which computes the overall output [45]. More detailed information about ANFIS technique can be found in [41, 42, 45].

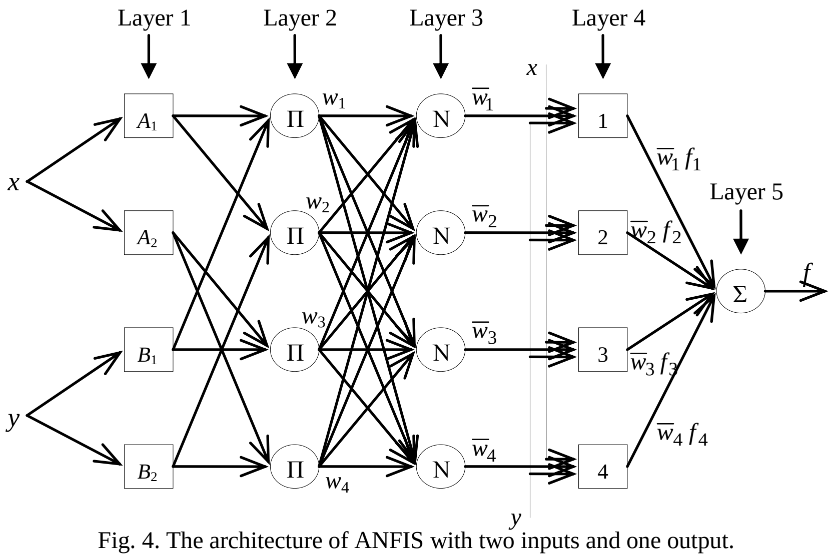

Fig. 4. The architecture of ANFIS with two inputs and one output.

3 Anti-Synchronization with ANFIS

In this paper, the model reference adaptive control technique with ANFIS is proposed for the anti-synchronization of chaotic systems. The goal of model reference adaptive control technique is to get a control system behaving like the reference model, which specifies the desired response of the system. The parameters of controllers are adjusted to minimize the error between the outputs of the model and the actual system. If the system has nonlinearity, intelligent algorithms such as ANFIS would rather be used in the model reference adaptive control due to getting better performance.

In the reference model, the master Lorenz system is described as

$$\begin{aligned}\dot{x}_1 &= \sigma(y_1 - x_1),\\ \dot{y}_1 &= -x_1 z_1 + r x_1 - y_1,\\ \dot{z}_1 &= x_1 y_1 - \beta z_1,\end{aligned} \tag{5}$$

and the slave Lorenz system is

$$\begin{aligned}\dot{x}_2 &= \sigma(y_2 - x_2) + u_1,\\ \dot{y}_2 &= -x_2 z_2 + r x_2 - y_2 + u_2,\\ \dot{z}_2 &= x_2 y_2 - \beta z_2 + u_3,\end{aligned} \tag{6}$$

where u_1, u_2, and u_3 are the nonlinear controllers. Sundarapandian obtained the controllers for anti-synchronization as

$$u_1 = \sigma e_2,$$
$$u_2 = -re_1 + x_2 z_2 + x_1 z_1, \quad (7)$$
$$u_3 = -x_2 y_2 - x_1 y_1,$$

where error dynamics are $e_1 = x_2 + x_1$, and $e_2 = y_2 + y_1$ [17].

The diagram of adaptive model reference ANFIS control technique for the anti-synchronization of chaotic systems is shown in Fig. 5. The adaptive model reference ANFIS controllers are trained to drive the slave system so that the differences between the anti-synchronization errors of ANFIS and the outputs of a reference model are minimized. If the ANFIS output error $e_c(t+1)$ is defined as $e_c(t+1) = y_s(t+1) + y_m(t+1)$, then the goal of the anti-synchronization is to determine the bounded input $u(t)$ as $\lim_{t\to\infty} e_c(t+1) \cong 0$. The parameters of ANFIS controllers are adjusted by backpropagating the differences between anti-synchronization errors of ANFIS and nonlinear control reference model if the distance of error $e_c(t+1)$ is greater than $e_r(t+1)$. This training path is shown as a dashed line in Fig. 5. Once the ANFIS controllers are trained successfully, they are ready to use for anti-synchronization and there is no need to the reference model anymore.

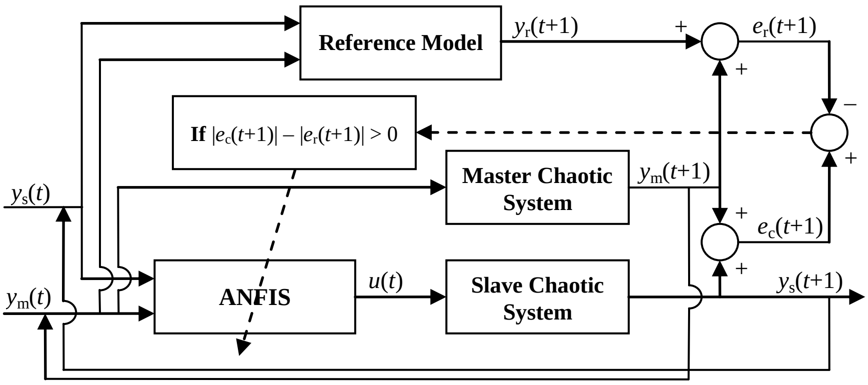

Fig. 5. The block diagram of adaptive model reference ANFIS control technique for chaos anti-synchronization.

The inputs of ANFIS include the state values of master and slave Lorenz chaotic systems. The output is the control signal. MATLAB is used for training the ANFIS controllers. This process is conducted with the command 'genfis1'. Triangular (trimf) type membership functions with the number of 3 are taken for all inputs. Therefore, the ANFIS controllers have 729 Sugeno type rules. The membership function of output variable is selected as linear type. The training process proceeded with the command 'anfis'. It identifies the parameters of Sugeno-type FISs. In the training stage, the hybrid learning rule, which is a combination of least-squared error and backpropagation gradient descent methods, with 5 epochs and zero error tolerance are preferred. Default values are used for all the other parameters. The training process finishes when the maximum epoch number is reached. In order to anti-synchronize successfully, the trained ANFIS controllers are employed to train again with a loop. After iterating the loop 10 times, the outputs of ANFIS controllers have 0.00025, 0.0007, and 0.00031 mean squared error as to x, y, and z states, respectively.

4 Simulation Results

In this section, numerical simulations are performed to show the anti-synchronization of two identical Lorenz chaotic systems having different initial conditions with adaptive model reference ANFIS control technique. The fourth-order Runge–Kutta method with variable time step is used in the numerical simulations. The above-mentioned parameter values of Lorenz system are considered to ensure the chaotic behaviour. The initial conditions are taken as $(x_1(0), y_1(0), z_1(0)) = (9, 15, 17)$ for the master system and $(x_2(0), y_2(0), z_2(0)) = (13, 8, 38)$ for the slave system. When the ANFIS controllers are activated at $t = 10$, the simulation results of anti-synchronization and error signals are demonstrated in Fig. 6 and Fig. 7, respectively.

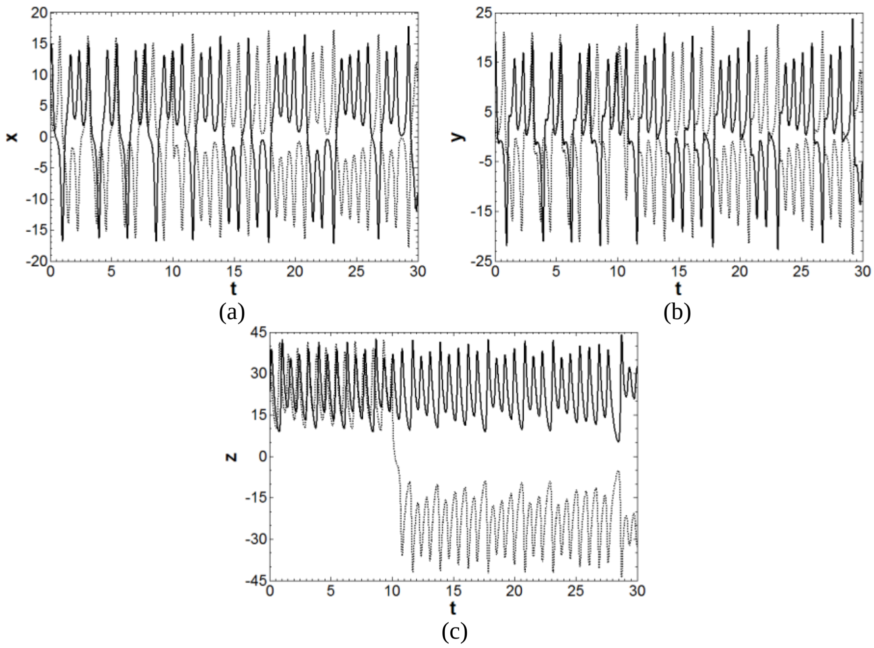

Fig. 6. Time responses of anti-synchronization of Lorenz chaotic systems when the ANFIS controllers are activated at $t = 10$ for (a) x, (b) y, and (c) z signals.

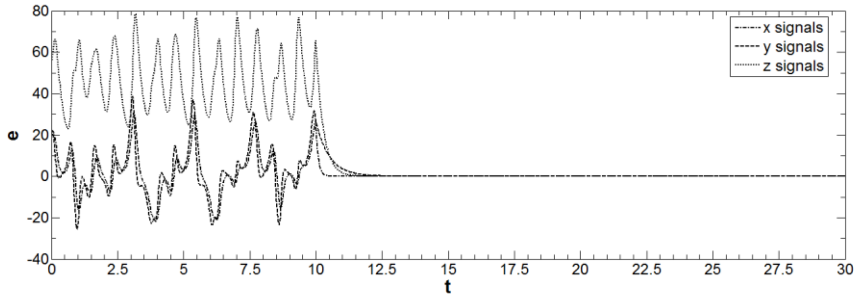

Fig. 7. The anti-synchronization error signals of Lorenz chaotic systems when the ANFIS controllers are activated at t = 10.

When the ANFIS controllers are activated at t = 20, the simulation results of anti-synchronization and error signals are demonstrated in Fig. 8 and Fig. 9, respectively.

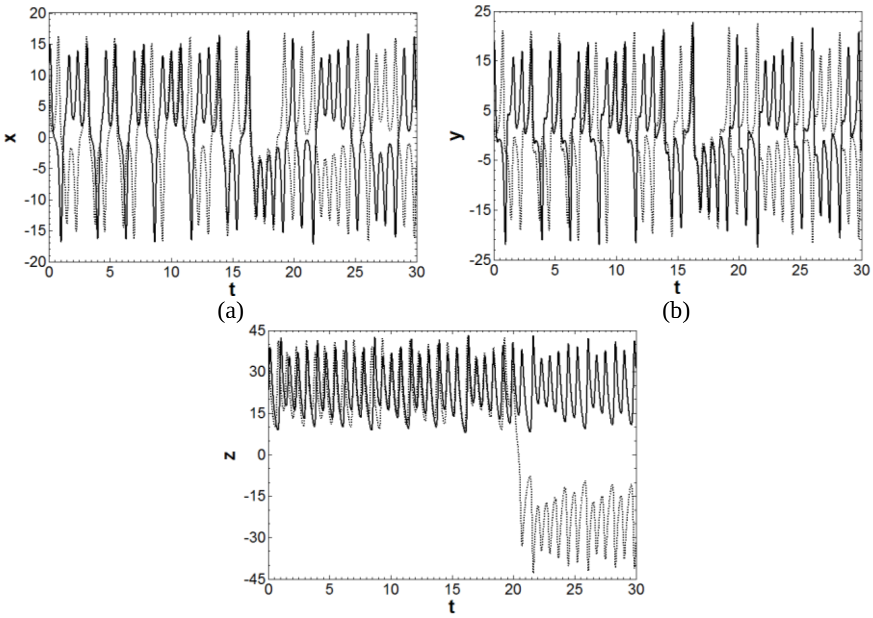

Fig. 8. Time responses of anti-synchronization of Lorenz chaotic systems when the ANFIS controllers are activated at t = 20 for (a) x, (b) y, and (c) z signals.

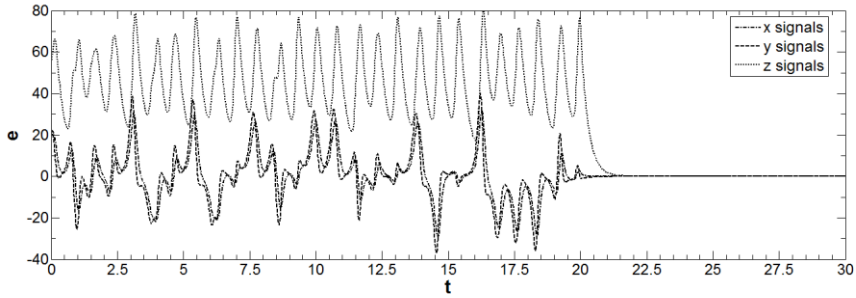

Fig. 9. The anti-synchronization error signals of Lorenz chaotic systems when the ANFIS controllers are activated at $t = 20$.

As expected, the anti-synchronization of two identical Lorenz chaotic systems starting from different initial conditions is achieved with the ANFIS controllers in Fig. 6 and Fig. 8. The anti-synchronization error signals that are shown in Fig. 7 and Fig. 9 converge asymptotically to zero. When the ANFIS controllers are activated at $t = 10$, the anti-synchronization is provided at $t \geq 12.5$. Also, the anti-synchronization is observed at $t \geq 21.5$, when the controllers are activated at $t = 20$. Hence, the computer simulations validate the effectiveness of proposed adaptive model reference ANFIS control technique.

5 Conclusions

In this paper, a novel approach to the anti-synchronization of a chaotic system is applied with an ANFIS technique. The ANFIS controllers are trained on the bases of adaptive model reference control technique. Famous Lorenz chaotic system is preferred for simulations and a nonlinear control method is considered as the reference system. Numerical simulations show that ANFIS controllers achieve the anti-synchronization of two identical Lorenz chaotic systems in an proper time period. As a future work, non-identical chaos anti-synchronization may be investigated with ANFIS.

References

1. L.M. Pecora and T.L. Carroll, Synchronization in chaotic systems. Physical Review Letters, 64(8), 821–824, 1990.
2. E.W. Bai and K.E. Lonngren. Synchronization of two Lorenz systems using active control. Chaos, Solitons & Fractals, 8(1), 51–58, 1997.
3. H.N. Agiza and M.T. Yassen. Synchronization of Rossler and Chen chaotic dynamical systems using active control. Physics Letters A, 278(4), 191–197, 2001.
4. G.H. Li. An active control synchronization for two modified Chua circuits. Chinese Physics, 14(3), 472–475, 2005.

5. M.C. Ho and Y.C. Hung. Synchronization of two different systems by using generalized active control. Physics Letters A, 301(5-6), 424–428, 2002.
6. G.H. Li. Synchronization and anti-synchronization of Colpitts oscillators using active control. Chaos, Solitons & Fractals, 26(1), 87–93, 2005.
7. A.N. Njah and U.E. Vincent. Synchronization and anti-synchronization of chaos in an extended Bonhöffer–van der Pol oscillator using active control. Journal of Sound and Vibration, 319(1-2), 41–49, 2009.
8. M.M. El-Dessoky. Synchronization and anti-synchronization of a hyperchaotic Chen system. Chaos, Solitons & Fractals, 39(4), 1790–1797, 2009.
9. A.A. Emadzadeh and M. Haeri. Anti-synchronization of two different chaotic systems via active control. Transactions on Engineering, Computing and Technology, 6, 62–65, 2005.
10. Q. Zheng, X.P. Zhang and Z.Z. Ren. Anti-synchronization between Lorenz and Liu hyperchaotic systems. Communications in Theoretical Physics., 50(3), 677–680, 2008.
11. V. Sundarapandian and R. Karthikeyan. Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control. International Journal of System Signal Control and Engineering Applications, 4(2), 18–25, 2011.
12. Z.W. Sun. Anti-synchronization of two novel four-scroll hyperchaotic systems with unknown parameters via adaptive control. Advances in Computer, Communication, Control & Automation, Lecture Notes in Electrical Engineering, 121, 377–384, 2012.
13. A. Jabbari and H. Kheiri. Anti-synchronization of a modified three-dimensional chaotic finance system with uncertain parameters via adaptive control. International Journal of Nonlinear Science, 14(2), 178–185, 2012.
14. S. Hammami, M. Benrejeb, M. Feki and P. Borne. Feedback control design for Rössler and Chen chaotic systems anti-synchronization. Physics Letters A, 374(28), 2835–2840, 2010.
15. Y. Chen, M.Y. Li and Z.F. Cheng. Global anti-synchronization of master-slave chaotic modified Chua's circuits coupled by linear feedback control. Mathematical and Computer Modelling, 52(3-4), 567–573, 2010.
16. X. Jian. Anti-synchronization of Liénard chaotic systems via feedback control. International Journal of the Physical Sciences, 5(18), 2757–2761, 2010.
17. V. Sundarapandian. Global chaos anti-synchronization of Tigan and Lorenz systems by nonlinear control. International Journal of Mathematical Sciences and Applications, 1(2), 679–690, 2011.
18. W.L. Li, X.Q. Chen and S. Zhiping. Anti-synchronization of two different chaotic systems. Physica A, 387(14), 3747–3750, 2008.
19. M.M. Al-Sawalha and M.S.M. Noorani. Anti-synchronization of two hyperchaotic systems via nonlinear control. Communications in Nonlinear Science and Numerical Simulation, 14(8), 3402–3411, 2009.
20. X. Jian. Anti-synchronization of uncertain Rikitake systems via active sliding mode control. International Journal of the Physical Sciences, 6(10), 2478–2482, 2011.
21. V. Sundarapandian and S. Sivaperumal. Anti-synchronization of the hyperchaotic Lorenz systems by sliding mode control. International Journal on Computer Science and Engineering, 3(6), 2450–2457, 2011.
22. S. Singh and B.B. Sharma. Sliding mode control based anti-synchronization scheme for hyperchaotic Lu systems. International Conference on Communication Systems and Network Technologies, IEEE, 382–386, 2011.
23. V. Sundarapandian and S. Sivaperumal. Anti-synchronization of hyperchaotic Qi systems by sliding mode control. International Journal of Engineering Science and Technology, 3(7), 5733–5739, 2011.

24. X.B. Zhou, B. Kong and H.Y. Ding. Synchronization and anti-synchronization of a new hyperchaotic Lü system with uncertain parameters via the passive control technique. Physica Scripta, 85(6), Article Number: 065004, 2012.
25. C.K. Ahn. An $H\infty$ approach to anti-synchronization for chaotic systems. Physics Letters A, 373(20), 1729–1733, 2009.
26. K.S. Ojo, A.N. Njah and G.A. Adebayo. Antisynchronization of identical and nonidentical Φ^6 van der Pol and Φ^6 Duffing oscillators with both parametric and external excitations via backstepping approach. International Journal of Modern Physics B, 25(14), 1957–1969, 2011.
27. M. Chen, C.S. Jiang, B. Jiang and Q.X. Wu. Sliding mode synchronization controller design with neural network for uncertain chaotic systems. Chaos, Solitons & Fractals, 39(4), 1856–1863, 2009.
28. H.K. Lam and L.D. Seneviratne. Synchronization of chaotic systems using neural-network-based controller. International Journal of Bifurcation and Chaos, 17(6), 2117–2125, 2007.
29. C.J. Cheng and J.R. Tsai. Adaptive control and synchronization of uncertain unified chaotic system by cellular neural networks, International Symposium on Computer, Communication, Control and Automation, IEEE, Tainan, Republic of China, vol. 2, 274–277, 2010.
30. C.K. Ahn. Neural network $H\infty$ chaos synchronization. Nonlinear Dynamics, 60(3), 295–302, 2010.
31. S. Kuntanapreeda. Synchronization of Duffing-Holmes oscillators using stable neural network controller. 2nd International Conference on Computational Collective Intelligence: Technologies and Applications, ICCCI 2010, Lecture Notes in Computer Science, vol. 6423, 242–251, 2010.
32. H.K. Lam, W.K. Ling, H.H.C. Iu and S.S.H. Ling. Synchronization of chaotic systems using time-delayed fuzzy state-feedback controller. IEEE Transactions on Circuits and Systems-I: Regular Papers, 55(3), 893–903, 2008.
33. H.T. Yau and C.S. Shieh. Chaos synchronization using fuzzy logic controller. Nonlinear Analysis: Real World Applications, 9(4), 1800–1810, 2008.
34. M. Ababneh, I. Etier, M. Smadi and J. Ghaeb. Synchronization of chaos systems using fuzzy logic. Journal of Computer Science, 7(2), 197–205, 2011.
35. V. Vembarasan and P. Balasubramaniam. Chaotic synchronization of Rikitake system based on T-S fuzzy control techniques. Nonlinear Dynamics, 74(1-2), 31–44, 2013.
36. D.Y. Chen, W.L. Zhao, J.C. Sprott and X.Y. Ma. Application of Takagi–Sugeno fuzzy model to a class of chaotic synchronization and anti-synchronization. Nonlinear Dynamics, 73(3), 1495–1505, 2013.
37. R.F. Zhang, D.Y. Chen, Y.H. Do and X.Y. Ma. Synchronization and anti-synchronization of fractional dynamical networks. Journal of Vibration and Control. Doi: 10.1177/1077546314522506.
38. C.S. Chen. Quadratic optimal neural fuzzy control for synchronization of uncertain chaotic systems. Expert Systems with Applications, 36(9), 11827–11835, 2009.
39. C.S. Chen and H.H. Chen. Robust adaptive neural-fuzzy-network control for the synchronization of uncertain chaotic systems. Nonlinear Analysis: Real World Applications, 10(3), 1466–1479, 2009.
40. E.N. Lorenz. Deterministic nonperiodic flow. Journal of Atmospheric Sciences, 20(2), 130-141, 1963.
41. J.S.R. Jang. Neuro-fuzzy modeling: architecture, analyses and applications. Ph.D. Thesis, Department of Electrical Engineering and Computer Science, University of California, Berkeley, USA, 1992.

42. J.S.R. Jang, C.T. Sun and E. Mizutani. Neuro fuzzy and soft computing: a computational approach to learning and machine intelligence. Prentice-Hall, New Jersey, 1997.
43. E.H. Mamdani and S. Assilian. An experiment in linguistic synthesis with a fuzzy logic controller. International Journal of Man Machine Studies, 7(1), 1–13, 1992.
44. T. Takagi and M. Sugeno. Fuzzy identification of systems and its application to modeling and control. IEEE Transactions on Systems, Man and Cybernetics, 15(1), 116–132, 1985.
45. H. Chaudhary, V. Panwar, R. Prasad and N. Sukavanam. Adaptive neuro fuzzy based hybrid force/position control for an industrial robot manipulator. Journal of Intelligent Manufacturing, 2014, doi:10.1007/s10845-014-0952-1.