

Control of Chaotic Finance System using Artificial Neural Networks

Uğur Erkin Kocamaz¹, Yılmaz Uyaroğlu², Gültekin Çağıl³, Harun Taşkın³, and Zeynep Çağıl³

¹ Department of Computer Technologies, Vocational School of Karacabey, Uludağ University, 16700 Karacabey, Bursa, Turkey
E-mail: ugurkocamaz@gmail.com

² Department of Electrical & Electronics Engineering, Faculty of Engineering, Sakarya University, 54187 Serdivan, Sakarya, Turkey
E-mail: uyaroglu@sakarya.edu.tr

³ Department of Industrial Engineering, Faculty of Engineering, Sakarya University, 54187 Serdivan, Sakarya, Turkey
E-mails: cagil@sakarya.edu.tr, taskin@sakarya.edu.tr, zcagil@sakarya.edu.tr

Abstract. In this paper, the control of a chaotic finance system is applied by using Artificial Neural Networks (ANNs). Economic systems become more complicated and have undesired nonlinear factors. It is difficult to control when a chaotic behaviour occurs. So that the ANNs have the ability of learning functional relations, they can achieve the control of chaotic systems more effectively. On-line neural training algorithms are used for regulating the chaotic finance system to its equilibrium points in the state space. For faster training in back-propagation, Levenberg-Marquardt algorithm is preferred. Numerical simulations are performed to demonstrate the effectiveness of the proposed control technique.

Keywords: Chaotic Finance System, Chaos Control, Neural Control, Artificial Neural Networks, Neural Networks.

1 Introduction

Nowadays, the financial systems are being more complicated and the markets are rising rapidly in an asymmetrical economic growth. The economic progresses are resulting in nonlinearity which leads difficult to control. Upon having some unpredictable nonlinear factors in interest rate, investment demand, price, per investment cost and stock, the financial systems can reveal chaotic behaviour. Nonlinearity and chaos in a financial system are undesired characteristics and traditional econometric approaches, strict assumptions, statistics based methods may be inadequate for a stable economic growth and control. Furthermore, the control of chaos in a financial system has significant importance from the management point of view of avoiding undesirable situations such as economical crises.



A nonlinear system under chaotic behaviour might have undesired trajectories; therefore it is required to control for eliminating chaos. Since the successful study of Ott, Grebogi, and Yorke named OGY control [1], various methods for the control and synchronization of chaotic systems have been presented. These approaches mainly include linear feedback [2], nonlinear feedback [3], time-delayed feedback [4], adaptive [5], sliding mode [6], impulsive [7], passive [8] control methods. In the most of these control methods, it is assumed that the dynamical model of the chaotic system is known. However, some of the chaotic system models do not exactly represent the real situation, the parameters may be unknown and many chaotic systems do not have any mathematical equations. Intelligent control techniques generally attempt to control the chaos by using the output values of states, so they can be more comprehensive solution. Recently, the control of Lorenz [9, 10], Rossler [11], Chen [12], Lü [13], unified [14], and unknown [15] chaotic systems have been implemented with Artificial Neural Networks (ANNs). Fuzzy logic, the other popular intelligent technique, is used in the control of Lorenz [16, 17], Chua [17, 18], Rossler [18], Chen [18, 19], unified [20], Mathieu–van der Pol [21] and some other chaotic systems. With the Adaptive Neuro-Fuzzy Inference Systems (ANFIS), which is a combination of ANN and fuzzy logic systems, there are only a few papers for the control of chaotic systems [22–25].

The first chaotic finance system has been introduced in 2001 [26, 27]. Then, a new chaotic finance attractor has been built in 2007 [28]. Afterwards, two different hyperchaotic finance systems have been presented respectively in 2009 and 2012 [29, 30]. Some papers have been published concerning the dynamic behaviours of these chaotic finance systems [30–33]. The synchronization of the chaotic finance systems have been implemented with active [34, 35], nonlinear feedback [36, 37], adaptive [38], sliding mode [39], and passive [39] control methods. For the control of the chaotic finance systems, several control methods have been proposed [29, 30, 40–45]. Yang and Cai have achieved the control of chaotic finance system via linear feedback, speed feedback, selection of gain matrix, and revision of gain matrix controllers in 2001 [40]. Chen has employed the time-delayed feedbacks to provide the control of this system in 2008 [41]. Emiroglu et al. have used a passive controller for controlling this nonlinear system in 2012 [42]. Cai et al. have constructed the control of modified chaotic finance system by means of linear feedback, speed feedback and adaptive control methods in 2011 [43]. The control of the former hyperchaotic finance system has been performed with the effective speed feedback control method by Ding et al. in 2009 [29], with the linear feedback control method by Uyaroglu et al. in 2012 [44], and with the time-delayed feedback control methods by Gelber et al. in 2012 [45]. Yu et al. have realized the control of the latter hyperchaotic finance system with the linear feedback and effective speed feedback control methods in 2012 [30].

Motivated by the previous intelligent chaos control papers, in this study, further investigations on the control of chaotic finance systems are explored. Although

there are several studies in financial chaos control, as to the knowledge of the authors there is no intelligent approach for controlling the chaotic finance systems. In this study, neural controllers are employed for achieving the control of a chaotic finance system. The effectiveness of the proposed ANN control technique has been presented visually by using simulation results.

2 Materials and Methods

2.1 Chaotic Finance System

The chaotic finance system is described by a set of three first-order differential equations as

$$\begin{aligned} \dot{x} &= z + (y - a)x, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \end{aligned} \tag{1}$$

where x, y, z are state variables and a, b, c are positive constant parameters, they represent the interest rate, investment demand, price exponent, saving amount, per investment cost, and elasticity of demands of commercials, respectively [26, 27]. It exhibits chaotic behaviour when the parameter values are chosen as $a = 3, b = 0.1,$ and $c = 1$ [41]. The time series, 2D phase portraits and 3D phase plane of the chaotic finance system under these parameter values and the initial conditions $x(0) = 1.5, y(0) = 4.5,$ and $z(0) = -0.5$ are illustrated in Fig. 1, Fig. 2, and Fig. 3, respectively.

The equilibria of chaotic finance system (1) can be found by solving the following equation:

$$\begin{aligned} z + (y - a)x &= 0, \\ 1 - by - x^2 &= 0, \\ -x - cz &= 0. \end{aligned} \tag{2}$$

Then, the chaotic finance system possesses three equilibrium points;

$$\begin{aligned} E_1 &(0, 1/b, 0), \\ E_2 &(-\sqrt{-c(b - c + abc)}/c, (1 + ac)/c, \sqrt{-c(b - c + abc)}/c^2), \\ E_3 &(\sqrt{-c(b - c + abc)}/c, (1 + ac)/c, -\sqrt{-c(b - c + abc)}/c^2). \end{aligned} \tag{3}$$

When the parameter values are taken as $a = 3, b = 0.1,$ and $c = -0.5,$ the equilibrium points become $E_1(0, 10, 0), E_2(-0.7746, 4, 0.7746),$ and $E_3(0.7746, 4, -0.7746).$

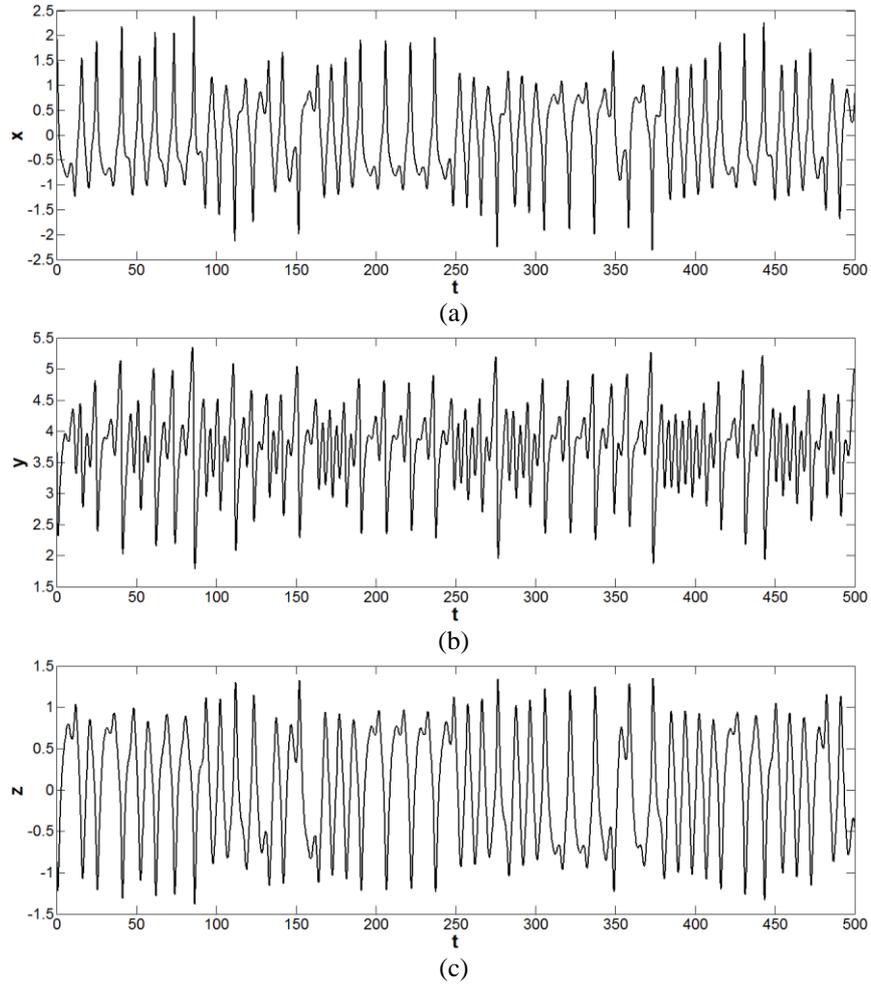


Fig. 1. Time series of the chaotic finance system for (a) x signals, (b) y signals, and (c) z signals

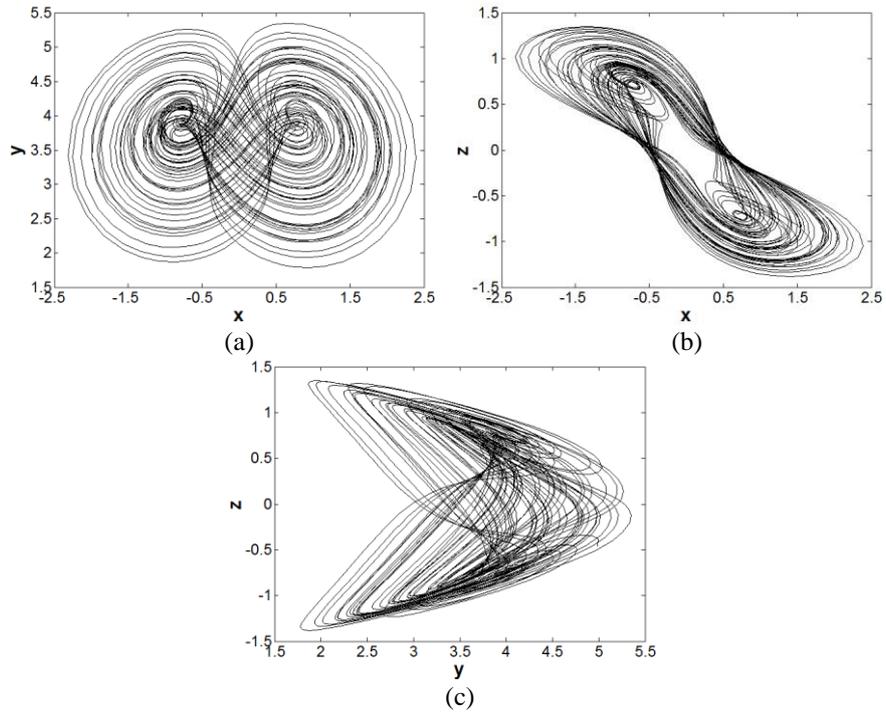


Fig. 2. 2D phase portraits of the chaotic finance system in (a) x - y phase plot, (b) x - z phase plot, and (c) y - z phase plot

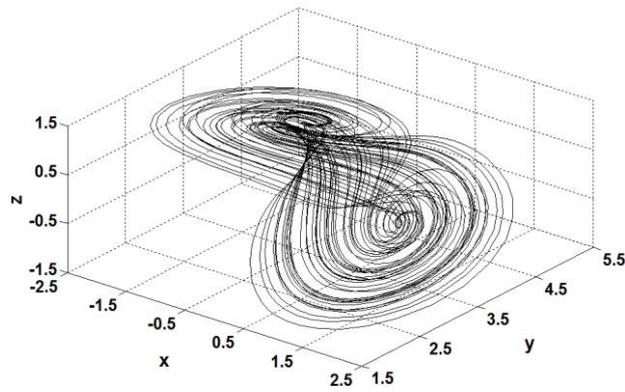


Fig. 3. 3D phase plane of the chaotic finance system

2.2 Artificial Neural Networks (ANNs)

ANNs, which are inspired from biological neural networks, are basically a parallel computing technique. An ANN consists of processing elements called neurons and connections between them with coefficients called weights. Each processing element makes its computation based upon a weighted sum of its inputs and an activation function is also used for determining the output value. ANNs adapt themselves to the given inputs and desired outputs with a learning algorithm, and then they respond to the unknown situations rationally. If using only input layer and output layer is not sufficient, increasing the number of layers called hidden layers can solve the learning problem. There are different kinds of ANNs, the most commonly preferred one is the three-layered Feed-Forward Neural Network (FFNN). As shown in Fig. 4, elementary FFNNs have three layers of neurons: input layer, hidden layer and output layer.

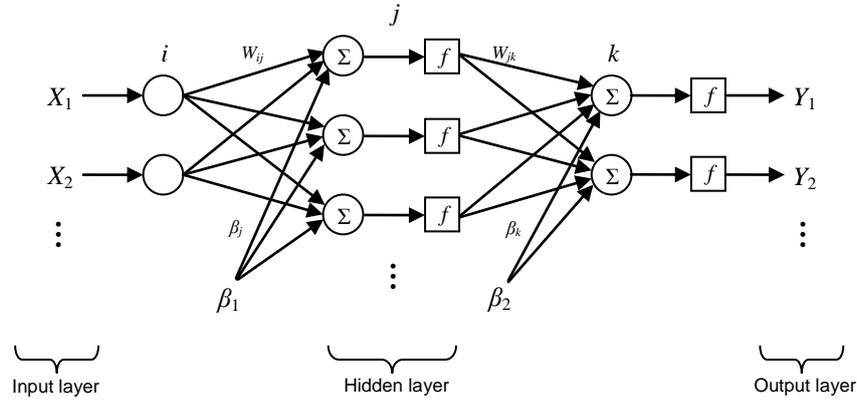


Fig. 4. Basic architecture of feed-forward neural networks

In Fig. 4, $X(i)$ and $Y(k)$ are the input-output data pairs, β_1 and β_2 are the bias values, w is the interconnection weight, and f is the activation function. i , j , and k represent the number of inputs, neurons in the hidden layer, and outputs, respectively. The values of each neuron in the hidden layer of FFNN can be calculated by

$$H(j) = f\left(\sum_{i=1}^n X(i)w(i, j) + \beta_1(j)\right), \quad (4)$$

and the output layer of FFNN can be found as

$$Y(k) = f\left(\sum_{j=1}^l H(j)w(j, k) + \beta_2(k)\right). \quad (5)$$

Sigmoid and tangent sigmoid functions are the commonly used activation functions in FFNNs. While the sigmoid function produces only positive numbers between 0 and 1, the tangent sigmoid function produces numbers between -1 and 1. The formula of sigmoid function is given by

$$f(x) = \frac{1}{1 + e^{-x}}, \quad (6)$$

and the tangent sigmoid function can be denoted as

$$f(x) = \frac{\sinh x}{\cosh x} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{2}{1 + e^{-2x}} - 1, \quad (7)$$

where e is the base of natural logarithm.

FFNNs must be trained for adapting themselves to the given inputs and desired outputs. Back-propagation can be used as a teaching method for FFNNs. It is also known as gradient descent training algorithm. But, it is often too slow. Therefore, several high-performance training algorithms such as scaled conjugate gradient; Fletcher–Reeves conjugate gradient; Powell–Beale restarts conjugate gradient; resilient back-propagation; Broyden, Fletcher, Goldfarb, Shanno quasi-Newton; one step secant and Levenberg–Marquardt algorithms are preferred in recent years.

3 Controlling Chaotic Finance System with ANNs

In this study, a direct ANN control technique is proposed to control the chaotic finance system. The direct control technique is widely used for controller design with intelligent methodologies. Its goal is to get a control system with backpropagating the errors. The parameters of controllers are adjusted to minimize the error between the plant's output and desired output. If the system includes nonlinearity, then linear controllers may not produce performance satisfactorily. In such cases, artificial intelligence techniques such as ANNs can be used in a direct control system. Its fast response time, general approximation, and learning abilities make the ANN an attractive method for nonlinear control. Fig. 5 shows the diagram of proposed direct ANN control technique for the control of chaotic finance system.

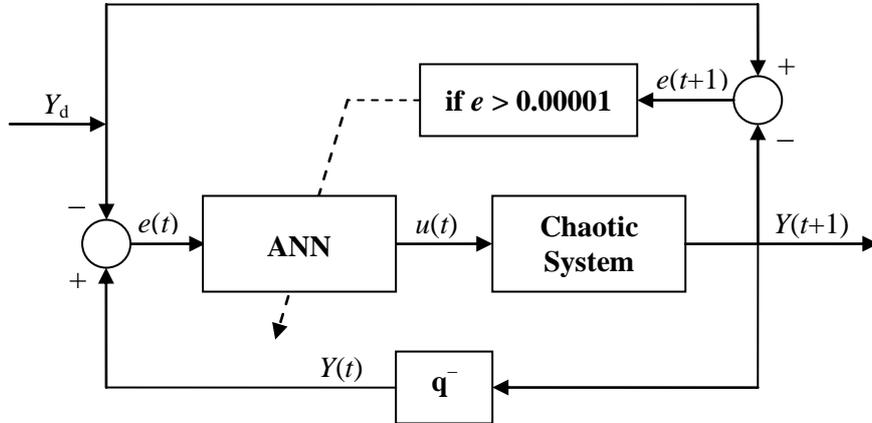


Fig. 5. The model of direct ANN control method for chaos control

In Fig. 5, Y_d is the desired process output (an equilibrium point of the chaotic system), $Y(t+1)$ is the actual process output, $e(t)$ equals to $Y(t) - Y_d$ is the input of the ANN, and $u(t)$ is the output of the ANN. The network's output error $e(t+1)$ is defined as $Y_d - Y(t+1)$. The goal of the control is to determine the bounded input $u(t)$ as $\lim_{t \rightarrow \infty} e(t+1) \cong 0$. The ANN controllers are trained to control the chaotic system by backpropagating the errors so that the differences between desired output and actual output are minimized. This training path is shown as a dashed line in Fig. 5. For each of the x , y , and z states, there exists different ANN controllers in the proposed control model. 27 neurons are assigned in the hidden layer of ANNs. The weights of ANNs are adjusted online without a specific pre-training stage. If the error between desired output and actual output is too small, the control is achieved at that moment and no need to train the ANN controllers for this situation. Instead of using back-propagation (gradient descent) as a training method, the Levenberg–Marquardt algorithm is preferred because of learning relatively very fast. The inputs and outputs of ANNs are normalized to values between -1 and 1. Tangent sigmoid function is taken as the activation function because it produces numbers between -1 and 1. The training parameters of ANN are considered as $epochs = 3$, $goal = 10^{-10}$ and $min_grad = 10^{-10}$. Default values are used for all the other parameters. In order to control successfully, the ANN controllers are employed to train again for the new $Y(t+1)$ situations by adjusting the weights of ANNs simultaneously.

4 Numerical Simulations

This section of the paper demonstrates the control results of chaotic finance system to verify the effectiveness of proposed ANN control technique. The simulation results are performed using the Matlab software. The numerical analyses are carried out using fourth-order Runge–Kutta method with variable time step. The same parameter values and initial conditions of finance system

described in Section 2 are taken to ensure the chaotic behaviour. When the ANN controllers are activated at $t = 50$, the simulation results for the control of chaotic finance system to $E_1(0, 10, 0)$, $E_2(-0.7746, 4, 0.7746)$, and $E_3(0.7746, 4, -0.7746)$ equilibrium points are shown in Fig. 6, Fig. 7, and Fig. 8, respectively.

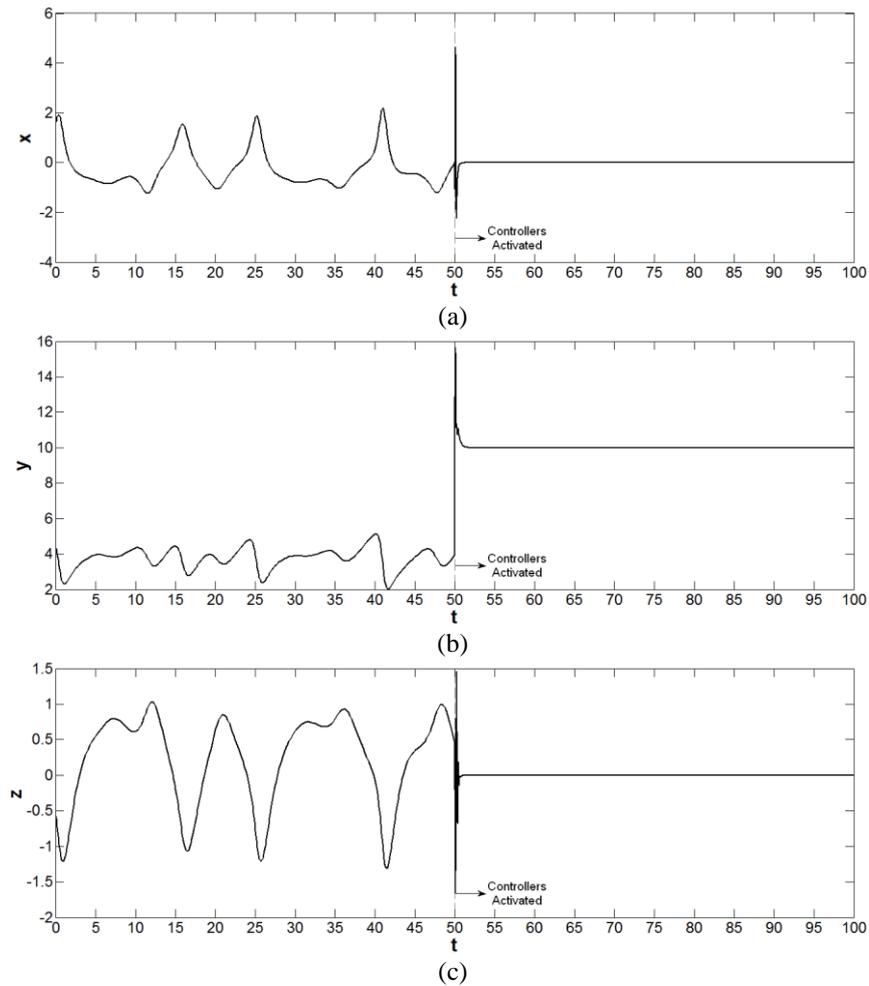


Fig. 6. Time responses of controlled chaotic finance system to $E_1(0, 10, 0)$ with the ANN controllers are activated at $t = 50$ for (a) x signals, (b) y signals, and (c) z signals

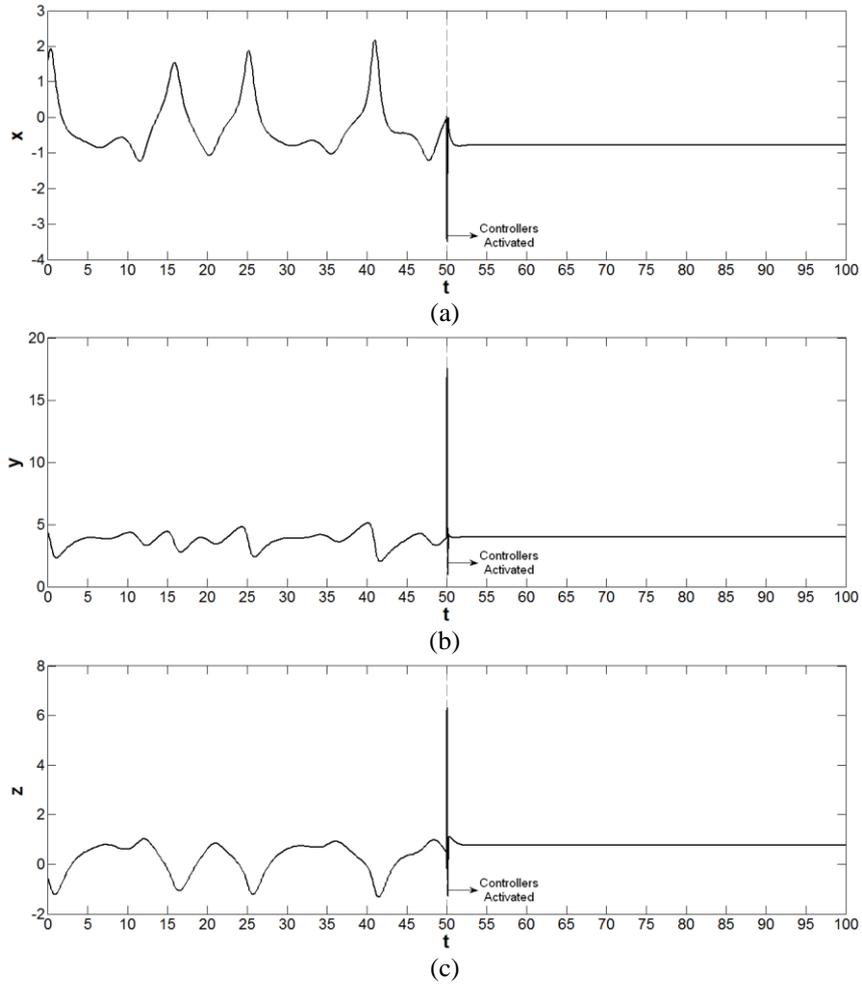


Fig. 7. Time responses of controlled chaotic finance system to $E_2(-0.7746, 4, 0.7746)$ with the ANN controllers are activated at $t = 50$ for (a) x signals, (b) y signals, and (c) z signals

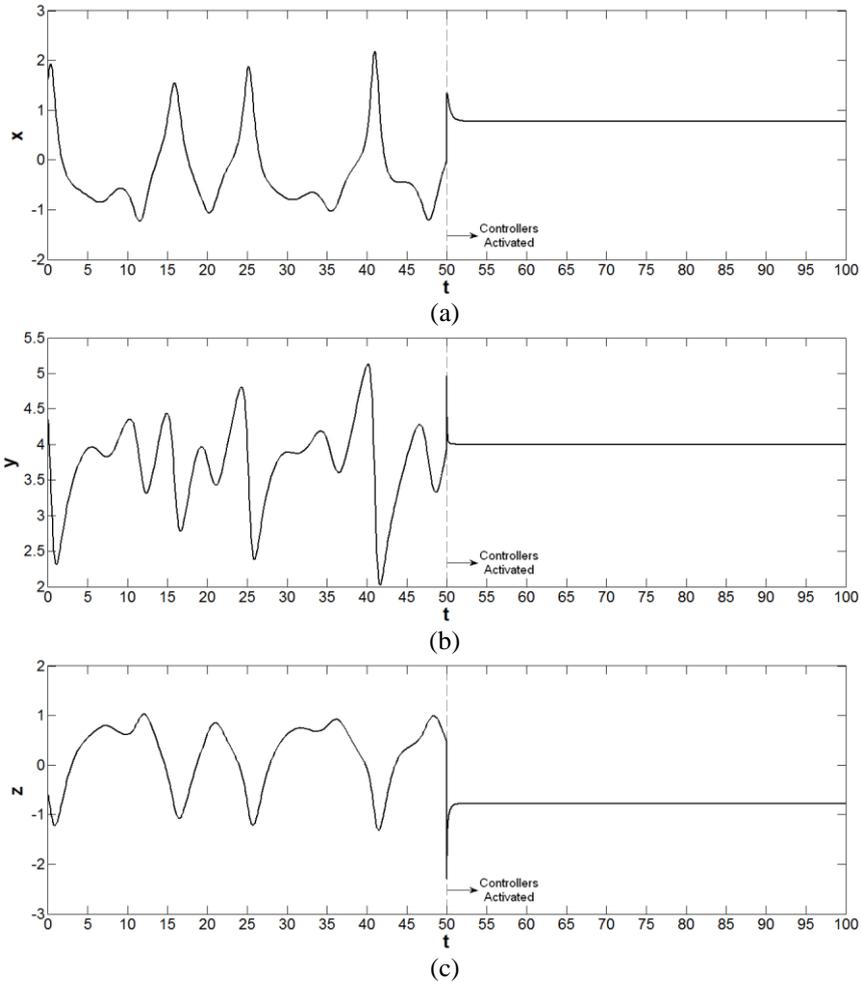


Fig. 8. Time responses of controlled chaotic finance system to $E_3(0.7746, 4, -0.7746)$ with the ANN controllers are activated at $t = 50$ for (a) x signals, (b) y signals, and (c) z signals

As expected, the Figs. 6–8 show that the proposed ANN controllers have stabilized the chaotic motion of the finance system towards its equilibrium points. When the controllers are activated at $t = 50$, the control is observed at $t \geq 52$ for all equilibrium points. The errors between desired output and actual output signals converge to zero with an appropriate time period. Hence, the simulation results verify the effectiveness of proposed online direct ANN control technique.

5 Conclusions

Although several papers have concerned on the control of chaotic finance system, it is the first time its control is investigated with an artificial intelligence methodology in this study. A direct ANN control technique is proposed to achieve the control. The weights of ANN controllers are adjusted online without a specific pre-training stage. Levenberg–Marquardt algorithm is preferred for faster training. The simulation results in Figs. 6–8 have shown that the chaotic finance system is stabilized towards its equilibrium points effectively owing to the ANN controllers. The proposed method differs from the previous finance chaos control techniques in that it is feasible even if the equations of the finance system is unknown. As a future work, the other intelligent techniques such as fuzzy logic, ANFIS and genetic algorithm can be applied for the control of chaotic finance system.

References

1. E. Ott, C. Grebogi and J.A. Yorke. Controlling chaos. *Physical Review Letters*, 64(11), 1196–1199, 1990.
2. X.Y. Wang and X.G. Li. Feedback control of the Liu chaotic dynamical system. *International Journal of Modern Physics B*, 24 (3), 397–404, 2010.
3. W.N. Zhou, L. Pan, Z. Li and W.A. Halang. Non-linear feedback control of a novel chaotic system. *International Journal of Control, Automation and Systems*, 7 (6), 939–944, 2009.
4. K. Pyragas. Continuous control of chaos by self-controlling feed back. *Physics Letters A*, 170 (6), 421–428, 1992.
5. A. Hubler. Adaptive control of chaotic systems. *Helvetica Physica Acta*, 62, 343–346, 1989.
6. H.T. Yau and J.J. Yan. Design of sliding mode controller for Lorenz chaotic system with nonlinear input. *Chaos, Solitons and Fractals*, 19 (4), 891–898, 2004.
7. X.Q. Wu, J.A. Lu, C.K. Tse, J.J. Wang and J. Liu. Impulsive control and synchronization of the Lorenz systems family. *Chaos, Solitons and Fractals*, 31 (3), 631–638, 2007.
8. W. Yu. Passive equivalence of chaos in Lorenz system. *IEEE Transaction on Circuits and Systems-I: Fundamental Theory and Applications*, 46 (7), 876–878, 1999.
9. H.J. Guo, S.F. Lin and J.H. Liu. A radial basis function sliding mode controller for chaotic Lorenz system. *Physics Letters A*, 351 (4-5), 257–261, 2006.
10. L.Q. Shen and M. Wang. Adaptive control of chaotic systems based on a single layer neural network. *Physics Letters A*, 368 (5), 379–382, 2007.
11. L.X. Yang, Z.R. Zhang and J.G. Zhang. Using a neural network controller to control chaos in the Rossler system. *Int. Conf. on Artificial Intelligence and Computational Intelligence*, IEEE, Shanghai, China, pp. 50–53, Nov. 7–8 2009.
12. Z. Lu, L.S. Shieh, G. Chen and N.P. Coleman. Adaptive feedback linearization control of chaotic systems via recurrent high-order neural networks. *Information Sciences*, 176, 2337–2354, 2006.
13. X. Yan and D. Liu. Chaos control of fractional order Lü system via RBF neural network sliding mode controller. *Proc. of the 8th World Congress on Intelligent Control and Automation*, Jinan, China, IEEE, pp. 5099–5104, July 6–9 2010.

14. C.J. Cheng and J.R. Tsai. Adaptive control and synchronization of uncertain unified chaotic system by cellular neural networks. *Int. Symp. on Computer, Communication, Control and Automation (3CA)*, IEEE, Tainan, Taiwan, vol. 2, pp. 274–277, May 5–7 2010.
15. A. Boukabou and N. Mansouri. Neural predictive control of unknown chaotic systems. *Nonlinear Analysis: Modelling and Control*, 10 (2), 95–106, 2005.
16. L. Wu and C. Wang. Adaptive fuzzy sliding mode control of Lorenz chaotic system. *Frontiers of Electrical and Electronic Engineering in China*, 2 (3), 277–281, 2007.
17. A.M. Harb and I.A. Smadi. On fuzzy control of chaotic systems. *Journal of Vibration and Control*, 10 (7), 979–993, 2004.
18. Y.G. Zheng and G.R. Chen. Fuzzy impulsive control of chaotic systems based on TS fuzzy model. *Chaos, Solitons and Fractals*, 39 (4), 2002–2011, 2009.
19. W. Chang, J.B. Park, Y.H. Joo and G.R. Chen. Static output-feedback fuzzy controller for Chen's chaotic system with uncertainties. *Information Sciences*, 151, 227–244, 2003.
20. B. Chen, X.P. Liu and S.C. Tong. Adaptive fuzzy approach to control unified chaotic systems. *Chaos, Solitons and Fractals*, 34 (4), 1180–1187, 2007.
21. S.Y. Li. Chaos control of new Mathieu–van der Pol systems by fuzzy logic constant controllers. *Applied Soft Computing*. 11 (8), 4474–4487, 2011.
22. I.M. Ginarsa, A. Soeprijanto and M.H. Purnomo. Controlling chaos using ANFIS-based composite controller (ANFIS-CC) in power systems. *Int. Conf. on Instrumentation, Communications, Information Technology, and Biomedical Engineering (ICICI-BME)*, Bandung, Indonesia, pp. 1–5, Nov. 25–27 2009.
23. I.M. Ginarsa, A. Soeprijanto and M.H. Purnomo. Controlling chaos and voltage collapse using an ANFIS-based composite controller-static var compensator in power systems. *International Journal of Electrical Power & Energy Systems*, 46, 79–88, 2013.
24. A. Bagheri and J.J. Moghaddam. Decoupled adaptive neuro-fuzzy (DANF) sliding mode control system for a Lorenz chaotic problem. *Expert Systems with Applications*, 36 (3), 6062–6068, 2009.
25. K. Yeh, C.W. Chen, D.C. Lo and K.F.R. Liu. Neural-network fuzzy control for chaotic tuned mass damper systems with time delays. *Journal of Vibration and Control*, 18 (6), 785–795, 2011.
26. J.H. Ma and Y.S. Chen. Study for the bifurcation topological structure and the global complicated character of a kind of non-linear finance system (I). *Applied Mathematics and Mechanics*, 22 (11), 1240–1251, 2001.
27. J.H. Ma and Y.S. Chen. Study for the bifurcation topological structure and the global complicated character of a kind of non-linear finance system (II). *Applied Mathematics and Mechanics*, 22 (11), 1375–1382, 2001.
28. G. Cai and J. Huang. A new finance chaotic attractor. *International Journal of Nonlinear Science*, 3 (3), 213–220, 2007.
29. J. Ding, W. Yang and H. Yao. A new modified hyperchaotic finance system and its control. *International Journal of Nonlinear Science*, 8 (1), 59–66, 2009.
30. H.J. Yu, G.L. Cai and Y.X. Li. Dynamic analysis and control of a new hyperchaotic finance system. *Nonlinear Dynamics*, 67 (3), 2171–2182, 2012.
31. J.H. Ma, Y.Q. Cui and L.X. Liu. Hopf bifurcation and chaos of financial system on condition of specific combination of parameters. *Journal of Systems Science and Complexity*, 21 (2), 250–259, 2008.
32. Q. Gao and J.H. Ma. Chaos and Hopf bifurcation of a finance system. *Nonlinear Dynamics*, 58 (1-2), 209–216, 2009.

33. C. Ma and X.Y. Wang. Hopf bifurcation and topological horseshoe of a novel finance chaotic system. *Communications in Nonlinear Science and Numerical Simulation*, 17 (2), 721–730, 2012.
34. X.S. Zhao, Z.B. Li and S.A. Li. Synchronization of a chaotic finance system. *Applied Mathematics and Computation*, 217 (13), 6031–6039, 2011.
35. S.O. Kareem, K.S. Ojo and A.N. Njah. Function projective synchronization of identical and non-identical modified finance and Shimizu–Morioka systems. *Pramana – Journal of Physics*, 79 (1), 71–79, 2012.
36. J. Jian, X. Deng and J. Wang. Globally exponentially attractive set and synchronization of a class of chaotic finance system. 6th International Symposium on Neural Networks, Wuhan, China, May 26-29, 2009, *Advances in Neural Networks – Lecture Notes in Computer Science*, 5551, 253–261, 2009.
37. G. Cai and M. Yang. Globally exponentially attractive set and synchronization of a novel three-dimensional chaotic finance system. *Third Int. Conf. on Information and Computing (ICIC)*, vol. 2, Wuxi, Jiang Su, China, IEEE, pp. 70–73, June 4–6 2010.
38. A. Jabbari and H. Kheiri. Anti-synchronization of a modified three-dimensional chaotic finance system with uncertain parameters via adaptive control. *International Journal of Nonlinear Science*, 14 (2), 178–185, 2012.
39. U.E. Kocamaz, A. Goksu, H. Taskin and Y. Uyaroglu. Synchronization of chaos in nonlinear finance system by means of sliding mode and passive control methods: A comparative study. *Information Technology and Control*, 44 (2), 172–181, 2015.
40. M. Yang and G. Cai. Chaos control of a non-linear finance system. *Journal of Uncertain Systems*, 5 (4), 263–270, 2011.
41. W.C. Chen. Dynamics and control of a financial system with time-delayed feedbacks. *Chaos, Solitons and Fractals*, 37 (4), 1198–1207, 2008.
42. S. Emiroglu, Y. Uyaroglu and E. Koklukaya. Control of a chaotic finance system with passive control. 3rd Int. Symp. on Sustainable Development, Sarajevo, Bosnia and Herzegovina, pp. 125–130, May 31 – June 01 2012.
43. G. Cai, H. Yu and Y. Li. Stabilization of a modified chaotic finance system. *Fourth Int. Conf. on Information and Computing (ICIC)*, Phuket Island, Thailand, IEEE, pp. 188–191, April 25–27 2011.
44. Y. Uyaroglu, R. Temel and H. Kirlioglu. Feedback control of chaos in a hyperchaotic finance system. 3rd Int. Symp. on Sustainable Development, Sarajevo, Bosnia and Herzegovina, pp. 135–138, May 31 – June 01 2012.
45. H. Gelberi, S. Emiroglu, Y. Uyaroglu and M.A. Yalcin. Time delay feedback control of chaos in a hyper chaotic finance system. 3rd Int. Symp. on Sustainable Development, Sarajevo, Bosnia and Herzegovina, pp. 139–144, May 31 – June 01 2012.