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Study of the Rate-and-State Equation Solutions for Different Critical Stresses by Grassberger-Procaccia Method

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Abstract: A problem of seismicity variation due to human action is considered. The widely used "stick-slip" model of the seismic regime with "rate-and-state" friction law was adopted for description of a sliding along tectonic faults. The main distinctions of used approach from the common one (Hobbs [7], Erickson et al. [4]) are the followings: we consider two-parameter friction law and vary the value of critical shear stress in the rate-and-state equation in suggestion that this is the value which can be varied by human impact (by mining, fluid injection and production, hydraulic fracturing and so on). Calculations were done for the critical stress varied from 5MPa up to 50 MPa with increment 5 MPa. For each value of the critical stress, the time series of the displacement along the fault, its rate and change of shear stress were calculated. Obtained time series were analyzed with the help of Grassberger-Procaccia method of correlation integral calculation for different embedding space dimensions. It was found that if the critical stress increases, the system behavior changes significantly. Oscillations of the fault sliding become inharmonic, and when the critical stress reaches 45 MPa, the oscillations become quasi-chaotic. An estimation of the obtained attractor dimensions by Grassberger-Procaccia method showed, that an increase of the critical stresses τ^* results in increase of the attractor correlation dimensionality C^*: τ^*=5MPa C^*=1.4; τ^*=15MPa C^*=1.6; τ^* = 30 MPa C^*=2.2; τ^*=45MPa C^*=2.5. It was found, that if the critical stress continue to increase, the correlation dimension would stop to increase. A comparison of the obtained results with real induced seismicity data analysis showed that in real case the correlation dimensionality is higher. This discrepancy can be explained by taking into account the presence of the seismic events, which are not related with human influence and which can be considered as a stochastic background. An addition of random component with signal/noise ratio 2 to the model data resulted in increase of the model correlation dimensionality to 4-5, which is in good correspondence with induced seismicity data.

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1 Introduction

Despite the fact that rate-and-state model of friction was proposed in the second half of the previous century, the interest to it has increased in recent years. The reason for that is a success in physics of nonlinear phenomena, in particular, in the area of chaotic system analysis. The rate-and-state model was adopted as quite appropriate basis for describing seismic processes in the Earth crust and for modeling relevant geophysical systems. Currently, it is believed that this model describes the seismic process most adequately.

In 60s, Brace and Byerlee [1] proposed to consider unstable frictional sliding along tectonic faults as a model of earthquakes. The model included a suggestion that a cohesion existing in some parts of the fault prevents free slipping along it and leads to an accumulation of shear stress to a critical level, after which the slip and the earthquake occur.

Peculiarities of the friction forces dependence on the duration of the stationary state of the contact and on the speed of the motion along the fault was examined by Dieterich [3]. Gu et al. [6] experimentally investigated various modes of the frictional movements and determined empirical constants which values are used in many modern variants of the rate-and-state equation.

The origin of the unstable sliding and its dynamics were studied by Ohnaka et al. [8]. The work was focused on the study of mechanism of the transition to instability.

The rate-and-state equation was considered by Hobbs [7] by means of nonlinear dynamics methods. Change of friction was studied as a function of displacement and velocity at a variation of the stiffness coefficient in the rate-and-state equation. The similar approach was implemented by Erickson et al. [4], they examined an appearance of chaotic solutions in the one-parameter velocity-dependent friction equation.

Turuntaev et al. [9] showed that the man-made impact on underground leads to an increase in the “regularity” of the seismic regime. To explain the increase in the seismic regime regularity, a model of fault motion defined by the two-parameter velocity dependent friction law was considered.

In the presented paper, we consider two-parameter type of the friction law and vary the value of critical shear stress in the rate-and-state equation in suggestion that this is the value varied by human impact (by mining, fluid injection and production, hydraulic fracturing and so on). The obtained solutions of the rate-and-state equation are analyzed by means of Grassberger-Procaccia method [5].

2 The model description

Abstracting from internal structure and genesis of the faults, it can be expected that the fault sliding will be governed by the friction law of one type or another,

and that change of the sliding state due to anthropogenic impacts will be resulted in the growth of regularity of the seismic process.
Measurements of the tectonic fault motions show that the motions look like a combination of slow sliding (so-called creep) and fast moves, which accompanied by tremors (earthquakes). This type of the motion can be described with the help of the model proposed by Burridge & Knopov [2], which looks like a system of blocks, elastically connected with each other (Fig. 1 - top view and Fig. 2 - general view of the model). Each block moves under net action of elastic forces from adjacent blocks and frictional force from the stationary substrate. To simplify the model it can be assumed that all the blocks have the same mass, the same area of contact with the surface and that elastic links between the blocks have the same modulus.
Let’s consider the rate-and-state motion equation with the two-parameter friction law and let’s assume that the man-made impact of any nature reduces the critical shear stress (for example, by increasing the pore pressure by fluid injection or by action of vibrations, etc.).

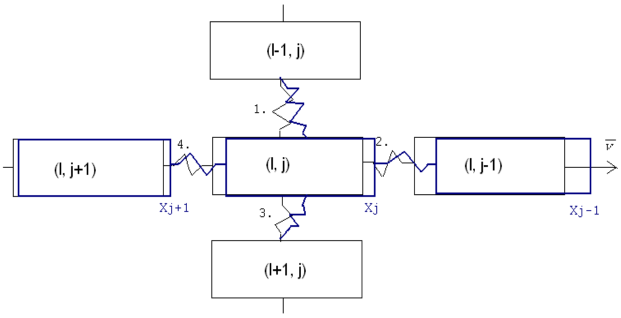


Fig. 1. The model of tectonic blocks (top view).

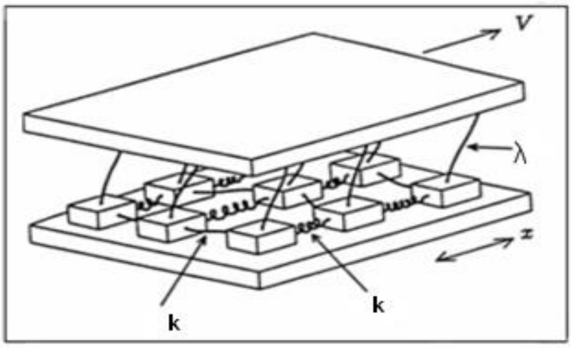


Fig. 2. The Burridge - Knopov (B- K) model of active tectonic faults (general view).

The motion equation for the single-chain of the blocks can be written as follows:

$$m\bar{x}_j \;=\; k\left(v_0 t \;-x_{j-1} + 2x_j - x_{j+1}\right) - \;\tau_j s \qquad (1)$$

where the first term defines the elastic forces from adjacent blocks, the second one is the fault friction: k – stiffness of the elastic links between blocks, υ_0 - speed at infinity, τ - shear stress occurs as a result of friction. In this paper we consider the two-parameter friction law in the form proposed by Hobbs [7]:

$$\tau \;=\; \tau^* + \;A\;ln\left(\frac{v}{v^*}\right) + \;\theta_1 + \;\theta_2 \qquad (2)$$

where v^* - constant velocity of the crustal block relative motion, τ^* - critical stress, which can be changed by external influences and can be written as

$$\tau^* \;= C + \;\mu(\sigma \;- p) \qquad (3)$$

where C - cohesion coefficient, μ - coefficient of friction, p - pore pressure, σ - normal stress; θ_i - state variable, which characterizes the state of the sliding surfaces, and which evolution over time is determined by the equation:

$$\dot{\theta}_i = -\frac{v}{L_i}\left[\theta_i + B_i\,ln\left(\frac{v}{v^*}\right)\right] \qquad (4)$$

here L_i - characteristic dimensions of the roughness of sliding surfaces, $i = \overline{1,2}$. Values of the constants v^*, A, B_i, τ^*, L_i were taken from experiments of Gu et al. [6].

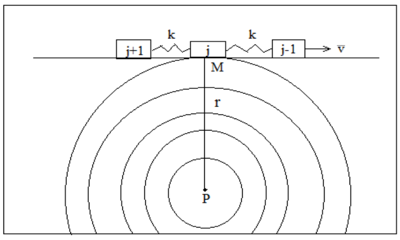


Fig.3. Changes of critical stress on the *j*-th block boundary at the point M due to change of pore pressure at the point P.

Figure 3 illustrates the way in which one of the parameters of equation (3) can be changed. Let's suppose that the pressure is increased at a point *P*. At some moment t_{cr} the pressure will change at the point *M*. It follows from (3) that the increase in the pore pressure will reduce the critical stress $\tau^*(t_{cr}) < \tau^*(\mathbf{0})$, and

consequently, it will reduce the value of the frictional force at which the *j*-th block begins to move.
According to the motion equation (1), it can cause the block "jump", and as a result, the redistribution of elastic forces in the links between the blocks. The whole system can come into motion in the result of a change even in one of the parameters. The resulting motion is complex. Turuntaev et al. [9] showed that to analyze such motions, it's reasonable to use the methods developed for the analysis of nonlinear dynamic systems.

3 Results

Numerical simulation of the block motions was carried out under the critical stress τ^* varied from 5 MPa to 50 MPa with increments 5 MPa. For each value of τ^*, time series of the block displacements, its velocity and shear stress at the block base were calculated. Complexity of the obtained time series were analyzed using algorithm for estimating the correlation dimension, based on the calculation of the correlation integral by Grassberger and Procaccia method [5].
Finite-difference scheme used to solve the equation of motion (1) was following

$$\frac{x_{i+1} - 2 \cdot x_i + x_{i-1}}{h^2} = \frac{k}{m} \cdot \left(\frac{x_i - x_{i-1}}{h} \cdot (ih) - x_i\right) - \frac{\tau_i}{s} \qquad (5)$$

with initial conditions $x(0) = 0, v(0) = 0$.
The values of the parameters *k*, *m*, *s* were taken from Hobbs [7].
To solve the equation we used the method of direct and reverse run with the following values of the preliminary factors

$$A = a(y_{i-1,j}, y_{i,j}) \cdot \frac{h}{h_x^2}$$
$$B = a(y_{i,j}, y_{i+1,j}) \cdot \frac{h}{h_x^2} \qquad (6)$$
$$C = \left(a(y_{i-1,j}, y_{i,j}) + a(y_{i,j}, y_{i+1,j})\right) \cdot \frac{h}{h_x^2} + 1$$
$$F = y_{i,j}$$

which were included in the calculation of the coefficients α_i, β_i in final formulas

$$\alpha_i = \frac{B}{C - A \cdot \alpha_{i-1}}$$

$$\beta_i = \frac{A \cdot \beta_{i-1} + F}{C - A \cdot \alpha_{i-1}} \qquad (7)$$
$$y_{i,j} = \alpha_i \cdot y_{i+1,j+1} + \beta_i$$

The values of the time step h, spatial grid h_x and the correction coefficients of approximation in the formulas (5) - (7) were the followings:

$$h_x = 0.01$$
$$h = 0.01$$
$$\delta = 0.01 \qquad (8)$$
$$\alpha = 0.5 \cdot \gamma \cdot \delta^2 \cdot y_{i-1,j}^{\gamma-1} + y_{i,j}^{\gamma-1}$$

The selected values of the coefficients give approximation error at the level of $O(h, h_x^2)$, that is enough accuracy for the considered problem.

The graphs of the displacements and the shear stresses for three values of the critical stress τ^* : 5 MPa, 20 MPa, 50 MPa are shown in Figures 4-6.

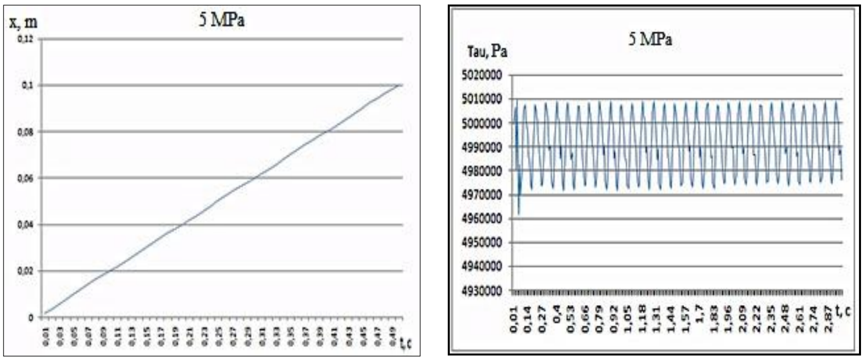


Fig. 4. Dependencies of displacement on time (left panel) and shear stress on time (right panel) at the critical stress equal to 5 MPa.

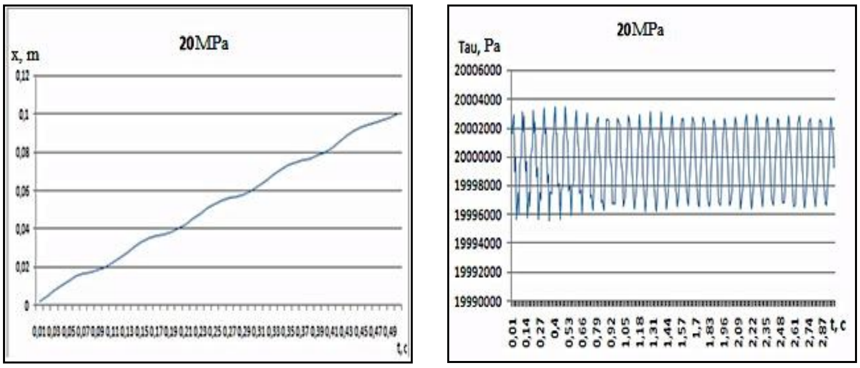


Fig. 5. Dependencies of displacement on time (left panel) and shear stress on time (right panel) at the critical stress equal to 20 MPa.

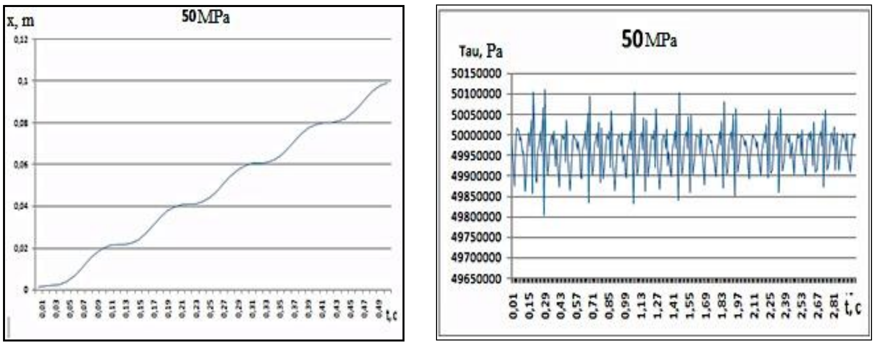


Fig. 6. Dependencies of displacement on time (left panel) and shear stress on time (right panel) at the critical stress equal to 50 MPa.

The graphs of the block motion respectively to motion with constant velocity at infinity v^* are shown in Fig. 7.

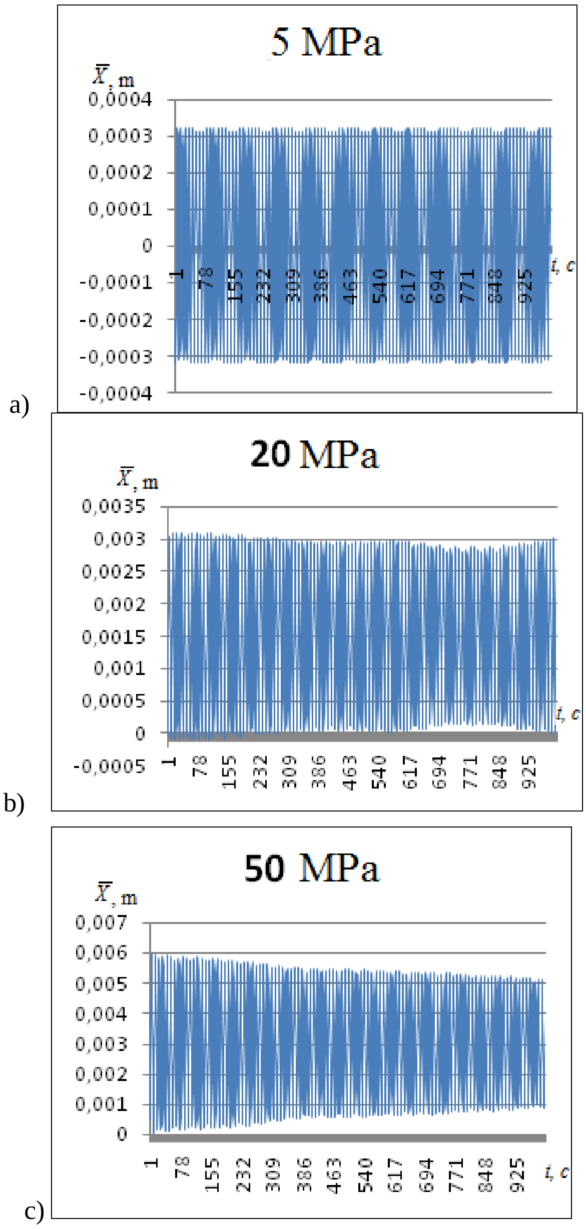
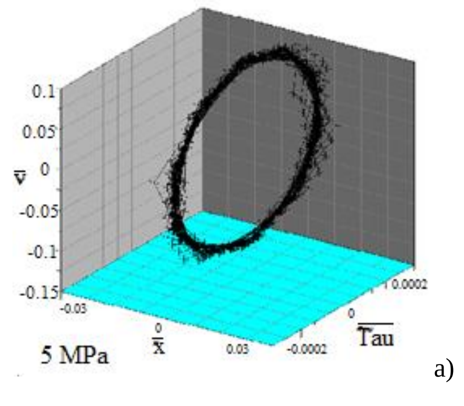
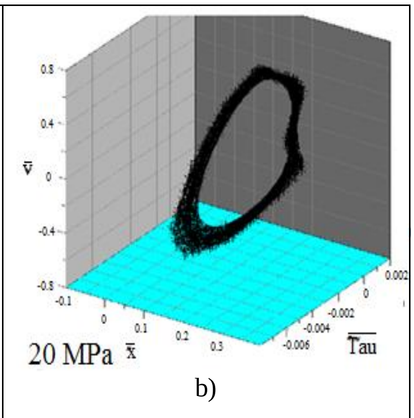


Fig . 7. Dependencies of displacements on time, calculated for the critical values of $\tau^* = 5$ MPa , $\tau^* = 20$ MPa, and $\tau^* = 50$ MPa.





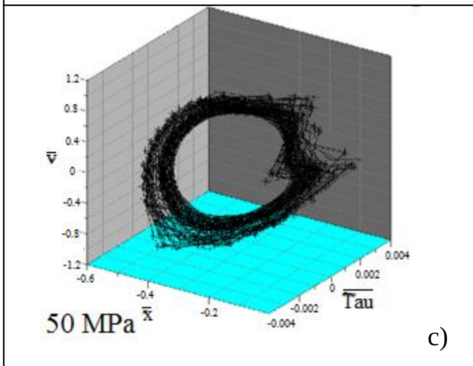


Fig. 8. Phase trajectories of a fault sliding in *x*-*v*-τ coordinates for values of critical shear stresses 5 MPa (a), 20 MPa (b) and 50 MPa (c).

Results of the numerical calculations for several values of the critical stresses are shown in Figure 8 as phase trajectories in *x*-*v*-τ coordinates. The values are normalized to the characteristic size L_1, v^* и $\tau^* = 5$ МПа for the values of the critical stress at 5 MPa, 20 MPa and 50 MPa (Fig. 8a, 8b and 8c, respectively).

An estimation of the obtained attractor dimensions by Grassberger-Procaccia method showed, that an increase of the critical stresses results in increase of the attractor correlation dimensionality: τ*=5MPa *C**=1.4; τ*=15MPa *C**=1.6; τ* = 30 MPa *C**=2.2; τ*=45MPa *C**=2.5. (Figure 9).

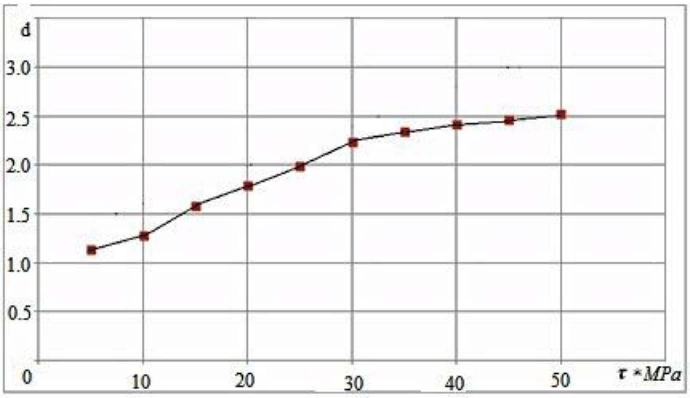


Fig. 9. The dependence of the correlation dimension on the critical stress.

4 Discussion and conclusions

Numerical analysis of the rate-and-state equation with the two-parameter friction law showed significant changes in the stick-slip motion when the critical shear stress varied.

Evaluation of the correlation dimension and the embedding space dimension by Grasbergera - Procaccia method for obtained time series has shown that both of these variables have small values. Change of critical stress from 5 MPa to 50 MPa resulted in variation of correlation dimension and embedded space dimension from 1.1 to 2.5 and from 3 to 5, respectively.

In the range of the critical stress from 5 MPa to 30 MPa the correlation dimension increases linearly with critical stress increase; at higher values of the critical stress there is a tendency of saturation of the correlation dimension dependence on the critical stress.

Values of dimensions obtained in the numerical modeling may differ from the values, which were obtained in the analysis of real seismicity (for example, in the area of the Bishkek geodynamic test site, see Turuntaev et al. [9]). We can assume that this difference is caused by significantly higher complexity of real seismic processes in comparison with the model one. This distinguish can be explained by taking into account the presence of the seismic events, which are not related with human influence and which can be considered as a stochastic background. An addition of random component with signal/noise ratio 2 to the model data resulted in increase of the model correlation dimensionality to 4-5, which is in good correspondence with induced seismicity data.

The existence of stable states in the equation solution allows us to specify the problem of seismic activity forecast and of seismic regime control technologies. According to the equation (3), the influence on the movement of the crustal blocks can be performed by changing coefficient of friction by fluid injection. The aim of further research is to study the minimal values of the pore pressure variations that can change the state of a system of interconnected blocks. We plan to investigate the solutions of the equations of motion (1) with more real parameters than obtained in laboratory experiments (the characteristic parameters of the contacting surfaces, the velocity of relative motion of the fault, stiffness, cohesion, etc.).

At the present stage of the research one can conclude that an increase of the critical stresses in the rate-and-state equation results in increase of the attractor correlation dimensionality: C^*: τ^*=5MPa C^*=1.4; τ^*=15MPa C^*=1.6; τ^* = 30 MPa C^*=2.2; τ^*=45MPa C^*=2.5. It was found, that if the critical stress continue to increase, the correlation dimension would stop to increase.

References

1. Brace W.F., Byerlee J.D. Stick-slip as mechanism for earthquakes. *Science.* 1966. V. 153. № 3739. P. 990-992.
2. Burridge R., Knopov L. Model and theoretical seismisity//Bull. Seism. Soc. Amer. 1967. V. 67. P.341-371.
3. Dieterich J.H. Earthquake nucleation on faults with rate and state-dependet friction. *Tectonophysics.* 1992. V. 211. P. 115-134.
4. Erickson B., Birnir B., Lavalle D. A model for aperiodicity in earthquakes. *Nonlinear Processes in Geophysics*. 2008.
5. Grassberger P., Procaccia I. Measuring the strangeness of strange attractors. *Physica. North-Holland Publishing Company.* 1983. V. 9D. P. 189-208.
6. Gu J. C., Rice J.R., Ruina A.L., Tse S.T. Slip motion and instability of a single degree of freedom elastic system with rate-and-state dependent friction. *J. Mech. Phys. Solids.* 1984. V. 32. P. 167-196.
7. Hobbs B.E. Chaotic behavior of frictional shear instabilities. *Rockbursts and Seismicity in Mines / Fairhurst (ed.).* 1990. Balkema, Rotterdam. P. 87-91.
8. Ohnaka, M., Kuwahara, Y., Yamamoto, K. and Hirasawa, T. In: Dynamic breakdown processes and the generating mechanism for high-frequency elastic radiation during stick-slip instabilities. American Geophysical Union, Washington D.C.. 37: 4. 1986. doi: 10.1029/GM037p0013.
9. Turuntaev S.B., Vorohobina S.V., Melchaeva O.Y. Identification of anthropogenic changes of seismic regime using methods of nonlinear dynamics. *Physics of the Earth*. 2012. № 3. P. 52-65.