# The Mechanisms of Chaotization in Switching Power Converters with Compensation Ramp

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**Abstract:** Recently much attention has been paid to investigation of nonlinear dynamics of switching power converters, as this kind of dynamical systems, being inherently hybrid, is capable of exhibiting a wide variety of well known smooth as well as novel non-smooth phenomena. This research shows the diversity of complex interactions of smooth bifurcations and border collisions in one of the most typical power circuits – boost converter under current mode control – applying the method of complete bifurcation groups. The effects of realistic parameters and implementation of compensation signals on the robustness of chaotic modes of operation are investigated and explained in details.
**Keywords:** Bifurcations, Chaos, Non-smooth phenomena, Switching power converters.

## 1. Introduction

It is common knowledge for the majority of engineers working in the field of power electronics, that the only acceptable operating regime of switching power converters (SPC) is the period-1 (P1) mode, when all waveforms repeat at the same rate as driving clock element. So, all the efforts of practicing engineers are directed to insurance of stable operation of DC-DC converters, eliminating the possibilities of occurrence of any subharmonic oscillations. On the other hand, recent investigations have shown that the operation of SPC in subharmonic or even chaotic modes allows achieving higher performance characteristics of these devices. In example, paper [1] presents the novel control strategy, allowing simple digital implementation and excellent transient response. The idea of the control is based on the use of various combinations of two different control pulses that from the point of view of nonlinear dynamics could be treated as operation in a variety of subharmonic regimes. The other research [2] shows the applicability of inherently arising chaotic modes of operation of switching converters to the reduction of high levels of electromagnetic noise, generated by this kind of devices. Thus it has been demonstrated, that contrary to generally accepted opinion, the non-linear operating modes of switching converters could be efficiently utilized, providing new progressive control perspectives.

The methodology of implementation of the control ramp is widely used as the

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compensation tool for the irregularities of the current loop, ensuring stable P1 operation of SPC. In general, the introduction of this compensating signal shifts the border of the first period-doubling bifurcation, estranging the appearance of subharmonic oscillations that are usually avoided. However, the same ramp also modifies the structure of the parameter space even after the period-doubling bifurcation, defining noticeable changes in the dynamics of the system. This research is dedicated to exploration of different mechanisms of chaotization and further aftereffects in the operation of SPC, defined by the implementation of mentioned compensation technique.

It has been demonstrated during several last decades, that the conventional models and methodologies used to predict the appearance of subharmonic oscillations in switching power converters are generally oversimplified and not capable of providing reliable data in many cases [3]-[6]. This fact determined the development of great number of scientific researches dedicated to possible improvements of already existing models and to the introduction of new promising approaches. Recently one innovative methodology – method of complete bifurcation groups (MCBG) [7]-[9] – has been applied to investigation of rare phenomena and chaos in SPC, allowing the detection and detailed analysis of previously unobserved operating regimes. MCBG is utilized within current research in order to provide the most complete analysis of the observed non-linear phenomena in the dynamics of SPC.

The structure of the paper is as follows. The second section presents the simplified discrete-time model of the boost SPC, introducing the compensating ramp in the current control loop. The results of the complete bifurcation analysis, including the construction of bifurcation map and various bifurcation diagrams, are presented and discussed in section 3. The concluding remarks and comments are given in the last section.

## 2. The Model of Boost Converter with Compensation Ramp

The SPC under study is widely used boost converter with peak current mode control, exhibiting unstable dynamics as the duty cycle exceeds 0.5 [3]. The main methodology of extending the region of stable P1 operation in this kind of devices is the introduction of compensation ramp signal, which is also included in the dynamical model.

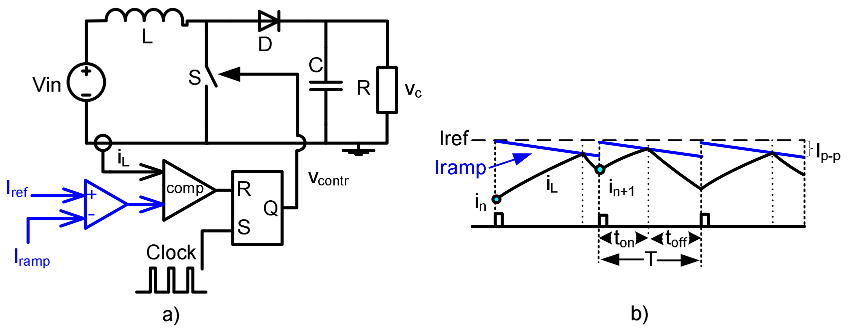


Fig. 1. (a) The simplified model of current - mode controlled boost converter; (b) waveforms of inductor current and compensation ramp.

The simplified model of boost SPC is shown in Fig. 1. The operation of converter is as follows: the switch is turned *ON* as the clock pulse arrives and turned *OFF* as the value of inductor current reaches the compensation ramp.

The dynamics of this energy conversion circuit could be described by two systems of difference equations, depending on the sequence of switching events. If the clock pulse arrives before the inductor current reaches the $I_{ramp,}$ the obtained discrete time model is the following:

$$v_{n+1} = v_n e^{-T/(RC)}$$
$$i_{n+1} = i_n + V_{in} TL\,. \tag{1}$$

If the inductor current reaches $I_{ramp}$ before the arrival of the next clock pulse the map would include the *ON* and *OFF* intervals:

$$\begin{aligned} v_{n+1} &= e^{-mt_{off}}\left[K_1\cos(\mu t_{off}) + K_2\sin(\mu t_{off})\right] + V_{in} \\ i_{n+1} &= e^{-mt_{off}}[C[-m(K_1\cos(\mu t_{off}) + K_2\sin(\mu t_{off})) + \\ &+ \mu(-K_1\sin(\mu t_{off}) + K_2\cos(\mu t_{off}))] + (K_1\cos(\mu t_{off}) \\ &+ K_2\sin(\mu t_{off}))/R] + V_{in}/R, \end{aligned} \tag{2}$$

where

$$\begin{aligned} &t_{on} = (I_{ref} - i_n)/(V_{in}/L + S_c);\ K_1 = v_n e^{-2mt_{on}} - V_{in}; \\ &t_{off} = T - t_{on};\ K_2 = \left[I_{ref}/C - \left(v_n e^{-2mt_{on}} + V_{in}\right)\right]/\mu; \\ &m = 1/(2RC);\ p = 1/\sqrt{LC};\ \mu = \sqrt{p^2 - m^2};\ S_c = I_{p-p}/T. \end{aligned} \tag{3}$$

The borderline $I_{border}$ defines the case, when the clock pulse arrives exactly at the time instance the inductor current reaches the control signal:

$$I_{border} = I_{ref} - T\left(V_{in}/L + S_c\right). \tag{4}$$

It has been shown that this discrete-time model could be efficiently applied to the study of nonlinear dynamics and estimation of stability boundaries of main period-1 and subharmonic modes of operation [8].

The provided model (1)-(4) is used in the process of the construction of bifurcation map, complete bifurcation diagrams, calculating parameters of different periodic regimes as well as estimating their stability.

The values of main parameters of boost converter under test are as follows: $R$=40 (Ω); $L$=1.5 (mH); $C$=5 (μF); $T$=100 (μS); $V_{in}$=5 (V); $I_{ref}$=[0.2…0.9] (A); $S_c$=[0…2000] (A/s).

## 3. Results of the complete bifurcation analysis

As it has been mentioned in the introduction, the analysis of the global dynamics of boost SPC will be provided by means of one of the most progressive techniques – method of complete bifurcation groups. This methodology has proved to be very useful during the complete analysis of nonlinear phenomena, observed in various classes of smooth as well as non-smooth dynamical systems [7]-[9].

The complete bifurcation analysis of the boost converter begins with the construction of bifurcation map, selecting reference current and compensation ramp as primary and secondary bifurcation parameters (see Fig. 2). As the mechanisms of chaotization are of special interest and the P1 orbits are not involved in the rapid transitions to chaotic modes of operation, the range of parameters defining stable P1 regime is disregarded in the constructed bifurcation map. As it could be seen from Fig. 2 for $S_c$=0 the classical period doubling route to chaos should be observed. As the value of compensation ramp is increased other subharmonic operation regimes (as well as periodic windows) just after the P2 appear, defining the formation and structure of chaotic regions.

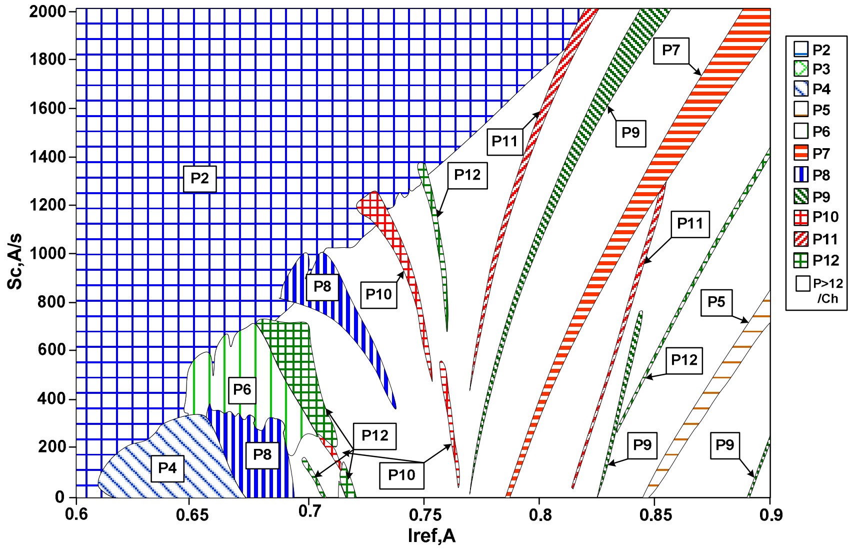


Fig.2. The bifurcation map of the boost converter.

This map will be referred to during the analysis of complete bifurcation diagrams, constructed as the horizontal cross-sections of Fig. 2.

It should be understood, that in this case the complete bifurcation diagrams are 3-dimensional graphs, depicting the sampled inductor current and capacitor voltage on two axes and the bifurcation parameter on the third one. For the clearness of analysis only the projection of this graph to the plane defined by the inductor current and the bifurcation parameter ($I_{ref}$) will be observed, as only in this plane the collisions with the border defined by (4) could be precisely detected and interpreted.

In order to analyze the influence of the incrementing compensation ramp on the chaotization of the system, first let's examine the complete bifurcation diagram for the boost converter without the compensation.

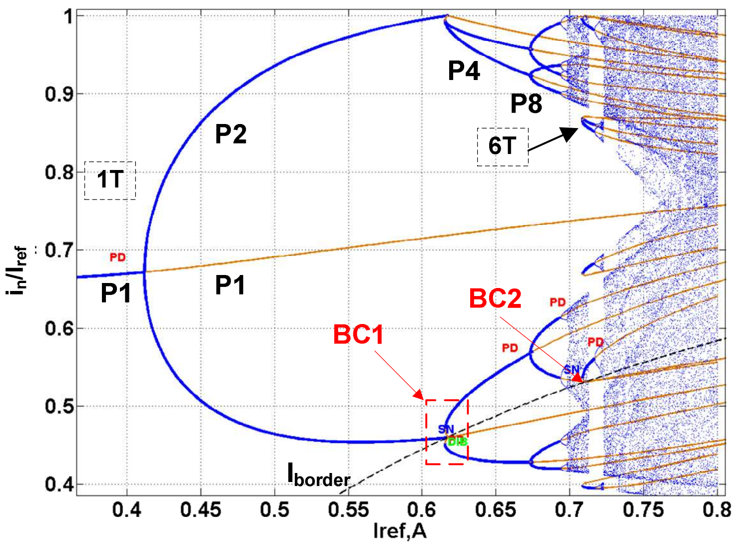


Fig. 3. The complete bifurcation diagram for $S_c$=0 (A/s).

The bifurcation diagram, depicting stable (dark lines) and unstable (light-colored lines) periodic regimes, as well as chaotic regions (shaded area), is shown in the Fig. 3. It could be seen, that for small values of reference current, the system operates in the stable P1 regime and moves to P2 mode through classical period doubling bifurcation. Further increment of $I_{ref}$ leads to the development of non-smooth phenomena, when stable or unstable orbits collide with the $I_{border}$ (dashed line), leading to non-smooth transition from P2 to P4 regimes (see point BC1), as well as change of shape of unstable branch of 6T bifurcation group (see point BC2). Thus it could be concluded, that for the selected set of system parameters, collisions with the border have slightly changed the topology of bifurcation diagram, without any noticeable rapid jumps between different modes of operation.

The second bifurcation diagram, constructed for $S_c$=200 (A/s) is shown in the Fig. 4. One of the most interesting features of the observed diagram is the presence of rather uncommon phenomena that will be referred as "cutting border collision" (CBC). It is well known, that the collision with borders in hybrid systems could lead to the appearance of non-smooth bifurcations, when the multipliers do not smoothly cross the unit circle, indicating the widely observed period-doubling or saddle-node bifurcations, but rather "jump" over the border of unit circle, depicting the rapid change in the stability of definite regime under investigation. In the mentioned case the periodic regime still continues to exist, but its stability suddenly changes. However the CBC leads to the "disappearance" of all stable as well as unstable periodic regimes that cross the defined borderline.

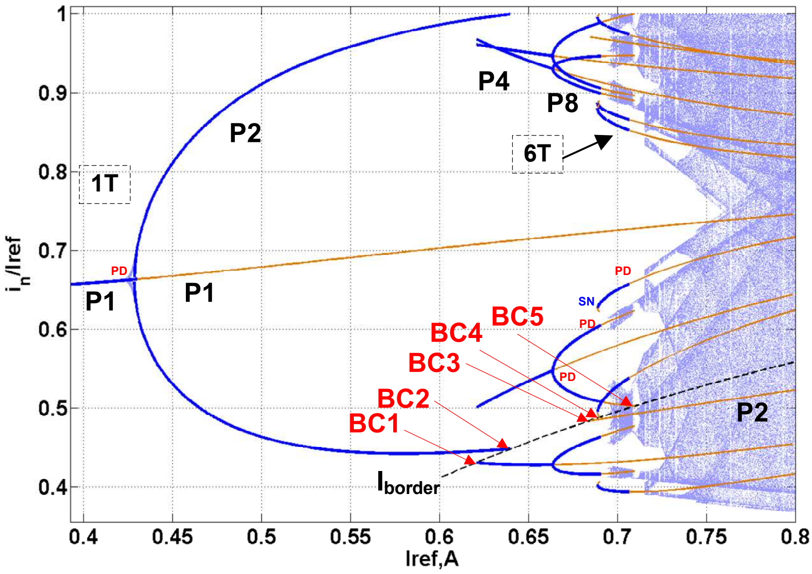


Fig. 4. The complete bifurcation diagram for $S_c$=200 (A/s).

This phenomenon could be observed, in example, in points BC1, BC2 and BC4 in the Fig. 4, where stable P2, P4 and unstable P8 regimes collide with the borderline defined in (4) and disappear without any signs of bifurcations. The route to chaos in this case is formed by rather uncommon period-doublings, leading to the infinite number of unstable periodic orbits and chaotic mode of operation.

It has been shown in the Fig. 3, that for $S_c$=0 (A/s) the border collision of 6T bifurcation group leads to some changes in the shape of bifurcation diagram. However the diagram in the Fig. 4 demonstrates that the same collision for $S_c$=200 (A/s) causes the disappearance of unstable branch of 6T bifurcation group, preserving the stable branch, leading to the development of independent chaotic regime.

It is interesting to note that the P2 orbit appears at point BC3 as unstable regime and continues to exist for larger values of $I_{ref}$. So it could be assumed that in the interval $I_{ref}$=[0.64…0.68] (A) this regime slides along the borderline (4). Other orbits after the CBC are not observed within the parameter range of interest.

As the transition from P2 to P4 in the Fig.4 is caused by highly non-smooth event, it would be interesting to investigate the topology of basins of attraction of coexisting P2 and P4 modes of operation for $I_{ref}$=0.63 (A).

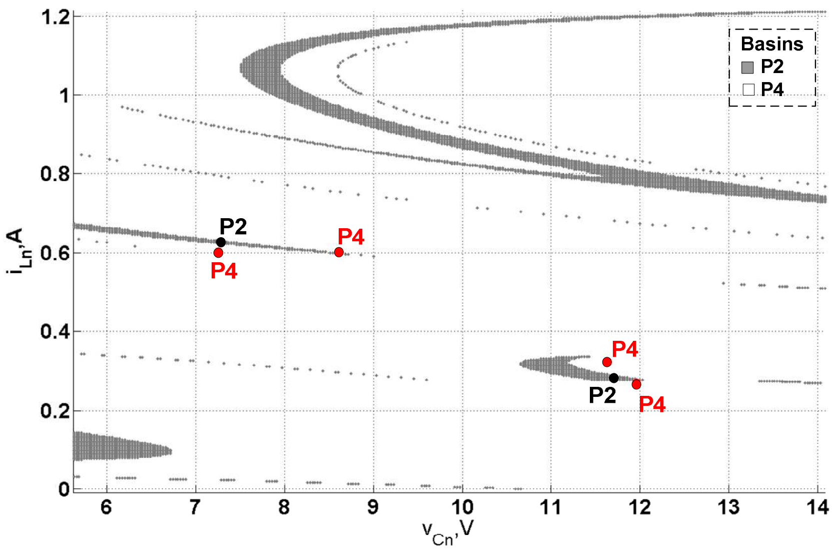


Fig. 5. Basins of attraction of P2 and P4 regimes for $S_c$= 200 (A/s) and $I_{ref}$=0.63 (A) with corresponding attractors.

As it could be seen from Fig. 5, despite the border collisions observed in the complete bifurcation diagrams, the basins of attraction of P2 and P4 regimes, forming rather complex structure, still remain smooth and no sign of uncommon topological peculiarities are observed.

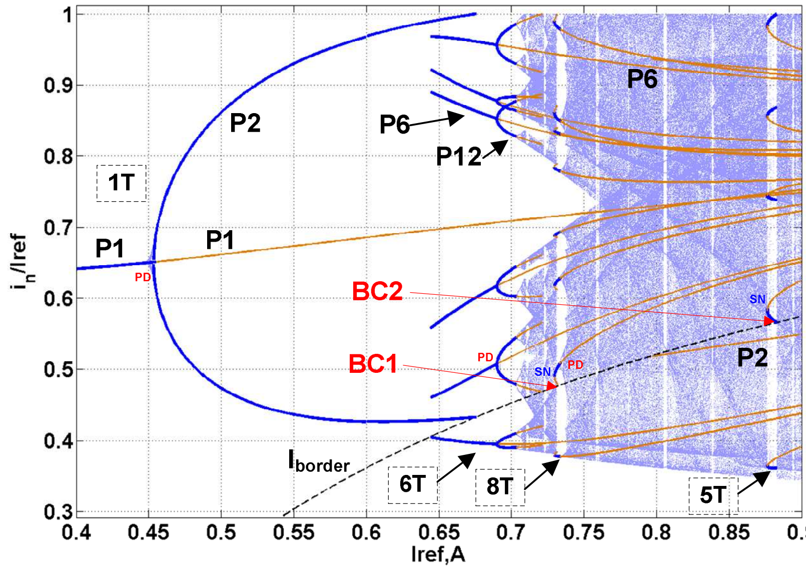


Fig. 6. The complete bifurcation diagram for $S_c$=500 (A/s).

Further increment of the compensation ramp practically leads to the enlargement of stable P1 region and essentially changes the sequence and types of bifurcations. Fig. 6 depicts the complete bifurcation diagram of the boost converter for $S_c$=500, showing that the first period doubling at $I_{ref}$=0.45 is followed by rapid transition from P2 to P6 operating regime with definite region of coexistence of both mentioned orbits. Further chaotization is governed by unstable orbits arising from the P6 regime through classical period-doublings.

Points BC1 and BC2 in the Fig. 6 demonstrate to possible interactions of periodic orbits, appearing from saddle-node bifurcations with the borderline (4). At point BC1 the cutting border collision eliminates only the unstable regime of 8T bifurcation group, allowing the gradual development of chaotic attractor. However at point BC2 the stable branch of 5T bifurcation group collides with the borderline and rapid chaotification is observed without the development of sequent period-doubling cascade.

The bifurcation map, shown in the Fig. 2, allows asserting that further increasing the value of $S_c$ leads to the implementation of direct P2-P6, P2-P8, P2-P10 etc. transitions as well as the development of increasingly wider periodic windows, excluding the possibility of existence of practically useful robust chaotic modes of operation.

## 4. Conclusions

The results of complete bifurcation analysis allow revealing some interesting changes in chaotification scenario of the compensated boost switching power converter. For the converters with small output capacitance and without compensating ramp (i.e. $S_c$=0) the classical smooth period doubling route to chaos could be observed. As the value of the compensating ramp signal is increased the non-smooth effects, emerging from the interaction of bifurcation branches of stable and unstable periodic regimes with the borderline (4), take place, determining the general appearance of bifurcation diagram and transition to chaotic mode of operation.

It should be noted that, taking into account the topology of complete bifurcation diagrams constructed within this research, the definition of bifurcation group in MCBG (see e.g. [7]) should be revised, as the P2 and P4 regimes in the Fig. 4, corresponding to the same 1T bifurcation group have no common bifurcation points and are not mutually connected with stable or unstable branches.

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