Ideal Chaotic Pattern Recognition Using the Modified Adachi Neural Network

Ke Qin[1] and B. J. Oommen[2,3]

[1] University of Electronic Science & Technology of China, Chengdu, China. 611731
(E-mail: `thinker_qk@hotmail.com`)
[2] Carleton University, Ottawa, ON, Canada. K1S 5B6
[3] University of Agder, Postboks 509, 4898 Grimstad, Norway.
(E-mail: `oommen@scs.carleton.ca`)

Abstract. The benchmark of a chaotic Pattern Recognition (PR) system is the following: First of all, one must be able to train the system with a set of "training" patterns. Subsequently, as long as there is no testing pattern, the system must be chaotic. However, if the system is, thereafter, presented with an unknown testing pattern, the behavior must ideally be as follows. If the testing pattern is not one of the trained patterns, the system must continue to be chaotic. As opposed to this, if the testing pattern is truly one of the trained patterns (or a noisy version of a trained pattern), the system must switch to being *periodic*, with the specific trained pattern appearing periodically at the output. This is truly an ambitious goal, with the requirement of switching from chaos to periodicity being the most demanding. The Adachi Neural Network (AdNN) [1–5] has properties which are *pseudo*-chaotic, but it also possesses *limited* PR characteristics. As opposed to this, the Modified Adachi Neural Network (M-AdNN) proposed by Calitoiu *et al* [7], is a fascinating NN which has been shown to possess the required periodicity property desirable for PR applications. In this paper, we shall tune the parameters of the M-AdNN for its weights, steepness and external inputs, to yield a new NN, which we shall refer to as the Ideal-M-AdNN. Using a rigorous Lyapunov analysis, we shall analyze the chaotic properties of the Ideal-M-AdNN, and demonstrate its chaotic characteristics. Thereafter, we shall verify that the system is also truly chaotic for untrained patterns. But most importantly, we demonstrate that it is able to *switch to being periodic* whenever it encounters patterns with which it was trained. Apart from being quite fascinating, as far as we know, the theoretical and experimental results presented here are both unreported and novel. Indeed, we are not aware of any NN that possesses these properties!

Keywords: Chaotic Neural Networks, Chaotic Pattern Recognition.

1 Introduction

Pattern Recognition (PR) has numerous well-established sub-areas such as statistical, syntactic, structural and neural. The field of *Chaotic* PR is, however,

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relatively new and is founded on the principles of chaos theory. It is also based on a distinct phenomenon, namely that of *switching* from *chaos* to *periodicity*. Indeed, Freeman's clinical work has clearly demonstrated that the brain, at the individual neural level and at the global level, possesses chaotic properties. He showed that the quiescent state of the brain is chaos. However, during perception, when attention is focused on any sensory stimulus, the brain activity becomes more periodic [10].

If the brain is capable of displaying both chaotic and periodic behavior, the premise of this paper is that it is expedient to devise artificial Neural Network (NN) systems that can display these properties too. Thus, the primary goal of *chaotic* PR is to develop a system which mimics the brain to achieve chaos and PR, and to consequently develop a new PR paradigm.

Historically, the initial and pioneering results concerning these CNNs were presented in [1–5]. Subsequently, the author of [11] proposed two methods of controlling chaos by introducing a small perturbation in continuous time, i.e., by invoking a combined feedback with the use of a specially-designed external oscillator or by a delayed self-controlling feedback without the use of any external force. The reason for the introduction of this perturbation was to stabilize the unstable periodic orbit of the chaotic system. Thereafter, motivated by the work of Adachi, Aihara and Pyragas, various types of CNNs have been proposed to solve a number of optimization problems (such as the Traveling Salesman Problem, (TSP)), or to obtain Associative Memory (AM) and/or PR properties. An interesting step in this regard was the work in [15], where the authors utilized the delayed feedback and the Ikeda map to design a CNN to mimic the biological phenomena observed by Freeman [10].

More recently, based on the AdNN, Calitoiu and his co-authors made some interesting modifications to the basic network connections so as to obtain PR properties and "blurring". In [8], they showed that by binding the state variables to those associated with *certain* states, one could obtain PR phenomena. However, by modifying the manner in which the state variables were bound, they designed a newly-created machine, the so-called Mb-AdNN, which was also capable of justifying "blurring" from a NN perspective. While all of the above are both novel and interesting, since most of these CNNs are *completely*-connected graphs, the computational burden is rather intensive. Aiming to reduce the computational cost, in our previous paper [12], we proposed a mechanism (the Linearized AdNN (L-AdNN)) to reduce the computational load of the AdNN. To complete this historical overview, we mention that in [12], we showed that the AdNN goes through a spectrum of characteristics (i.e., AM, *quasi*-chaotic, and PR) as one of its crucial parameters, α, changes. It can even recognize masked or occluded patterns!

Although it was initially claimed that the AdNN and M-AdNN possessed "pure" (i.e., periodic) PR properties, in actuality, this claim is not as precise as the authors claimed – the output can be periodic for both trained and untrained input patterns – which is where our present paper is relevant. The primary aim of this paper is to show that the M-AdNN, when tuned appropriately, is capable of demonstrating ideal PR capabilities. Thus, the primary contributions of this paper are:

1. We formalize the requirements of a PR system which is founded on the theory of chaotic NNs.
2. We enhance the M-AdNN to yield the Ideal-M-AdNN, so that it does, indeed, possess *Chaotic* PR properties.
3. We show that the Ideal-M-AdNN does switch from chaos to periodicity when it encounters a trained pattern, but that it is truly chaotic for all other input patterns.
4. We present results concerning the stability of the network and its transient and dynamic retrieval characteristics. This analysis is achieved using eigenvalue considerations, and the Lyapunov exponents.
5. We provide explicit experimental results to justify our claims.

In the interest of brevity and space, the details of the theoretical results reported here are not included. They are found in the unabridged version of the paper [13].

2 The Ideal-M-AdNN

The goal of the field of Chaotic PR systems can be expressed as follows: We do not intend a chaotic PR system to report the identity of a testing pattern with a class "proclamation" as in a traditional PR system. Rather, what we want to achieve for a the chaotic PR system are the following phenomena:

- The system must yield a strong *periodic* signal when a trained pattern, which is to be recognized, is presented.
- Further, between two consecutive recognized patterns, none of the trained patterns must be recalled.
- On the other hand, and most importantly, if an untrained pattern is presented, the system must give a chaotic signal.

Calitoiu *et al* were the first researchers who recorded the potential of chaotic NNs to achieve PR. But unfortunately, their model, as presented in [7], named the M-AdNN, was not capable of demonstrating *all* the PR properties mentioned above.

The topology of the Ideal-M-AdNN is exactly the same as that of the M-AdNN. Structurally, it is also composed of N neurons, topologically arranged as a completely connected graph. Each neuron has two internal states $\eta_i(t)$ and $\xi_i(t)$, and an output $x_i(t)$. Just like the M-AdNN, the Present-State/Next-State equations of the Ideal-M-AdNN are defined in terms of only a *single* global neuron (and its corresponding two global states), which, in turn, is used for the state updating criterion for *all* the neurons. Thus,

$$\eta_i(t+1) = k_f \eta_m(t) + \sum_{j=1}^{N} w_{ij} x_j(t),$$

$$\xi_i(t+1) = k_r \xi_m(t) - \alpha x_i(t) + a_i,$$

$$x_i(t+1) = f(\eta_i(t+1) + \xi_i(t+1)),$$

where m is the index of this so-called "global" neuron.

We shall now concentrate on the differences between the two models, which are the parameters: $\{w_{ij}\}$, ε and a_i, which were previously set arbitrarily. Rather, we shall address the issue of how these parameters must be assigned their respective values so as to yield pure *Chaotic* PR properties.

2.1 The Weights of The Ideal-M-AdNN

The M-AdNN uses a form of the Hebbian Rule to determine the weights of the connections in the network. This rule is defined by the following equation:

$$w_{ij} = \frac{1}{p} \sum_{s=1}^{p} P_i^s P_j^s, \tag{1}$$

where $\{P\}$ are the training patterns, P_i^s denotes the i^{th} neuron of the s^{th} pattern P^s, and where p is the number of known training patterns. This rule is founded on two fundamental premises, namely:

1. Each element of the learning vectors should be either 1 or -1;
2. Any pair of learning vectors, P and Q, must be orthogonal.

In this regard, we note that:

1. In [7], the elements of the corresponding learning vectors are restricted to be either 0 and 1, which implies that the connection between any two neurons, say, A and B, will be increased only if they are both positive. Further, the connection weights are not changed otherwise.
2. The formal rationale for orthogonality is explained in [13]. Although it is not so stringent, when the number of neurons is much larger than the number of patterns, and the learning vectors are randomly chosen from a large sample set, the probability of having the learning vectors to be orthogonal is very high. Consequently, generally speaking, the Hebbian rule is true, albeit in a probabilistic sense.

Based on the above observations, we conclude that for the M-AdNN, we should not use the Hebbian rule as dictated by the form given in Equation (1), since the data sets used by both Adachi *et al* and Calitoiu *et al* are defined on $\{0,1\}^N$, and the output is further restricted to be in $[0,1]$ by virtue of the logistic function. In fact, this is why Adachi and Aihara computed the weights by scaling all the patterns to be in -1 and 1 using the formula given by Equation (2) instead of Equation (1):

$$w_{ij} = \frac{1}{p} \sum_{s=1}^{p} (2P_i^s - 1)(2P_j^s - 1). \tag{2}$$

By virtue of this argument, in this paper, we advocate the use of this formula, i.e., Equation (2), to determine the connection weights of the network.

It is pertinent to mention that since the patterns $\{P\}$ are scaled to be in the range between -1 and 1, it *does* change the corresponding property of orthogonality. This is clearly demonstrated in [13], but omitted here in the interest of space.

2.2 The Steepness, Refractory and Stimulus Parameters

Significance of ε for the AdNN: The next issue that we need to consider concerns the value of the steepness parameter, ε, of the output function. As explained earlier, we see that the output function is defined by the Logistic function $f(x) = \frac{1}{1+e^{-x/\varepsilon}}$, which is a typical sigmoid function.

One can see that ε controls the steepness of the output. If $\varepsilon = 0.01$, then $f(x)$ is a normal sigmoid function. If ε is two small, for example, 0.0001, the Logistic function almost degrades to become a unit step function, as shown in [13]. The question is one of knowing how to set the "optimal" value for ε.

To provide a rationale for determining the best value of ε, we concentrate on the Adachi's neural model [2] defined by:

$$y(t+1) = ky(t) - \alpha f(y(t)) + a, \tag{3}$$

where $f(\cdot)$ is a continuous differentiable function, which as per Adachi *et al* [2], is the Logistic function.

The properties of Equation (3) greatly depend on the parameters k, α, a and $f(\cdot)$. In order to obtain the full spectrum of the properties represented by Equation (3), it is beneficial for us to first consider $f(\cdot)$ in terms of a unit step function, and to work with a fixed point analysis. In [13] this analysis has been in great detail for the case of: (a) a single fixed point, (b) period-2 fixed points, and (c) period-n orbits. By a lengthy argument, we have explained *how* the parameter ε should be set.

In our experiment, indeed, if the parameters are set to be $\alpha = 1$ and $k = 0.5$, the "tipping point" for ε is $1/6 \approx 0.1667$. As shown in [13] if $\varepsilon = 0.18 > 0.1667$, all of the fixed points are stable. Otherwise, if $\varepsilon = 0.15 < 0.1667$, there exist period-doubling bifurcations As ε is further decreased, one can observe chaotic windows.

We conclude this section by emphasizing that ε cannot be too small, for if it were, the Adachi neural model would degrade to the Nagumo-Sato model, which does not demonstrate any chaotic behavior. This is also clearly demonstrated in the figures shown in [13].

Our arguments show that the value of ε as set in [8] to be $\varepsilon = 0.00015$, is not appropriate. Rather, to develop the Ideal-M-AdNN, , we have opted to use a value of ε which is *two* orders of magnitude larger, i.e., $\varepsilon = 0.015$.

3 Lyapunov Exponents Analysis of the Ideal-M-AdNN

We shall do a Lyapunov Exponents (LE) analysis of the Ideal-M-AdNN, both from the perspective of a *single* neuron and of the network in its entirety.

As is well known, the LEs describe the behavior of a dynamical system. There are many ways, both numerically and analytically, to compute the LEs of a dynamical system. Generally, for systems with a small dimension, the best way is to analytically compute it using its formal definition. As opposed to this, for systems with a high dimension, it is usually not easy to obtain the entire LE spectrum in an analytic manner. In this case, we have several other alternatives to judge whether a system is chaotic. One of these is to merely determine the largest LE (instead of computing the entire spectrum) since the existence of a single positive LE indicates chaotic behavior. Algorithmically, the basic idea is to follow two orbits that are close to each other, and to calculate their average logarithmic rate of separation [9,16].

In practice, this algorithm is both simple and convenient if we have the right to access the equations that govern the system. Furthermore, if it is easy to obtain the partial derivatives of the system, we can also calculate the LE spectrum by QR decomposition [9,14,16].

The unabridged version of the paper [13] also contains a detailed analysis for obtaining the LE spectrum using the QR decomposition. It is omitted here due to the space limitations.

We have also, in [13], undertaken a Lyapunov Analysis of the Ideal-M-AdNN. Indeed, it can be easily proven that a single neuron is chaotic when the parameters are properly set.

Also, for the Ideal-M-AdNN (i.e., the entire network), we can show [13] that the Lyapunov Exponents are: $\lambda_1 = \cdots \lambda_{N-1} = -\infty$, $\lambda_N = \log N + \log k_f > 0$, $\lambda_{N+1} = \cdots \lambda_{2N-1} = -\infty$, $\lambda_{2N} = \log N + \log k_r > 0$. In conclusion, the Ideal-M-AdNN has two positive LEs, which indicates that the network is truly a chaotic network!

It's very interesting to compare this result with the one presented for the AdNN. Indeed, as we can see from [6], the AdNN has two different LEs: $\log k_f$ and $\log k_r$. The difference is that by binding the states of all the neurons to a single "*global*" neuron, we force the Ideal-M-AdNN to have two positive LEs. The LE spectrum of the two networks are compared in [13].

4 Chaotic and PR Properties of the Ideal-M-AdNN

We shall now report the properties of the Ideal-M-AdNN. These properties have been gleaned as a result of examining the Hamming distance between the input pattern and the patterns that appear at the output. In this regard, we mention that the experiments were conducted using two data sets, namely the figures used by Adachi *et al* given in Figure 1 (a), and the numeral data sets used by Calitoiu *et al* [7,8] give in Figure 1 (b). In both the cases, the patterns were described by 10×10 pixel images, and the networks thus had 100 neurons.

Before we proceed, we remark that although the experiments were conducted for a variety of scenarios, in the interest of brevity, we present here only a few typical sets of results – essentially, to catalogue the overall conclusions of the investigation.

We discuss the properties of the Ideal-M-AdNN in three different settings. In all of the three cases, the parameters were set to be $k_f = 0.2$, $k_r = 0.9$, and $\varepsilon = 0.015$, and all the internal states, $\eta_i(0)$ and $\xi_i(0)$, start from 0.

AM Properties: We now examine whether the Ideal-M-AdNN possesses any AM-related properties for certain scenarios, i.e., if we fix the external input $a_i = 2$ for all neurons. The observation that we report is that during the first 1,000 iterations (due to the limitations of the file size, we present here only the

(*a*) (*b*)

Fig. 1. The 10×10 patterns used by Adachi *et al* (on the left) and Calitoiu *et al* (on the right). In both figures (a) and (b), the first four patterns are used to train the network. The fifth patterns are obtained from the corresponding fourth patterns by including 15% noise in (a) and (b) respectively. In each case, the sixth patterns are the untrained patterns.

first 36 images), the network only repeats black and white images. This can be seen in Figure 2. The reason for this phenomenon is explained in detail in [13].

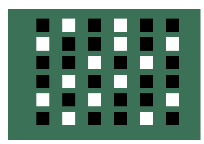
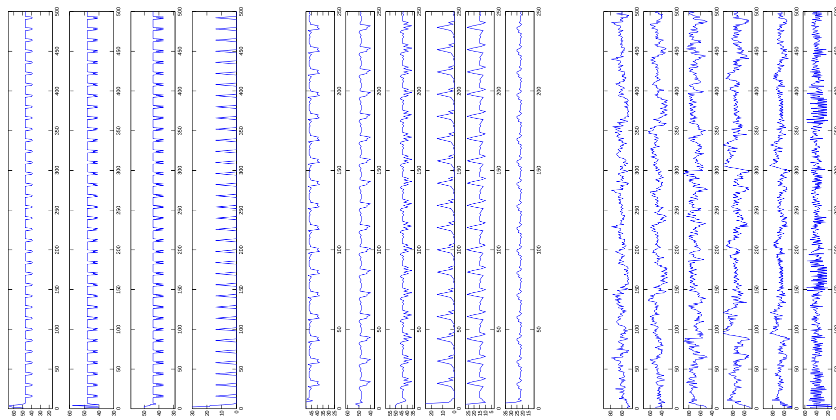


Fig. 2. The visualization of the output of the Ideal-M-AdNN under the external input $a_i = 2$. We see that the output switches between images which are entirely only black or only white.

PR Properties: The PR properties of the Ideal-M-AdNN are the main concern of this paper. As illustrated in Section 2, the goal of a chaotic PR system is the following: The system should respond periodically to trained input patterns, while it should respond chaotically (with chaotic outputs) to untrained input patterns. We now confirm that the Ideal-M-AdNN does, indeed, possess such phenomena. We present an in-depth report of the PR properties of the Ideal-M-AdNN's by using a Hamming distance-based analysis. The parameters that we used were: $k_f = 0.2$, $k_r = 0.9$, $\varepsilon = 0.015$ and $a_i = 2 + 6x_i$. The PR-related results of the Ideal-M-AdNN are reported for the three scenarios, i.e., for trained inputs, for noisy inputs, and for untrained (unknown) inputs respectively.

1. The external input of the network corresponds to a known pattern, P4.
   To report the results for this scenario, we request the reader to observe Figure 3 (a), where we can find that P4 is retrieved periodically as a response to the input pattern. This occurs 391 times in the first 500 iterations. On the other hand, the other three patterns never appear in the output sequence. The phase diagrams of the internal states that correspond to Figure 3 (a) are shown in [13], whence we verify that the periodicity is 14, because all the phase plots have exactly 14 points.
2. The external input of the network corresponds to a noisy pattern, in this case P5, which is a noisy version of P4.
   Even when the external stimulus is a garbled version of a known pattern (in this case P5 which contains 15% noise), it is interesting to see that *only* the original pattern P4 is recalled periodically. In contrast, the others



(*a*) (*b*) (*c*)

Fig. 3. PR properties: The Hamming distance between the output and the trained patterns. The input was the pattern P4, P5 and P6 in (*a*), (*b*) and (*c*) respectively. Note that P4 appears periodically in Case (*a*). Also, note that P4 (not P5) appears periodically in Case (*b*). Finally, note that *none* of the trained patterns appear at the output in Case (*c*).

three known patterns are *never* recalled. This phenomenon can be seen from the Figure 3 (*b*). By comparing Figures 3 (*a*) and (*b*), we can draw the conclusion that the Ideal-M-AdNN can achieve chaotic PR even in the presence of noise and distortion. As in the previous case, the phase diagrams of the internal states that correspond to Figure 3 (*a*) are shown in [13], whence we again verify that the periodicity is 14, because all the phase plots have exactly 14 points. Indeed, even if the external stimulus contains some noise, the Ideal-M-AdNN is still able to recognize it correctly, by resonating periodically!

3. The external input corresponds to an unknown pattern, P6.

   In this case we investigate whether the Ideal-M-AdNN is capable of distinguishing between known and unknown patterns. Thus, we attempt to stimulate the network with a completely unknown pattern. In our experiments, we used the pattern P6 of Figure 1 (a) initially used by Adachi *et al*. From Figure 3 (*c*) we see that neither those known patterns nor the presented unknown pattern appear at the output. As in the previous two cases, the phase diagrams of the internal states that correspond to Figure 3 (*c*) are shown in [13], whence, the lack of periodicity can be observed since the plots themselves are dense.

   In other words, the Ideal-M-AdNN responds intelligently to the various inputs with correspondingly different outputs, each resonating with the input that excites it – which is the crucial golden hallmark characteristic of a *Chaotic* PR system. Indeed, the switch between "order" (resonance) and "disorder" (chaos) seems to be consistent with Freeman's biological results – which, we believe, is quite fascinating!

5 Conclusions

In this paper we have concentrated on the field of Chaotic Pattern Recognition (PR), which is a relatively new sub-field of PR. Such systems, which have only recently been investigated, demonstrate chaotic behavior under normal conditions, and "resonate" (i.e., by presenting at the output a specific pattern frequently) when it is presented with a pattern that it is trained with. This ambitious goal, with the requirement of switching from chaos to periodicity is, indeed, most demanding, and has been achieved by the design of the so-called Ideal-M-AdNN.

Using a rigorous Lyapunov analysis, we have shown the chaotic properties of the Ideal-M-AdNN, and demonstrated its chaotic characteristics. We have also verified that the system is truly chaotic for untrained patterns. But most importantly, we have shown that it is able to *switch to being periodic* whenever it encounters patterns with which it was trained (or noisy versions of the latter).

Apart from being quite fascinating, as far as we know, the theoretical and experimental results presented here are both unreported and novel.

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