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# Modelling of Multiphase Flow as Random Process

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**Abstract:** Basic equation array of differential equations was proposed for a multiphase flow considering the probability of each phase in a flow. The main analysis focused on a two-phase flow. The conservation equations for the mass, momentum and energy were obtained under the assumption that all parameters of the interacting phases play the statistical process. In parallel, dynamical system by the Kolmogorov's theorem for two states of a statistical system (phases of two-phase mixture) was considered. Probability of phases was taken further for comparison with the probability and parameters of a two-phase flow from the equations of flow dynamics. Analysis of the parameters was performed for the available flow regimes and on the condition that probability varies from 0 to 1. Correspondence of parameters by the equation array for flow dynamics and by solution of the dynamical system of two interacting statistical states revealed the values of coefficients for dynamical system, expressed in terms of the flow parameters. The results obtained were stated for further discussion and comparison with the experimental data and with results of other researchers of multiphase flows.
**Keywords:** Two-Phase, Flow, Mixing, Kolmogorov Theorem, Chaotic, Modeling.

## 1 Introduction

The mathematical models of continua are most often based on the continuity hypotheses of medium and continuous *n*-times differentiability of all functions almost everywhere except for separate points, lines or surfaces, where some discontinuities are allowed. Then phenomenological approach and classic calculus are applied. But many processes in continua don't satisfy this physical model, e.g. in turbulent flow the velocity acceleration is not in a class of continuous or nearly everywhere continuous functions; and the film flow compressed in the direction of a tangent to a free surface is nowhere differentiable in this direction even if remains continuous (a surface saw-tooth with teeth, perpendicular to it). The other examples are: spraying and cavitation, where individualization is impossible even for volumes since the continuum turns into a set of free points.

In heterogeneous media, the fields of velocities, temperatures, etc. are fractured and the combination of two various fields in one continuum, a polysemy of parameters belong not to the individualized point of a medium but to a spatial point in which the individualized points of the various parameters are combined in. Statistical approach and variation methods [1, 2] are applied to the non-

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classical problems of the continuum mechanics, which aren't satisfying to hypotheses of the phenomenological theory. Due to mathematical complexity, the statistical (microscopic) approach is most often used for justification of the phenomenological (macroscopic) models of continua if only it isn't unique: the discharged gases, plasma, etc. Strictly speaking, in nature there are no real continua, however, the continuity hypothesis describes them well at the macro level and the model of continua allows using the powerful differential calculus. For the systems, which do not answer continuity hypothesis of an occupied space, the fractal theory [3] and the integro-differential calculus of arbitrary order [4] are more adequate. Here we use phenomenological approach at which creation of mathematical models of continua is based on assumption that each point of the medium (physically infinitesimal volume) is characterized by a set of the defining parameters introduced on the basis of experimental data and theoretical investigations. Such set of mathematical models for the various tasks have been developed taking into account their specific features that a need ripened for their systematization and development the basic principles of mathematical modeling of processes in continua.

General equations of the dynamics of continua by any structure (including the heterogeneous mix considered without phase interaction, which can be not taken into account when studying the movement of the heterogeneous system as a uniform complex continuous medium), may be represented in the form:

$$\partial\rho / \partial t = -div\left(\rho\vec{v}\right), \ \rho\left(\partial\vec{v} / \partial t + \vec{v}\nabla\vec{v}\right) = divP + \rho\vec{F} - \sum_{j=1}^{N} \vec{v}\nabla\left(\rho_j \vec{v}_j\right), \qquad (1),$$

(2)

$$\rho\left(\partial e / \partial t + \vec{v}\nabla e\right) = div\left(\vec{q} + P\vec{v}\right) + \rho\vec{F}\vec{v} + \sum_{j=1}^{N}\left[\rho_j \vec{F}_j \vec{v} - div\left(\rho_j e_j \vec{v}\right)\right], \qquad (3)$$

where $\rho$ - the density of the heterogeneous medium, $\vec{v}$ - velocity vector, *t*- time, *P*- stress tensor, $\vec{F}$ - volumetric force, $\rho_j, \vec{v}_j$ - parameters of the components, *e*-specific density of energy, *q*- specific volumetric energy influx. There are the mass, momentum and energy conservation equations. For reversible processes, the uncompensated warmth is zero. Except for internal energy and entropy, the other functions of state and additional thermodynamic relations are used. In a heterogeneous medium, the exchange of mass, momentum and energy between phases inside the volume must be accounted in the heterogeneous mix in (1)-(3). This is the main problem because it is indefinite in the most cases.

## 2 Probability Model for the Multiphase Flows

Each phase is present in given point of continuum with a probability $P_i$. The interaction of the phases is treated according to the theorem of Kolmogorov [5]. Locally, interaction of the phases 1 and 2 can be considered according to Fig. 1:

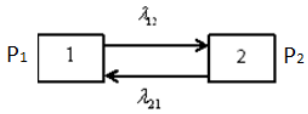


Fig. 1. Interacting phases as the Markov process

Here $\lambda_{ij}$ is the transition intensity from the state $i$ to the state $j$. If a few phases are interacting, they are represented as the actors of the Markov process. The intensities of the phases' interaction are determined by the physical properties and the flow regime. According to the Kolmogorov theorem:

$$\frac{dP_i}{dt} = -\sum_{j=1}^{m} \lambda_{ij} P_i + \sum_{j=1}^{m} \lambda_{ji} P_j , \qquad \sum_{i=1}^{m} P_i = 1 \cdot \tag{4}$$

In case (1)-(3) for the turbulent multiphase-phase flow, the function-indicator can be treated as a mathematical expectation. Then solution of the equations (4) averaged by time on the time interval [0,T] results in the functions-indicators of the phases. In case of two phases, (4) with the corresponding initial conditions is

$$\frac{dP_1}{dt} = -\lambda_{12} P_1 + \lambda_{21} P_2 , \ \frac{dP_2}{dt} = -\lambda_{21} P_2 + \lambda_{12} P_1 , \ \ P_1 + P_2 = 1 ; \ t = 0 , \ P_1 = 1 , \ P_2 = 0 . \tag{5}$$

Any other starting point for the phases can be considered as well. Equation array (5) contains three equations for two functions, thus, it is over defined. A system of any two equations can be solved and then the solution obtained is put into the third one. The solution of the Cauchy problem (5) is

$$P_1 = \left[ \lambda_{12} e^{-(\lambda_{12}+\lambda_{21})t} + \lambda_{21} \right] / \left( \lambda_{12} + \lambda_{21} \right) . \tag{6}$$

After averaging by the time it yields

$$P_1 = \frac{\lambda_{21}}{\lambda_{12} + \lambda_{21}} + \frac{\lambda_{12}}{\lambda_{12} + \lambda_{21}} \cdot \frac{1 - e^{-(\lambda_{12}+\lambda_{21})T}}{\left( \lambda_{12} + \lambda_{21} \right) T} \cdot \tag{7}$$

With running of time it yields the following simpler expression

$$P_1 = \frac{\lambda_{21}}{\lambda_{12} + \lambda_{21}} = \frac{1}{\lambda_{12} / \lambda_{21} + 1} = \frac{1}{\gamma + 1} , \ \frac{dP_1}{dz} = -\frac{1}{\left( \gamma + 1 \right)^2} \frac{d\gamma}{dz} , \ \gamma = \lambda_{12} / \lambda_{21} .$$

The function-indicator $B_1$ depends on the interval of averaging as far as the phases are interacting. By long time interval $P_1 = \lambda_{21} / \left(\lambda_{12} + \lambda_{21}\right)$, so that it is determined by intensities of the phases' interaction. These coefficients differ at each point of the flow, therefore the function-indicator changes from point to point. For small time interval, $B_1$=1 with accuracy to the linear terms by $T$. Each phase occupies some part of elementary volume of the heterogeneous medium: the volumetric contents of $N$ phases satisfy $\sum_{j=1}^{N} \alpha_j = 1$, the density of the medium is expressed through the real densities of the phases by $\rho = \sum_{j=1}^{N} \rho_j \alpha_j$. At each point of the heterogeneous medium, $N$ parameters are defined for the continuum (densities, velocities, temperatures, etc.). The set of continuums, each of which corresponds to its phase and fills the same volume, is called the multi-speed continuum. Such approach is applied in the most multiphase methods, except a few of them like the one by A.I. Nakorchevski [6].

Using the above approach, with introduction of the probability of the phases at each point of a flow, all characteristics of the mixture $a^l(t)$ in multiphase flow are expressed through the corresponding characteristics of the phases $a_i^l(t)$:

$$a^l(t) = \sum_{i=1}^{m} P_i(t) a_i^l(t), \tag{8}$$

where $P_i(t)$ was introduced as the above-stated and is connected with the so-called function-indicator $B_i(t)$ [6]. Summation of the equations of the mix by all phases gives the equations of the heterogeneous mixture, without account of internal structure. Such model doesn't show features of the interfacial interaction. Contrary, accounting interaction of the phases' macroscopic inclusions results in the conditions of joint deformation, movement of phases, influences of form and number of inclusions, their distributions in space, phase transformations, etc.

If physic-mechanical processes in the continua are rather precisely described by continuous or nearly everywhere continuous functions of coordinates and time, it is possible to replace the system of integral conservation equations with the corresponding differential equations. However, for the real continua prone to the external influences, the classical methods can be unacceptable owing to what the variation and numerical methods based on the use of the integral correlations are remaining useful in case of fractured fields and media if integration by Riemann is replaced by integration by Lebesgue. In a region of continuous or nearly everywhere continuous flow, the integral equations are transformed to the differential equation array taking into account the joint movement of phases and the interfacial mass, momentum, and energy exchange, using the Gauss-Ostrogradsky's formula. The main obstacle by mathematical modeling of the

heterogeneous media is caused by the need of a specification of the phases' interactions that is extremely difficult.
The features of heterogeneous medium depend not only on the velocity fields, pressure, and temperature of phases; therefore, the determination of regularities of interfacial interaction even for special cases is a complex challenge. And still, the accounting of fields' ruptures on the boundary interfaces is absolutely necessary for some important tasks. For a weak manifestation of the interfacial interaction, the differential equation array obtained from the conservation equations by each component of heterogeneous mix through the summation on all mix may be used. But remarkably the balance equations depend on the relative movement of phases inside the heterogeneous mix. From (8), for the two-phase flow is got

$$\begin{aligned} a^l(t) &= P_1(t)a_1^l(t) + P_2(t)a_2^l(t) = P_1(t)a_1^l(t) + (1-P_1(t))a_2^l(t) = \\ &= a_2^l(t) + P_1(t)(a_1^l(t) - a_2^l(t)) \,. \end{aligned} \tag{9}$$

## 3 The equation array for two-phase flow

The equation array for two-phase incompressible flow of immiscible liquids without the twisting $\left(\partial\varphi / \partial r = 0\right)$ in the cylindrical coordinate system $(r, \varphi, z)$ is

$$\begin{aligned}
&\frac{\partial}{\partial r}\left\{\left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right]\left[u_2 + P_1\left(u_1-u_2\right)\right]\right\} + \frac{1}{r}\left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right]\left[u_2 + P_1\left(u_1-u_2\right)\right] + \\
&\quad + \frac{\partial}{\partial z}\left\{\left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right]\left[w_2 + P_1\left(w_1-w_2\right)\right]\right\} + \frac{\partial}{\partial t}\left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right] = 0 \,, \\
&\left\{\frac{\partial}{\partial t}\left[u_2 + P_1\left(u_1-u_2\right)\right] + \frac{1}{2}\frac{\partial}{\partial r}\left[u_2 + P_1\left(u_1-u_2\right)\right]^2 + \left[w_2 + P_1\left(w_1-w_2\right)\right]\frac{\partial}{\partial z}\left[u_2 + P_1\left(u_1-u_2\right)\right]\right\}\cdot \\
&\cdot\left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right] + \frac{\partial}{\partial r}\left[p_2 + P_1\left(p_1-p_2\right)\right] = \frac{\partial}{\partial r}\left\{\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial}{\partial r}\left[u_2 + P_1\left(u_1-u_2\right)\right]\right\} + \\
&+ \frac{\partial}{\partial z}\left\{\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial}{\partial z}\left[u_2 + P_1\left(u_1-u_2\right)\right]\right\} - \frac{1}{r^2}\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\left[u_2 + P_1\left(u_1-u_2\right)\right] + \\
&\quad + \frac{\partial}{r\partial r}\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\left[u_2 + P_1\left(u_1-u_2\right)\right] , \\
&\left\{\frac{\partial}{\partial t}\left[w_2 + P_1\left(w_1-w_2\right)\right] + \frac{1}{2}\frac{\partial}{\partial r}\left[w_2 + P_1\left(w_1-w_2\right)\right]^2 + \left[u_2 + P_1\left(u_1-u_2\right)\right]\frac{\partial}{\partial r}\left[w_2 + P_1\left(w_1-w_2\right)\right]\right\}\cdot \\
&\cdot\left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right] + \frac{\partial}{\partial z}\left[p_2 + P_1\left(p_1-p_2\right)\right] = \frac{\partial}{\partial r}\left\{\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial}{\partial r}\left[w_2 + P_1\left(w_1-w_2\right)\right]\right\} + \\
&+ \frac{\partial}{\partial z}\left\{\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial}{\partial z}\left[w_2 + P_1\left(w_1-w_2\right)\right]\right\} + \frac{\partial}{r\partial r}\left\{\left[\mu_2 + P_1\left(\mu_1-\mu_2\right)\right]\left[w_2 + P_1\left(w_1-w_2\right)\right]\right\} + \\
&\quad + \left[\rho_2 + P_1\left(\rho_1-\rho_2\right)\right] g \,,
\end{aligned} \tag{10}$$

$$\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]\left[c_{V2}+P_1\left(c_{V1}-c_{V2}\right)\right]\left\{\frac{\partial}{\partial t}\left[T_2+P_1\left(T_1-T_2\right)\right]+\left[u_2+P_1\left(u_1-u_2\right)\right]\cdot\right.$$

$$\left.\cdot\frac{\partial}{\partial r}\left[T_2+P_1\left(T_1-T_2\right)\right]+\left[w_2+P_1\left(w_1-w_2\right)\right]\frac{\partial}{\partial z}\left[T_2+P_1\left(T_1-T_2\right)\right]\right\}+\left[p_2+P_1\left(p_1-p_2\right)\right]\cdot$$

$$\cdot\left\{\frac{\partial}{\partial r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{1}{r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{\partial}{\partial z}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}=$$

$$=\frac{\partial}{\partial r}\left\{\left[\kappa_2+P_1\left(\kappa_1-\kappa_2\right)\right]\frac{\partial}{\partial r}\left[T_2+P_1\left(T_1-T_2\right)\right]\right\}+\frac{\partial}{r\partial r}\left\{\left[\kappa_2+P_1\left(\kappa_1-\kappa_2\right)\right]\left[T_2+P_1\left(T_1-T_2\right)\right]\right\}+$$

$$+\frac{\partial}{\partial z}\left\{\left[\kappa_2+P_1\left(\kappa_1-\kappa_2\right)\right]\frac{\partial}{\partial z}\left[T_2+P_1\left(T_1-T_2\right)\right]\right\}+\frac{1}{2}\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\left\{\frac{\partial}{\partial z}\left[u_2+P_1\left(u_1-u_2\right)\right]+\right.$$

$$\left.+\frac{\partial}{\partial r}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}^2,$$

where $r$ is the radial coordinate, $z$ is directed along the axis, $u$, $w$ are the corresponding velocity components, $p$ and $T$ are pressure and temperature; $c,\kappa,\mu$ are the heat capacity, heat conductivity, and dynamic viscosity coefficients, respectively. Here the gravitational force is directed along the coordinate $z$, so that to act in the same direction as the momentum of a flow. If the flow is going vertically up, the sign of $g$ is negative (gravity acceleration).
Accounting (6) for probabilities of the phases, from (10) follows for the constant densities of the phases and physical properties:

$$\left[u_2+P_1\left(u_1-u_2\right)\right]\left(\rho_1-\rho_2\right)\frac{\partial P_1}{\partial r}+\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]\left\{\frac{\partial}{\partial r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{u_2+P_1\left(u_1-u_2\right)}{r}\right\}+$$

$$+\left[w_2+P_1\left(w_1-w_2\right)\right]\left(\rho_1-\rho_2\right)\frac{\partial P_1}{\partial z}+\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]\frac{\partial}{\partial z}\left[w_2+P_1\left(w_1-w_2\right)\right]+\left(\rho_1-\rho_2\right)\frac{\partial P_1}{\partial t}=0,$$

$$\left\{\frac{\partial}{\partial t}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{1}{2}\frac{\partial}{\partial r}\left[u_2+P_1\left(u_1-u_2\right)\right]^2+\left[w_2+P_1\left(w_1-w_2\right)\right]\frac{\partial}{\partial z}\left[u_2+P_1\left(u_1-u_2\right)\right]\right\}\cdot$$

$$\cdot\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]+\frac{\partial}{\partial r}\left[p_2+P_1\left(p_1-p_2\right)\right]=\left(\mu_1-\mu_2\right)\frac{\partial P_1}{\partial r}\frac{\partial}{\partial r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\cdot$$

$$\cdot\frac{\partial^2}{\partial r^2}\left[u_2+P_1\left(u_1-u_2\right)\right]$$

$$+\left(\mu_1-\mu_2\right)\frac{\partial P_1}{\partial z}\frac{\partial}{\partial z}\left[u_2+P_1\left(u_1-u_2\right)\right]-\frac{\mu_2+P_1\left(\mu_1-\mu_2\right)}{r^2}\left[u_2+P_1\left(u_1-u_2\right)\right]+$$

$$+\frac{\left(\mu_1-\mu_2\right)}{r}\left[u_2+P_1\left(u_1-u_2\right)\right]\frac{\partial P_1}{\partial r}+\frac{\left(\mu_1-\mu_2\right)}{r}\left[u_2+P_1\left(u_1-u_2\right)\right]\frac{\partial P_1}{\partial r}+$$

$$+\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial^2}{\partial z^2}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{1}{r}\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial}{\partial r}\left[u_2+P_1\left(u_1-u_2\right)\right],$$

$$\left\{\frac{\partial}{\partial t}\left[w_2+P_1\left(w_1-w_2\right)\right]+\frac{1}{2}\frac{\partial}{\partial r}\left[w_2+P_1\left(w_1-w_2\right)\right]^2+\left[u_2+P_1\left(u_1-u_2\right)\right]\frac{\partial}{\partial r}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}\cdot$$

$$\cdot\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]+\frac{\partial}{\partial z}\left[p_2+P_1\left(p_1-p_2\right)\right]=\left(\mu_1-\mu_2\right)\frac{\partial P_1}{\partial r}\frac{\partial}{\partial r}\left[w_2+P_1\left(w_1-w_2\right)\right]+$$

$$+\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial^2}{\partial r^2}\left[w_2+P_1\left(w_1-w_2\right)\right]+\left(\mu_1-\mu_2\right)\frac{\partial P_1}{\partial z}\frac{\partial}{\partial z}\left[w_2+P_1\left(w_1-w_2\right)\right]+$$

$$+\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\frac{\partial^2}{\partial z^2}\left[w_2+P_1\left(w_1-w_2\right)\right]+\frac{\partial}{r\partial r}\left\{\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}+$$

$$+\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]g\,, \tag{11}$$

$$\left[\rho_2+P_1\left(\rho_1-\rho_2\right)\right]\left[c_{V2}+P_1\left(c_{V1}-c_{V2}\right)\right]\left\{\frac{\partial}{\partial t}\left[T_2+P_1\left(T_1-T_2\right)\right]+\left[u_2+P_1\left(u_1-u_2\right)\right]\cdot\right.$$

$$\left.\cdot\frac{\partial}{\partial r}\left[T_2+P_1\left(T_1-T_2\right)\right]+\left[w_2+P_1\left(w_1-w_2\right)\right]\frac{\partial}{\partial z}\left[T_2+P_1\left(T_1-T_2\right)\right]\right\}+\left[p_2+P_1\left(p_1-p_2\right)\right]\cdot$$

$$\cdot\left\{\frac{\partial}{\partial r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{1}{r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{\partial}{\partial z}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}=$$

$$=\left(\kappa_1-\kappa_2\right)\frac{\partial P_1}{\partial r}\frac{\partial}{\partial r}\left[T_2+P_1\left(T_1-T_2\right)\right]+\left[\kappa_2+P_1\left(\kappa_1-\kappa_2\right)\right]\frac{\partial^2}{\partial r^2}\left[T_2+P_1\left(T_1-T_2\right)\right]+$$

$$+\frac{\kappa_1-\kappa_2}{r}\left[T_2+P_1\left(T_1-T_2\right)\right]\frac{\partial P_1}{\partial r}+\frac{\kappa_2+P_1\left(\kappa_1-\kappa_2\right)}{r}\frac{\partial}{\partial r}\left[T_2+P_1\left(T_1-T_2\right)\right]+$$

$$+\left(\kappa_1-\kappa_2\right)\frac{\partial}{\partial z}\left[T_2+P_1\left(T_1-T_2\right)\right]\frac{\partial P_1}{\partial z}+\left[\kappa_2+P_1\left(\kappa_1-\kappa_2\right)\right]\frac{\partial^2}{\partial z^2}\left[T_2+P_1\left(T_1-T_2\right)\right]+$$

$$+\frac{1}{2}\left[\mu_2+P_1\left(\mu_1-\mu_2\right)\right]\cdot\left\{\frac{\partial}{\partial z}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{\partial}{\partial r}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}^2.$$

The simplest 1-D flow is described according to (11) with the following system ($\partial/\partial r=0$, the channel is narrow and characteristics are mainly depending on $z$):

$$\frac{u_2+P_1\left(u_1-u_2\right)}{r}+\frac{w_2+P_1\left(w_1-w_2\right)}{\rho_2+P_1\left(\rho_1-\rho_2\right)}\left(\rho_1-\rho_2\right)\frac{\partial P_1}{\partial z}+\frac{\partial}{\partial z}\left[w_2+P_1\left(w_1-w_2\right)\right]+\frac{\left(\rho_1-\rho_2\right)}{\rho_2+P_1\left(\rho_1-\rho_2\right)}\frac{\partial P_1}{\partial t}=0\,,$$

$$\frac{\partial^2}{\partial z^2}\left[u_2+P_1\left(u_1-u_2\right)\right]+\left\{\frac{\mu_1-\mu_2}{\mu_2+P_1\left(\mu_1-\mu_2\right)}\frac{\partial P_1}{\partial z}-\frac{\rho_2+P_1\left(\rho_1-\rho_2\right)}{\mu_2+P_1\left(\mu_1-\mu_2\right)}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}\cdot$$

$$\cdot\frac{\partial}{\partial z}\left[u_2+P_1\left(u_1-u_2\right)\right]=\frac{u_2+P_1\left(u_1-u_2\right)}{r^2}+\frac{\rho_2+P_1\left(\rho_1-\rho_2\right)}{\mu_2+P_1\left(\mu_1-\mu_2\right)}\frac{\partial}{\partial t}\left[u_2+P_1\left(u_1-u_2\right)\right],$$

$$\frac{\partial^2}{\partial z^2}\left[w_2+P_1\left(w_1-w_2\right)\right]+\left\{\frac{\mu_1-\mu_2}{\mu_2+P_1\left(\mu_1-\mu_2\right)}\frac{\partial P_1}{\partial z}\right\}\frac{\partial}{\partial z}\left[w_2+P_1\left(w_1-w_2\right)\right]= \tag{12}$$

$$=\frac{\rho_2+P_1\left(\rho_1-\rho_2\right)}{\mu_2+P_1\left(\mu_1-\mu_2\right)}\frac{\partial}{\partial t}\left[w_2+P_1\left(w_1-w_2\right)\right]+\frac{1}{\mu_2+P_1\left(\mu_1-\mu_2\right)}\frac{\partial}{\partial z}\left[p_2+P_1\left(p_1-p_2\right)\right]-\frac{\rho_2+P_1\left(\rho_1-\rho_2\right)}{\mu_2+P_1\left(\mu_1-\mu_2\right)}g\,,$$

$$\left[c_{V2}+P_1\left(c_{V1}-c_{V2}\right)\right]\left\{\frac{\partial}{\partial t}\left[T_2+P_1\left(T_1-T_2\right)\right]+\left[w_2+P_1\left(w_1-w_2\right)\right]\frac{\partial}{\partial z}\left[T_2+P_1\left(T_1-T_2\right)\right]\right\}+$$

$$+\frac{p_2+P_1(p_1-p_2)}{\rho_2+P_1(\rho_1-\rho_2)}\left\{\frac{1}{r}\left[u_2+P_1(u_1-u_2)\right]+\frac{\partial}{\partial z}\left[w_2+P_1(w_1-w_2)\right]\right\}=\frac{\kappa_2+P_1(\kappa_1-\kappa_2)}{\rho_2+P_1(\rho_1-\rho_2)}\frac{\partial^2}{\partial z^2}\left[T_2+P_1(T_1-T_2)\right]+$$

$$+\frac{\kappa_1-\kappa_2}{\rho_2+P_1(\rho_1-\rho_2)}\frac{\partial}{\partial z}\left[T_2+P_1(T_1-T_2)\right]\frac{\partial P_1}{\partial z}+\frac{\mu_2+P_1(\mu_1-\mu_2)}{2\left[\rho_2+P_1(\rho_1-\rho_2)\right]}\left\{\frac{\partial}{\partial z}\left[u_2+P_1(u_1-u_2)\right]\right\}^2.$$

The stationary regime of the flow described by the equation array (12) is

$$\frac{d^2}{dz^2}\left[u_2+P_1(u_1-u_2)\right]+\left\{\frac{d}{dz}\ln\left[\mu_2+P_1(\mu_1-\mu_2)\right]-\frac{\rho_2+P_1(\rho_1-\rho_2)}{\mu_2+P_1(\mu_1-\mu_2)}\left[w_2+P_1(w_1-w_2)\right]\right\}\cdot$$

$$\cdot\frac{d}{dz}\left[u_2+P_1(u_1-u_2)\right]=\frac{u_2+P_1(u_1-u_2)}{r^2},$$

$$\frac{u_2+P_1(u_1-u_2)}{r}+\frac{w_2+P_1(w_1-w_2)}{\rho_2+P_1(\rho_1-\rho_2)}(\rho_1-\rho_2)\frac{dP_1}{dz}++\frac{d}{dz}\left[w_2+P_1(w_1-w_2)\right]=0,$$

$$\frac{d^2}{dz^2}\left[w_2+P_1(w_1-w_2)\right]+\left\{\frac{\mu_1-\mu_2}{\mu_2+P_1(\mu_1-\mu_2)}\frac{dP_1}{dz}\right\}\frac{d}{dz}\left[w_2+P_1(w_1-w_2)\right]=$$

$$=\frac{1}{\mu_2+P_1(\mu_1-\mu_2)}\frac{d}{dz}\left[p_2+P_1(p_1-p_2)\right]-\frac{\rho_2+P_1(\rho_1-\rho_2)}{\mu_2+P_1(\mu_1-\mu_2)}g,$$

$$\left[c_{V2}+P_1(c_{V1}-c_{V2})\right]\left[w_2+P_1(w_1-w_2)\right]\cdot d\left[T_2+P_1(T_1-T_2)\right]/dz+$$

$$\frac{p_2+P_1(p_1-p_2)}{\rho_2+P_1(\rho_1-\rho_2)}\left\{\frac{1}{r}\left[u_2+P_1(u_1-u_2)\right]+\frac{d}{dz}\left[w_2+P_1(w_1-w_2)\right]\right\}=$$

$$=\frac{\kappa_1-\kappa_2}{\rho_2+P_1(\rho_1-\rho_2)}\frac{d}{dz}\left[T_2+P_1(T_1-T_2)\right]\frac{dP_1}{dz}+\frac{\kappa_2+P_1(\kappa_1-\kappa_2)}{\rho_2+P_1(\rho_1-\rho_2)}\cdot \qquad (14)$$

$$\cdot\frac{d^2}{dz^2}\left[T_2+P_1(T_1-T_2)\right]+\frac{\mu_2+P_1(\mu_1-\mu_2)}{2\left[\rho_2+P_1(\rho_1-\rho_2)\right]}\left\{\frac{d}{dz}\left[u_2+P_1(u_1-u_2)\right]\right\}^2.$$

## 4 Dimensionless form of the 1-D stationary two-phase flow

The equation array is presented in a dimensionless form for generalization of the model. The scales for the velocity, length, pressure and temperature are taken as: $u_0$, $R_0$, $p_0$, $T_0$ and for the dimensionless parameters the same assignments are kept as before for the dimensional ones. The dimensionless system is:

$$\left\{\frac{d}{dz}\ln\left[\mu_{21}+P_1(1-\mu_{21})\right]-\frac{\rho_{21}+P_1(1-\rho_{21})}{\mu_{21}+P_1(1-\mu_{21})}\left[w_2+P_1(w_1-w_2)\right]\mathrm{Re}\right\}\cdot$$

$$\cdot\frac{d}{dz}\left[u_2+P_1(u_1-u_2)\right]+\frac{d^2}{dz^2}\left[u_2+P_1(u_1-u_2)\right]=\frac{u_2+P_1(u_1-u_2)}{r^2},$$

$$\frac{w_2+P_1\left(w_1-w_2\right)}{\rho_{21}+P_1\left(1-\rho_{21}\right)}\left(1-\rho_{21}\right)\frac{dP_1}{dz}+\frac{d}{dz}\left[w_2+P_1\left(w_1-w_2\right)\right]+ +\frac{u_2+P_1\left(u_1-u_2\right)}{r}=0,$$

$$\frac{d^2}{dz^2}\left[w_2+P_1\left(w_1-w_2\right)\right]+\frac{d\ln\left[\mu_{21}+P_1\left(1-\mu_{21}\right)\right]}{dz}\frac{d}{dz}\left[w_2+P_1\left(w_1-w_2\right)\right]=$$

$$=\frac{Eu\cdot\mathrm{Re}}{\mu_{21}+P_1\left(1-\mu_{21}\right)}\frac{d}{dz}\left[p_2+P_1\left(p_1-p_2\right)\right]-\frac{\rho_{21}+P_1\left(1-\rho_{21}\right)}{\mu_{21}+P_1\left(1-\mu_{21}\right)}\frac{Ga}{\mathrm{Re}}, \qquad (15)$$

$$\left[c_{V21}+P_1\left(1-c_{V21}\right)\right]\left[w_2+P_1\left(w_1-w_2\right)\right]\frac{d}{dz}\left[T_2+P_1\left(T_1-T_2\right)\right]+c_{pv1}\frac{p_2+P_1\left(p_1-p_2\right)}{\rho_{21}+P_1\left(1-\rho_{21}\right)}\cdot Eu\cdot Ec\cdot$$

$$\cdot\left\{\frac{1}{r}\left[u_2+P_1\left(u_1-u_2\right)\right]+\frac{d}{dz}\left[w_2+P_1\left(w_1-w_2\right)\right]\right\}=\frac{\left(1-\kappa_{21}\right)c_{pv1}/Pe}{\rho_{21}+P_1\left(1-\rho_{21}\right)}\frac{d}{dz}\left[T_2+P_1\left(T_1-T_2\right)\right]\frac{dP_1}{dz}+$$

$$+\frac{\kappa_{21}+P_1\left(1-\kappa_{21}\right)}{\rho_{21}+P_1\left(1-\rho_{21}\right)}\frac{d^2}{dz^2}\left[T_2+P_1\left(T_1-T_2\right)\right]\frac{c_{pv1}}{Pe}+\frac{\left[\mu_{21}+P_1\left(1-\mu_{21}\right)\right]c_{pv1}}{2\left[\rho_{21}+P_1\left(1-\rho_{21}\right)\right]}\left\{\frac{d}{dz}\left[u_2+P_1\left(u_1-u_2\right)\right]\right\}^2\cdot\frac{Ec}{\mathrm{Re}}\cdot$$

Here are: $\nu_1=\mu_1/\rho_1$, $\kappa_{21}=\kappa_2/\kappa_1$, $\rho_{21}=\rho_2/\rho_1$, $\mu_{21}=\mu_2/\mu_1$, $a_1=\mu_1/\left(\rho_1 c_{p1}\right)$, $c_{pv1}=c_{p1}/c_{v1}$, $Ec=u_0^2/\left(c_{p1}T_0\right)$, $Ga=gR_0^3/\nu_1^2$, $Eu=p_0/\left(\rho_1 u_0^2\right)$, $\mathrm{Re}=u_0R_0/\nu_1$, $Pe=u_0R_0/a_1$. The Eckert number *Ec* plays an important role, representing the ratio of kinetic energy at the wall to the specific enthalpy of a fluid, the Galileo number is representing a ratio of the forces in a flow (gravitation*momentum to viscous forces), *Eu*– the Euler criterion (characteristic pressure $p_0$ to kinetic energy $\rho_1 u_0^2$), *Re*, *Pe* – the Reynolds and Peclet numbers, correspondingly.

System (15) contains 4 differential equations for 9 functions: $P_1$, $u_1$, $u_2$, $w_1$, $w_2$, $T_1$, $T_2$, $p_1$, $p_2$. Thus, the multiphase system is unclosed because it has no closing relations, which are the mass, momentum and energy interaction between the phases. This is the main obstacle in any multiphase system. It is not solved in general statement, only for specific cases through the structural approach. The system (15) is written for the heterogeneous mixture, which does not "feel" interaction of the phases. It can be used for the study of the multiphase system having some experimental data allowing the closing (15) for further its solution. The following methodology for solving this task is proposed. First the known temperature distribution for the mixture $\overline{T}=T_2+P_1\left(T_1-T_2\right)=\alpha_0+\alpha_1 z+\alpha_2 z^2$ is supposed. Then solution of (15) is sought for 4 functions: $\overline{w}=w_2+P_1\left(w_1-w_2\right)$, $\overline{u}=u_2+P_1\left(u_1-u_2\right)$, $\overline{p}=p_2+P_1\left(p_1-p_2\right)$, $P_1$. Only $P_1(z)$ shows a multiphase nature of the system. A few terms depend on *r*. As far as solution is assumed depending only on *z*, then only $P_1(r, z)$ is causing this. Thus, from (15) follows:

$$\left(\frac{d}{dz}\ln\overline{\mu}-\frac{\overline{\rho}}{\overline{\mu}}\overline{w}\mathrm{Re}\right)\frac{d\overline{u}}{dz}+\frac{d^2\overline{u}}{dz^2}=\frac{\overline{u}}{r^2}, \quad \frac{\overline{w}}{\overline{\rho}}\left(1-\rho_{21}\right)\frac{dP_1}{dz}+\frac{d\overline{w}}{dz}+\frac{\overline{u}}{r}=0,$$

$$\frac{d^2\overline{w}}{dz^2}+\frac{d\ln\overline{\mu}}{dz}\frac{d\overline{w}}{dz}=\frac{Eu\cdot\mathrm{Re}}{\overline{\mu}}\frac{d\overline{p}}{dz}-\frac{\overline{\rho}}{\overline{\mu}}\frac{Ga}{\mathrm{Re}},\quad \overline{\mu}=\mu_{21}+P_1\left(1-\mu_{21}\right),\tag{16}$$

$$\overline{c}_{21}\overline{w}\left(\alpha_1+2\alpha_2 z\right)+c_{pv1}\frac{\overline{p}}{\overline{\rho}}Eu\cdot Ec\left(\frac{\overline{u}}{r}+\frac{d\overline{w}}{dz}\right)=\frac{1-\kappa_{21}}{\overline{\rho}Pe}c_{pv1}\left(\alpha_1+2\alpha_2 z\right)\frac{dP_1}{dz}+2\alpha_2\frac{\overline{\kappa}}{\overline{\rho}}\frac{c_{pv1}}{Pe}+\frac{\overline{\mu}c_{pv1}}{2\overline{\rho}}\left(\frac{d\overline{u}}{dz}\right)^2\frac{Ec}{\mathrm{Re}},$$

$$\overline{c}_{21}=c_{V21}+P_1\left(1-c_{V21}\right),\ \overline{\kappa}=\kappa_{21}+P_1\left(1-\kappa_{21}\right),\ \overline{\rho}=\rho_{21}+P_1\left(1-\rho_{21}\right).$$

For 1-D system $\overline{u}=0$, with account of temperature effect in a simplest way as for the linear function by $z$ ($\alpha_2=0$), the equation array (16) yields:

$$\frac{\left(\rho_{21}-1\right)\overline{w}}{\rho_{21}+P_1\left(1-\rho_{21}\right)}\frac{dP_1}{dz}=\frac{d\overline{w}}{dz},\qquad \frac{d^2\overline{w}}{dz^2}+\frac{d\ln\overline{\mu}}{dz}\frac{d\overline{w}}{dz}=\frac{Eu\cdot\mathrm{Re}}{\overline{\mu}}\frac{d\overline{p}}{dz}-\frac{\overline{\rho}}{\overline{\mu}}\frac{Ga}{\mathrm{Re}},$$

$$\overline{c}_{21}\overline{w}\alpha_1+c_{pv1}\frac{\overline{p}}{\overline{\rho}}Eu\cdot Ec\frac{d\overline{w}}{dz}=\frac{1-\kappa_{21}}{\overline{\rho}Pe}c_{pv1}\alpha_1\frac{dP_1}{dz}.\tag{17}$$

Now from the first equation of the system (17) follows

$$\frac{1}{\overline{w}}\frac{d\overline{w}}{dz}=\frac{\left(\rho_{21}-1\right)}{\rho_{21}+P_1\left(1-\rho_{21}\right)}\frac{dP_1}{dz},\quad \rightarrow \ln\overline{w}+\ln\left[\rho_{21}+P_1\left(1-\rho_{21}\right)\right]=const,$$

where from considering the following boundary condition:

$$z=0,\quad \overline{w}_0=w_{20}+P_{10}\left(w_{10}-w_{20}\right),\quad \overline{\rho}_0=\rho_{21}+P_{10}\left(1-\rho_{21}\right),\tag{18}$$

the final expression with account of (18) is

$$\overline{w}=\frac{\overline{w}_0\overline{\rho}_0}{\rho_{21}+P_1\left(1-\rho_{21}\right)},\ \text{or}\ \ P_1=\frac{1}{1-\rho_{21}}\left(\frac{\overline{w}_0\overline{\rho}_0}{\overline{w}}-\rho_{21}\right),\ \frac{dP_1}{dz}=-\frac{\overline{w}_0\overline{\rho}_0}{\left(1-\rho_{21}\right)\overline{w}^2}\frac{d\overline{w}}{dz}.\tag{19}$$

Zero index means that values are taken by $z$=0. From (19) the explicit expression for the probability $P_1$ with account of the $\overline{w}=w_2+P_1\left(w_1-w_2\right)$ is:

$$\left(1-\rho_{21}\right)\left(w_1-w_2\right)P_1^2+\left[\left(1-\rho_{21}\right)w_2+\rho_{21}\left(w_1-w_2\right)\right]P_1+\rho_{21}w_2-\overline{\rho}_0\overline{w}_0=0.\tag{20}$$

## 5 Correctness analysis of equation array and probability function

Analysis of (20) shows that for the one-speed two-phase flow when $w_1= w_2= w$, the phase probability is expressed $P_1= P_{10}w_0/w$, where from the condition $P_1<1$ is satisfied by $w>P_{10}w_0$, so that if the first phase is presented in a large amount in a two-phase flow, the flow velocity cannot fall in such flow. It must grow with a coordinate increasing involving a second phase. But if a first phase is far from the maximal value (e.g. close to 0), then with decrease of a flow velocity the amounts of the first phase grows in a mixture. Then for the two phases of nearly the same densities ($\rho_{21}$=1), the condition 0<$P_1$<1 yields the requirements: $w_1$< $\bar{\rho}_0\bar{w}_0$, $w_2$> $\bar{\rho}_0\bar{w}_0$, or $w_1$> $\bar{\rho}_0\bar{w}_0$, $w_2$< $\bar{\rho}_0\bar{w}_0$. A general solution of (20) is

$$\left(P_1\right)_{1,2}=\frac{\left(\rho_{21}-1\right)w_2+\rho_{21}\left(w_2-w_1\right)\pm\sqrt{\left(\rho_{21}w_1-w_2\right)^2+4\bar{\rho}_0\bar{w}_0\left(1-\rho_{21}\right)\left(w_1-w_2\right)}}{2\left(1-\rho_{21}\right)\left(w_1-w_2\right)}, \quad (21)$$

where from follows $\left(\rho_{21}w_1-w_2\right)^2\geq 4\bar{\rho}_0\bar{w}_0\left(1-\rho_{21}\right)\left(w_2-w_1\right)$ because $P_1$ must be real. Obviously, it is always satisfied by $\rho_{21}\leq 1$, $w_2\leq w_1$ (lighter phase has lower velocity), and $\rho_{21}\geq 1$, $w_2\geq w_1$ (heavier phase has higher velocity). But in case of a two-phase jet when lighter liquid is going from the nozzle into surroundings with heavier liquid ejected by the first phase [6-8] it is not satisfied. Therefore, $\left(\rho_{21}w_1-w_2\right)^2\geq 4\bar{\rho}_0\bar{w}_0\left(1-\rho_{21}\right)\left(w_2-w_1\right)$ for the 2 cases: $\rho_{21}\leq 1$, $w_2\geq w_1$ and $\rho_{21}\geq 1$, $w_2\leq w_1$. The last inequality solved with regard to velocity components yields:

$$\left(\rho_{21}w_1-w_2\right)^2\geq 4\bar{\rho}_0\bar{w}_0\left(1-\rho_{21}\right)\left(w_2-w_1\right),\quad w_2\leq\left(w_2\right)_1,\quad w_2\geq\left(w_2\right)_2;$$
$$\left(w_2\right)_{1,2}=\rho_{21}w_1+2\bar{\rho}_0\bar{w}_0\left(1-\rho_{21}\right)\pm 2\left|1-\rho_{21}\right|\sqrt{\bar{\rho}_0\bar{w}_0\left(\bar{\rho}_0\bar{w}_0-w_1\right)}. \quad (22)$$

Here the square root must be real, therefore $w_1\leq\bar{\rho}_0\bar{w}_0$. And from (22) follows

$$\begin{gathered}\rho_{21}<1,\quad w_2\leq\rho_{21}w_1+2\left(1-\rho_{21}\right)\bar{\rho}_0\bar{w}_0\left(1-\sqrt{1-w_1/\left(\bar{\rho}_0\bar{w}_0\right)}\right),\\ w_2\geq\rho_{21}w_1+2\left(1-\rho_{21}\right)\bar{\rho}_0\bar{w}_0\left(1+\sqrt{1-w_1/\left(\bar{\rho}_0\bar{w}_0\right)}\right);\\ \rho_{21}>1,\quad w_2\leq\rho_{21}w_1+2\left(1-\rho_{21}\right)\bar{\rho}_0\bar{w}_0\left(1+\sqrt{1-w_1/\left(\bar{\rho}_0\bar{w}_0\right)}\right),\\ w_2\geq\rho_{21}w_1+2\left(1-\rho_{21}\right)\bar{\rho}_0\bar{w}_0\left(1-\sqrt{1-w_1/\left(\bar{\rho}_0\bar{w}_0\right)}\right).\end{gathered} \quad (23)$$

For the two-phase flow by $z$>0, the conditions (23) are satisfied with the positive values of $w_2$, $w_1$. From the other two equations of the system (17) is got

$$\frac{d\bar{p}}{dz}=\frac{\left(\mu_{21}-\rho_{21}\right)\bar{w}+\left(1-\mu_{21}\right)\bar{w}_0\bar{\rho}_0}{Eu\cdot\mathrm{Re}\left(1-\rho_{21}\right)\bar{w}}\frac{d^2\bar{w}}{dz^2}+\frac{\bar{w}_0\bar{\rho}_0}{Eu\cdot\mathrm{Re}\,\bar{w}}\left[\frac{Ga}{\mathrm{Re}}+\frac{1}{\bar{w}}\left(\frac{d\bar{w}}{dz}\right)^2\right], \quad (24)$$

$$\frac{d\bar{w}}{dz}=Pe\cdot\frac{\alpha_1\bar{w}_0\bar{\rho}_0\bar{w}\left[\left(c_{V21}-\rho_{21}\right)\bar{w}+\left(1-c_{V21}\right)\bar{w}_0\bar{\rho}_0\right]}{c_{pv1}\left[\bar{p}\bar{w}^2Pe\cdot Eu\cdot Ec\left(1-\rho_{21}\right)+\alpha_1\left(1-\kappa_{21}\right)\bar{w}_0\bar{\rho}_0\right]},\quad P_1=\frac{1}{1-\rho_{21}}\left(\frac{\bar{w}_0\bar{\rho}_0}{\bar{w}}-\rho_{21}\right).$$

$$z=0,\quad \bar{w}_0=w_{20}+P_{10}\left(w_{10}-w_{20}\right),\quad \bar{\rho}_0=\rho_{21}+P_{10}\left(1-\rho_{21}\right),\quad \bar{p}=1. \quad (25)$$

The Cauchy problem (24), (25) can be solved numerically by the specified dimensionless criteria *Eu, Re, Ga, Pe, Ec*. It gives the pressure and velocity of the two-phase mixture and probability of the first phase $P_1$. Then from the expressions $\overline{w} = w_2 + P_1\left(w_1 - w_2\right)$, $\overline{p} = p_2 + P_1\left(p_1 - p_2\right)$, $\overline{T} = T_2 + P_1\left(T_1 - T_2\right)$ the influence of interacting phases is estimated using the function $P_1$.

## 6 Comparison of phase probability from the Kolmogorov theorem

Compare the function (7) by the Kolmogorov theorem to the solution above:

$$P_1 = \frac{\lambda_{21}}{\lambda_{12} + \lambda_{21}}, \; P_1 = \frac{1}{1-\rho_{21}}\left(\frac{\overline{w}_0\overline{\rho}_0}{\overline{w}} - \rho_{21}\right), \text{ or } \; P_1 = \frac{1}{1+\gamma}, \; \gamma = \frac{\lambda_{12}}{\lambda_{21}}.$$

Consider formally the correspondence of them and determine $\lambda_{12}, \lambda_{21}$, which reflect the interaction of the phases ($\lambda_{12}$ - intensity of transition from phase 1 to phase 2 and, inversely, $\lambda_{21}$ - from phase 2 to phase 1). It results the following

$$\lambda_{21} = \overline{w}_0\overline{\rho}_0 - \rho_{21}\overline{w}, \; \lambda_{12} = \overline{w} - \overline{w}_0\overline{\rho}_0, \; \gamma = \frac{\lambda_{12}}{\lambda_{21}} = \frac{\overline{w} - \overline{w}_0\overline{\rho}_0}{\overline{w}_0\overline{\rho}_0 - \rho_{21}\overline{w}}. \tag{26}$$

The correlation (26) shows that analogs of the interaction coefficients from the Kolmogorov theorem contain the common part $\overline{w}_0\overline{\rho}_0$ with the opposite sign (like in the third Newton's law concerning the two acting bodies) and similar terms $\overline{w}$, $\rho_{21}\overline{w}$, where density ratio in one of them underlines influence of the density ratio on the interaction of the phases in a two-phase flow. It looks reasonable from the physical point of view; therefore, solution (7) may be used for analysis in non-stationary case too, with the coefficients (26).

The probability ranges between 0 and 1, therefore from the above follows that always $\gamma > 0$, which yields from the above the following conditions:

$$\overline{w}_0\overline{\rho}_0 < \overline{w} < \frac{\overline{w}_0\overline{\rho}_0}{\rho_{21}}, \; \rho_{21} < 1; \quad \text{or} \quad \frac{\overline{w}_0\overline{\rho}_0}{\rho_{21}} < \overline{w} < \overline{w}_0\overline{\rho}_0, \; \rho_{21} > 1.$$

The last one shows that two-phase flow cannot be arbitrary from the considered expressions. If density of the phase 2 is less than density of phase 1, the velocity of two-phase mixture is varied in a range, which is expanding with a decrease of the density of a second phase. Similarly, in case of denser second phase, the flow velocity is supposed to be "allowed" for change in a specified range from $\overline{w}_0\overline{\rho}_0$ to a value aiming to zero with an increase of the density ratio.

## 7 Conclusions

The random model for multiphase flow was proposed. The conditions for correctness of the model in a form the limitations on correlation between density ratio of the phases and velocity of the two-phase flow were obtained from the analysis performed. The results obtained need comparison with the results of computations available from the other authors in a literature, and, the most important, with the experimental data. This is a subject for further investigation.

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