Chaotic Modeling and Simulation (CMSIM) 3: 307-316, 2017

Build Of The Compound Chaotic Multiattractors With The Variable Composite Structure

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Abstract: The way of building of compound chaotic multiattractors with the reconstructed composite structure defining a relative positioning of local chaotic attractors as a part of a multiattractor is presented.
Keywords: Multiattractor, Chaotic attractor, Replication operator, Dynamic system.

1 Introduction

Any change in the parameters of chaotic oscillations is associated, eventually, with the restructuring of the corresponding chaotic attractor [1,2]. In dynamic systems with a single chaotic attractor such restructuring is impossible without changing the conditions of existence of chaotic motion [3]. In contrast, in systems that have composite chaotic multiattractor, represents the union of the set of local chaotic attractors, the changes of the chaotic signal can be obtained by variations in the number and relative location of local attractors – without changing the conditions of existence of chaotic oscillations on the local attractors [4,5].
However, the ability of this approach is still limited due to the lack of methods of restructuring of the internal structure of the composite multiattractor. Well-known dynamical systems that has this multiattractor, allow the change only the total number of elements in the composition of multiattractor and its position relative to the origin at a constant order of local chaotic attractors inside multiattractor [6-12]. So, when you rebuild a "two-dimensional" multiattractor composite with the maximum dimension of 5x5 local attractors can be obtained, for example, the configuration shown in Fig.1,a, b, c. But the configuration shown in Fig.1,d,e at the present time it is impossible to implement.
Obviously, the possibility of restructuring the order of local attractors within a compound multiattractor greatly expands the capabilities of this method of control chaotic fluctuations, because the number of available configurations multiattractor will increase significantly. Therefore, the search for ways of restructuring the composite structure of the composite chaotic multiattractor is legitimate interest.

Received: 8 October 2016 / Accepted: 10 July 2017

ISSN 2241-0503

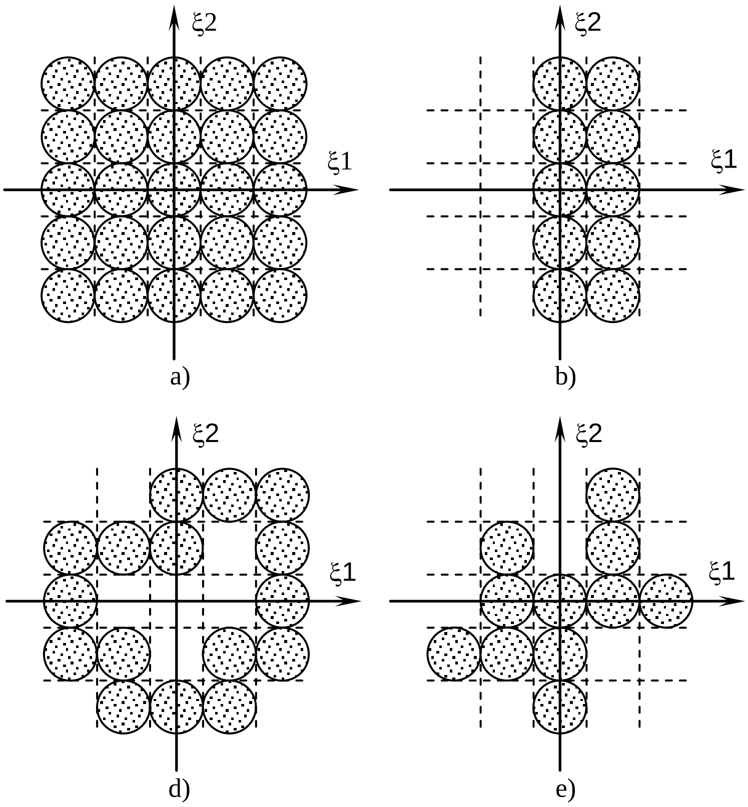

Fig.1. Examples of various types of structure of compound chaotic multiattractor with the compositional dimension of 5x5 elements.

2 Method for changing the compositional structure of compound chaotic multiattractor

Consider the following dynamic system with "two-dimensional" composite chaotic multiattractor [12]:

$$\begin{cases} \dfrac{dx}{d\tau} = -A\left[f\left(H_2(y) - H_1(x)\right) + CH_1(x)\right]; \\ \dfrac{dy}{d\tau} = f\left(H_2(y) - H_1(x)\right) + z; \\ \dfrac{dz}{d\tau} = B\left[f\left(H_2(y) - H_1(x)\right) - H_1(x)\right], \end{cases} \qquad (1)$$

where $f(\xi) = b\xi + (a-b)\dfrac{|\xi+1| - |\xi-1|}{2}$; A, B, C, a, b – are constants;

$$H_k(\xi_k) = \xi_k + (d_k + 1)\{P(\xi_k + s_k + h_k c_k) + P(\xi_k + s_k - h_k c_k) -$$
$$- \sum_{j=0}^{M_k}\left[P(\xi_k + s_k - (2j-1)h_k c_k) + \frac{h_k}{d_k}\right] - \sum_{j=0}^{N_k}\left[P(\xi_k + s_k + (2j-1)h_k c_k) - \frac{h_k}{d_k}\right]\Bigg\}, \quad (2)$$
$$P(\xi_k) = \frac{1}{2}\left(\left|\xi_k + \frac{h_k}{d_k}\right| - \left|\xi_k - \frac{h_k}{d_k}\right|\right), \quad c_k = 1 + \frac{1}{d_k},$$

The functions $H_1(x)$ and $H_2(y)$ provide a replication of the original attractor of a dynamical system and combining it with their copies in a single multiattractor. The constants M_k, N_k define the number of elements in the composition of multiattractor; constants h_k is equal to half of the length of local attractors along variable ξ_k; the constants s_k take into account the asymmetry of the local attractors relative to the centers of the local coordinate systems; the constants d_k, set the steepness of the intermediate segments of the functions $H_k(\xi_k)$; all the constants with index $k=1$, refer to functions $H_1(x)$, the constants with index $k=2$ refer to functions $H_2(y)$, $\xi_1=x$, $\xi_2=y$.
Multiattractor system (1), (2) consists of (1+M1+N1)×(1+M2+N2) chaotic attractors are identical to the attractor of the original system. Its structure can be

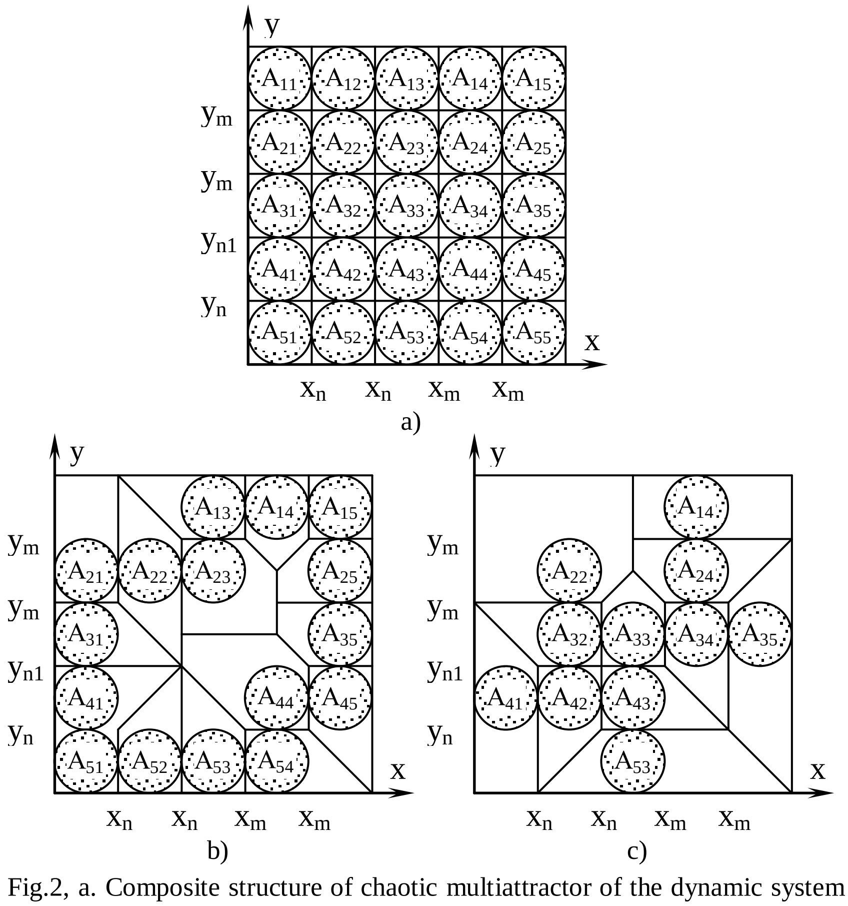

Fig.2, a. Composite structure of chaotic multiattractor of the dynamic system (1), (2) when $M_1=N_1=M_2=N_2=2$. $A_{11}...A_{55}$ – local chaotic attractors; b, c. Examples of types of composite structure of composite chaotic multiattractor, which can be implemented in the dynamic system (3), (4) when $M_1=N_1=M_2=N_2=2$.

represented as a two-dimensional array of elements (cells) of the phase space, the same region of the phase space of the original system, in which is its chaotic attractor [4].

When you use replicate operators of the form (2) all possible positions in the structure of multiattractor system (1) are already occupied by local attractors. Therefore, to be able to rebuild its composite structure, it should be possible to remove from multiattractor some local attractors. Delete local attractor is possible only together with the cell of the phase space containing it. This can be done by extending and locking each other in the adjacent cell. In result of the region of phase space previously contained a local attractor, will be replaced by "blank" areas adjacent cells.

For example, when M_1=N_1=M_2=N_2=2 multiattractor of system (1), (2) has a composite structure, shown in Fig.1. To obtain, for example, the structure shown in Fig.2,b, it is necessary to remove the local attractors A_{11}, A_{12}, A_{24}, A_{32}, A_{33}, A_{34}, A_{42}, A_{43}, A_{55}. To obtain the structure shown in Fig.2,c, you should remove the elements A_{11}, A_{12}, A_{13}, A_{15}, A_{21}, A_{23}, A_{25}, A_{31}, A_{44}, A_{45}, A_{51}, A_{52}, A_{54}, A_{55}. In the first case, this problem can be solved by expanding the cell of the phase space that contains the local attractors A_{13}, A_{14}, A_{21}, A_{22}, A_{23}, A_{25}, A_{31}, A_{35}, A_{41}, A_{44}, A_{45}, A_{52}, A_{53}, A_{54}, for example, as shown in Fig.2,b. In the second – extending cells that contain local attractors A_{14}, A_{22}, A_{24}, A_{32}, A_{33}, A_{34}, A_{35}, A_{41}, A_{42}, A_{43}, A_{53}, for example, as shown in Fig.2,c.

Since the formation of the phase cells provide the replicate functions, change the configuration of the cell requires a corresponding change of these functions. In their equation it is necessary to enter the members ensuring the displacement of cell boundaries relative to the starting position. For this equation (2), must be converted to the following form:

$$
\begin{aligned}
&H_k(\xi_k,\xi_\eta)=\xi_k+(d_k+1)\Bigg\{P\left(\xi_k+s_k+h_k+\frac{h_k}{d_k}-\Delta m(\xi_\eta)_{k,0}\right)+\\
&P\left(\xi_k+s_k-h_k-\frac{h_k}{d_k}-\Delta n(\xi_\eta)_{k,0}\right)-\\
&-\sum_{j=0}^{M_k}\left[P\left(\xi_k+s_k-(2j-1)\left(h_k+\frac{h_k}{d_k}\right)\right)+\frac{h_k}{d_k}-\Delta m(\xi_\eta)_{k,j}\right]-\\
&-\sum_{j=0}^{N_k}\left[P\left(\xi_k+s_k+(2j-1)\left(h_k+\frac{h_k}{d_k}\right)\right)-\frac{h_k}{d_k}-\Delta n(\xi_\eta)_{k,j}\right]\Bigg\},
\end{aligned}
\tag{3}
$$

where $\Delta m(\xi_n)_{k,j}$, $\Delta n(\xi_n)_{k,j}$ is the function defining the deviation of the boundaries of the expanding cells from their position, defined by the equations (2);

$\xi_\eta=\xi_{\eta 1}\ldots\xi_{\eta L}$ – variables that are subject to shifting boundaries, L is the number of these variables.
After that, the system (1) is converted to the form:

$$\begin{cases} \dfrac{dx}{d\tau} = -A\left[f\left(H_2(y,x) - H_1(x,y)\right) + CH_1(x,y)\right]; \\ \dfrac{dy}{d\tau} = f\left(H_2(y,x) - H_1(x,y)\right) + z; \\ \dfrac{dz}{d\tau} = B\left[f\left(H_2(y,x) - H_1(x,y)\right) - H_1(x,y)\right], \end{cases} \tag{4}$$

In Fig.3 and Fig.4 shown graphs of functions $\Delta m(y)_{1,j}$, $\Delta n(y)_{1,j}$, $\Delta m(x)_{2,j}$, $\Delta n(x)_{2,j}$, providing the formation in the system (4) configurations of phase cells, shown in Fig.2,b (solid lines) and in Fig.2,c (dashed lines).
$\Delta m(y)_{1,0} = \Delta n(y)_{1,0} = \Delta m(x)_{2,0} = \Delta n(x)_{2,0} = 0$. Other deflecting functions, shown in Fig.3 and Fig.4, are piecewise linear dependence can be expressed the following expression:

$$\Delta(\xi) = \begin{cases} V_1\xi + W_1, & \xi < \Xi_1; \\ V_2\xi + W_2, & \Xi_1 \le \xi < \Xi_2; \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ V_q\xi + W_q, & \Xi_{q-1} \le \xi < \Xi_q; \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ V_Q\xi + W_Q, & \xi > \Xi_{Q-1}, \end{cases} \tag{5}$$

where Q is the number of linear segments in the function.
The values of the parameters of equation (5) corresponding to the deflecting functions needed for the formation in the system (4) configurations of the phase cells is shown in Fig.2,b and Fig.2,c, shown in table 1 and table 2, respectively.

Table 1

Replication operator	$H_1(x,y)$				$H_2(y,x)$			
Deflecting function	$\Delta m(y)_{1,1}$	$\Delta m(y)_{1,2}$	$\Delta n(y)_{1,1}$	$\Delta n(y)_{1,2}$	$\Delta m(x)_{2,1}$	$\Delta m(x)_{2,2}$	$\Delta n(y)_{2,1}$	$\Delta n(x)_{2,2}$
ξ	y	y	y	y	x	x	x	x
$V_1;W_1;\Xi_1$	$0;0;-3U_2$	$-R_2;-3;-3U_2$	$0;0;3U_2$	$0;0;-3U_2$	$0;0;-3U_1$	$-R_1;-1;-U_1$	$0;0;-U_1$	$0;0;-3U_1$
$V_2;W_2;\Xi_2$	$-R_2;-3;-U_2$	$0;0;-U_2$	$-R_2;3;$ –	$R_2;3;-U_2$	$-R_1;-3;-U_1$	$0;0;U_1$	$0;1;2U_1$	$R_1;3;-U_1$
$V_3;W_3;\Xi_3$	0;-2;0	$-R_2;-1;0$	–	$-R_2;1;U_2$	$0;-1;2U_1$	$-R_1;1;2U_1$	$-R_1;3;3U_1$	$-R_1;1;U_1$
$V_4;W_4;\Xi_4$	$0;1;2U_2$	$0;-1;2U_2$	–	$0;0;3U_2$	0;0; –	$R_1;-3;3U_1$	0;0; –	$0;0;3U_1$
$V_5;W_5;\Xi_5$	$-R_2;3;3U_2$	$R_2;-3;3U_2$	–	$-R_2;-3;$ –	–	0;0;–	–	$-R_1;3;$–
$V_6;W_6;\Xi_6$	0;0; –	0;0; –	–	–	–	–	–	–

Table 2

Replication operator	$H_1(x,y)$				$H_2(y,x)$			
Deflecting function	$\Delta m(y)_{1,1}$	$\Delta m(y)_{1,2}$	$\Delta n(y)_{1,1}$	$\Delta n(y)_{1,2}$	$\Delta m(x)_{2,1}$	$\Delta m(x)_{2,2}$	$\Delta n(y)_{1,1}$	$\Delta n(x)_{2,1}$
ξ	y	y	y	y	x	x	x	x
$V_1;W_1;\Xi_1$	$-R_2;-1;-U_2$	$-R_2;-3;-3U_2$	$-R_2;-3;-3U_2$	$0;0;-U_2$	$0;0;-U_1$	0;2;0	$-R_1;-3;-3U_1$	$R_1;1;-U_1$
$V_2;W_2;\Xi_2$	$0;0;U_2$	$0;0;U_2$	$0;0;U_2$	$R_2;1;0$	$R_1;1;0$	0;0; –	$0;0;U_1$	$0;0;3U_1$
$V_3;W_3;\Xi_3$	$-R_2;1;2U_2$	$R_2;-1;$ –	$R_2;-1;2U_2$	$-R_2;1;U_2$	$-R_1;1;U_1$	–	$-R_1;-1;-U_2$	$-R_1;3;$ –
$V_4;W_4;\Xi_4$	0;-1; –	–	0;1; –	$0;0;3U_2$	$0;0;3U_1$	–	$-R_2;1;$ –	–
$V_5;W_5;\Xi_5$	–	–	–	$R_2;-3;$ –	$R_1;-3;$ –	–	–	–

In tables $U_k = h_k\left(1+\dfrac{1}{d_k}\right)$; $R_k = \dfrac{1}{U_k}$.

In Fig.5 and 6 represented multiattractors of dynamic system (3), (4) observed at A=7, B=3, C=0.25, a=0.5, b=-0.35, M1=N1=M2=N2=2, h1=1.58, s1=0, d1=30, h2=5.35, s2=0, d2=5. Multiattractor shown in Fig.5, has a composite structure, shown in Fig.2,b. Multiattractor depicted in Fig.6 has a composite structure, shown in Fig.2,c.

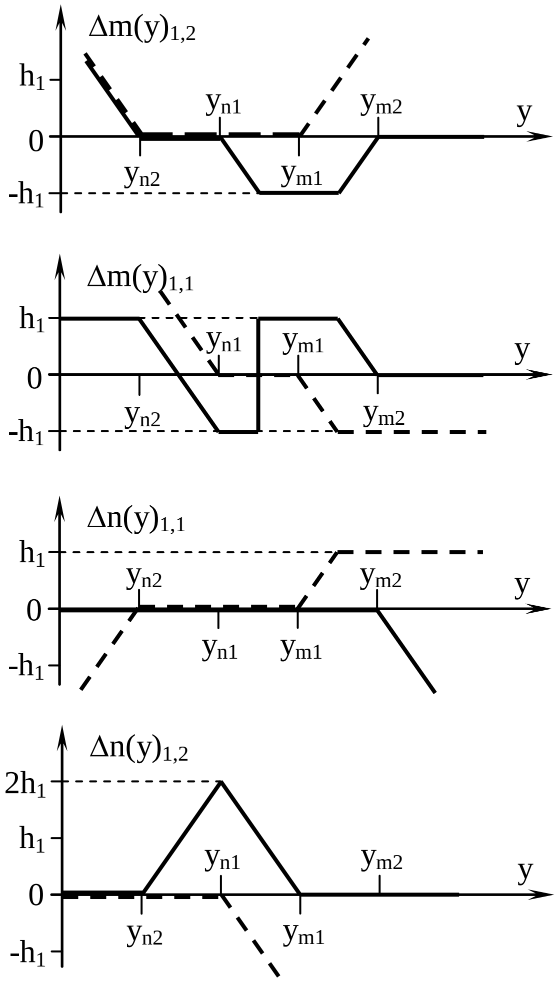

Fig.3. The functions defining the shifting boundaries between the cells of the phase space generated by the replicate operator $H_1(x,y)$ necessary for the formation of the composite structure shown in Fig. 2,b (solid lines) and in Fig.2,c (dashed lines).

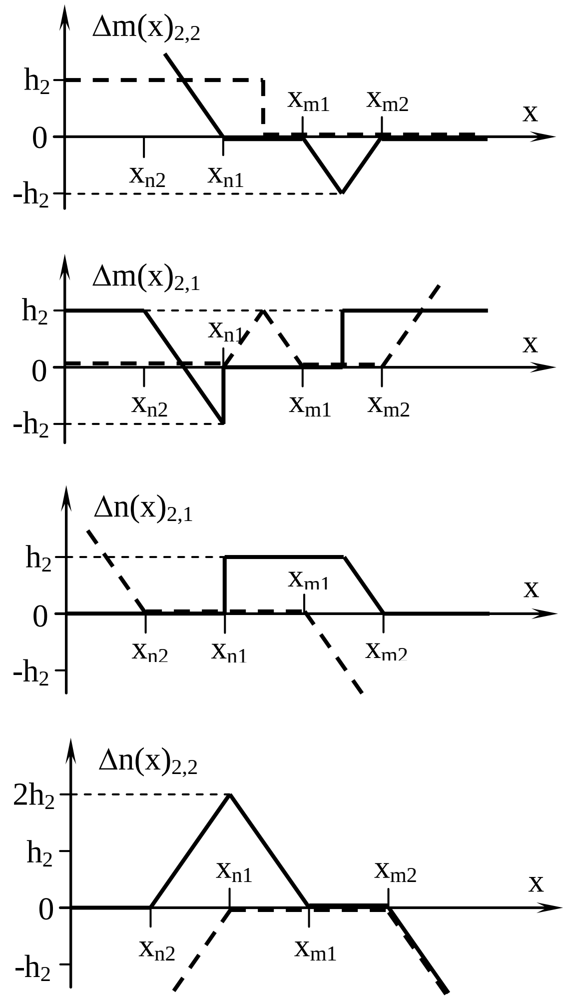

Fig.4. The functions defining the shifting boundaries between the cells of the phase space generated by the replicate operator $H_2(y,x)$ necessary for the formation of the composite structure shown in Fig. 2,b (solid lines) and in Fig.2,c (dashed lines).

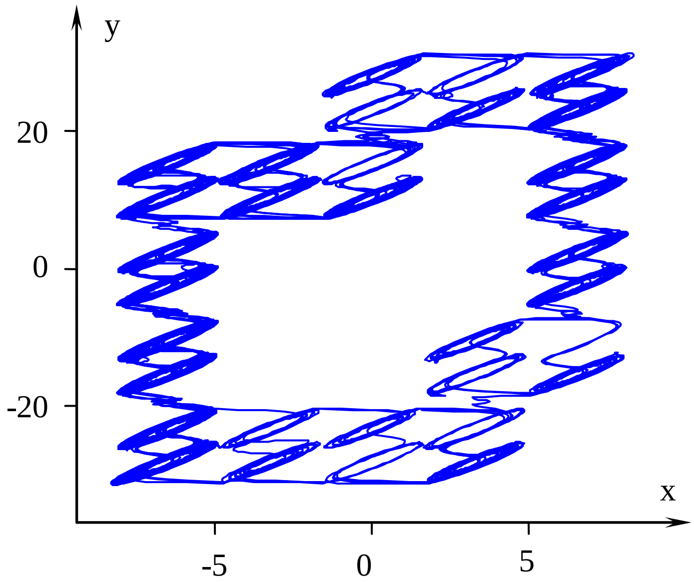

Fig.5. Example of projection onto the plane (x,y) of chaotic multiattractor of the system (3), (4) having a composite structure as shown in Fig.2,b.

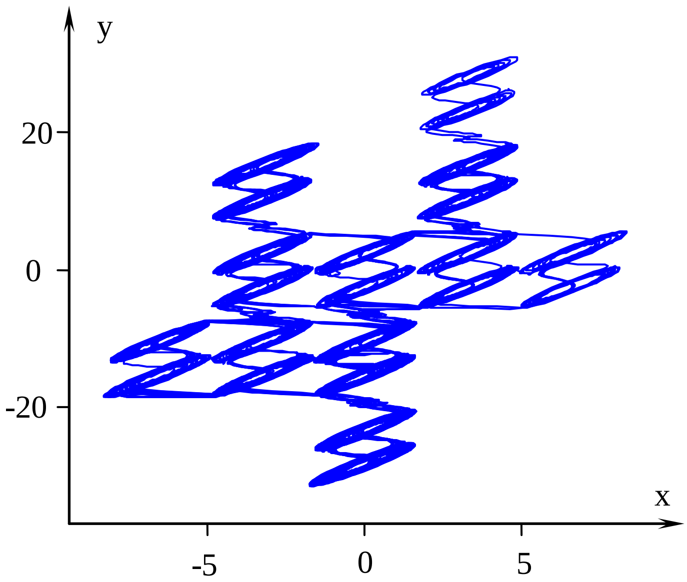

Fig.6. Example of projection onto the plane (x,y) of chaotic multiattractor of the system (3), (4) having a composite structure as shown in Fig.2,c.

Conclusions

The method provides the possibility of completely rebuilding common position structure of compound chaotic multiattractor that allows you to get all the possible configurations of these objects. The restructuring is due to the exclusion from multiattractor part of the elements. This is achieved by modifying the replicate operators, which allows them to generate the set configuration of the each phase cell contains a local attractor. This gives you the opportunity to remove some cells through their absorption advanced adjacent cells. Modification of replicate operators is introducing in the equation the additional members that specify the local offset of the phase boundaries between the cells relative to the starting position.

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