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Modeling of Chaotic Behavior in the Economic Model

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Abstract. Considered duopoly Cournot model, which allows analyze the different possible behavior strategies of each of the market participants. Detected conditions of the market equilibrium under which the income of both market participants – maximum. Also consider getting one duopolist additional revenue, on the condition that another go with the market.
The control method of economic Cournot model and simulation results are presented.
Keywords: Chaotic modelling, model, chaotic behaviour, control.

1 Introduction

In recent years, the chaos becomes more and more development and application in various fields. A large number of real systems have a nonlinear behavior despite the idealized linear behavior used in modeling. Some changes in nonlinear systems can lead to a complex and erratic behavior called chaos. The nonlinearity is one of the conditions needed by a system in order to develop chaos. The term chaos is used to describe the behavior of a system that is aperiodic and apparently random. The chaos and the concepts related to the dynamics of the systems and the their modeling using differential equations is named the chaos theory and is tightly related with the notion of nonlinearity [1]. The nonlinearity implies the loss of the causality correlation between the perturbation and effect propagated in time.
Strotz et al. [2] demonstration that chaotic dynamics can exist in an economics model, tremendous efforts have been devoted to investigating this kind of complex behaviors in various economic systems. Recently, it has also been shown that even oligopolistic markets may become chaotic under certain conditions [3, 4].
Oligopoly, with a few firms in the market, is an intermediate structure between the two opposite cases of monopoly and perfect competition. Even the duopoly situation in an oligopoly of two producers can be more complex than one might imagine since the duopolists have to take into account their actions and reactions when decisions are made. Oligopoly theory is one of the oldest

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branches of mathematical economics dated back to 1838 when its basic model was proposed by Cournot [4]. Research reported in Ref. [5] suggests that a Cournot adjustment process of output might be chaotic if the reaction functions are non-monotonic. This result was purely mathematical without substantial economic implication until Puu [6] provided one kind of economic circumstances, i.e., iso-elastic demand with different constant marginal costs for the competitor, under which meaningful unimodal reaction functions were developed. Since then, various modifications have been made by numerous economists. Rosser has a good state-of-the-art review of the theoretical development of complex oligopoly dynamics.

Unstable fluctuations have always been regarded as unfavorable phenomena in traditional economics.

Because chaos means unpredictable events in the long time, it is considered to be harmful by decision-makers in the economy. Research on controlling chaos in economic models has already begun and several methods have been applied to the Cournot model, such as the OGY chaos control method [7, 8]. These approaches require exact system information before their implementation [9].

2 Cournot duopoly model

Cournot model is one of the basic models non-cooperative quantitative oligopoly. It has great methodological value because it allows not only to analyze the various possibilities of strategic cooperation of market participants, but also to understand the basic problems of models application.

On the market of two competing firms producing homogeneous products and independently (without collusion), they take decision of the volume of its release. In classical Cournot model, each duopolist suggests that production volume opponent is constant and independent from changes in the scope of his own release [10].

Expected that the market demand is known and is given a decreasing linear function expressing the dependence of the market price of P on the number of products Q:

$$P = a - bQ,$$

where $a > 0$, $b > 0$, and supply volume Q equals the sum of the supply volumes the first and second companies:

$$Q_1 = q_1 + q_2$$

Thus,

$$P = a - b\left(q_1 + q_2\right)$$

Also assume that both firms have equal production costs:

$$TC_1 = TC_2 = cq_1 = cq_2$$

Define the conditions of equilibrium - this market state, in which both firms profits maximize possible. The profit of each of the companies is the difference between revenues and costs:

$$P_1 = TR_1 - TC_1 = Pq_1 - cq_1,$$

$$P_1 = \left(a - bq_1 - bq_2\right) q_1 - cq_1 ,$$
$$P_2 = TR_2 - TC_2 = Pq_2 - cq_2 ,$$
$$P_2 = \left(a - bq_1 - bq_2\right) q_2 - cq_2 .$$

Lines-level of profit function are called isoprofits. They represent a set of points at each of which one of the oligopolists have the same profit. A necessary condition for extremum income function is follows:

$$\frac{\partial P_1}{\partial q_1} = a - bq_2 - 2bq_1 - c = 0 ,$$
$$\frac{\partial P_2}{\partial q_2} = a - bq_1 - 2bq_2 - c = 0 .$$

Hence, we find the equations expressing the duopolist's optimal production level through optimum release of his rival:

$$\begin{cases} q_1 = -\dfrac{1}{2} q_2 + \dfrac{a-c}{2b} \\ q_2 = -\dfrac{1}{2} q_1 + \dfrac{a-c}{2b} \end{cases} \qquad (1)$$

The lines defined by the equations (1) are called lines the reaction Cournot duopolists. (Lines of reaction is also called response curves or the best answer curves). At each point of the reaction line the profit value of i-th oligopolists are maximum for the corresponding competitor issue volume. The point of equilibrium, if it exists, is the point of intersection of the reactions lines of all market participants.

Both equations (1) must be carried out simultaneously at the equilibrium point $\left(q_1^0 ; q_2^0\right)$. The solving of an appropriate system of equations let find the coordinates of this point:

$$q_1^0 = q_2^0 = \frac{a-c}{3b}$$

Lines reaction of duopolists (Fig. 1) given by the equations (1) are shown by straight 1 and 2:

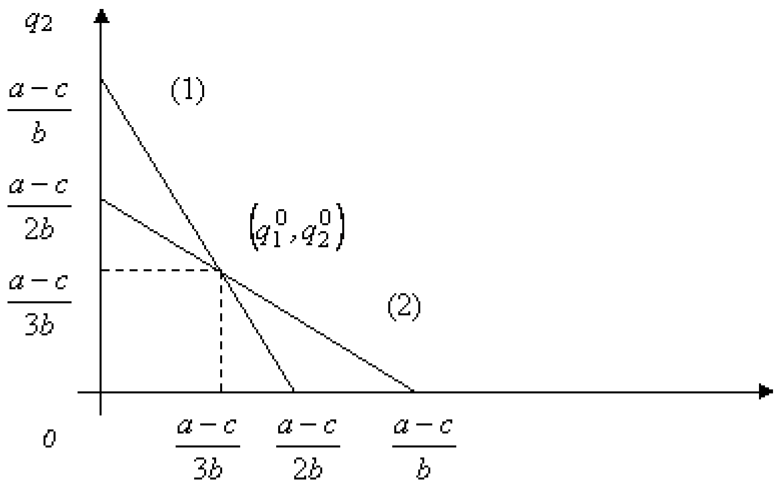

Fig. 1. Reaction of duopolists

Note that a sufficient condition for an extremum (negative second derivative) confirms duopolists maximum profit at the equilibrium point:

$$\frac{\partial^2 P_1}{\partial q_1^2} = -2b < 0, \ \frac{\partial^2 P_2}{\partial q_2^2} = -2b < 0$$

The resulting equilibrium values of output volumes are desirable for both companies, since provide each duopolist optimal profit of $P_i^0 = \frac{(a-c)^2}{9b}$ at the equilibrium price $p^0 = \frac{a+2c}{3}$.

It is interesting graphic interpretation of the model (Fig. 2). The profit function of the first duopolist is a function of two variables

$$P_1(q_1, q_2) = (a - bq_1 - bq_2)q_1 - cq_1 .$$

The levels line - isoprofits - are given by equations of the form $P_1(q_1, q_2) = K$, where K is an arbitrary constant.

Thus:

$$(a - bq_1 - bq_2)q_1 - cq_1 = K ,$$

$$q_2 = -q_1 - \frac{K}{b} \cdot \frac{1}{q_1} + \frac{a-c}{b} .$$

To study the shape of the curve find derivatives q_2 on q_1:

$$q_2' = \frac{K}{bq_1^2} - 1; \ q_2'' = \frac{2K}{bq_1^3} .$$

It is easy to see that in a positive half-plane isoprofit are a concave curve having a maximum point

$$q_1 = \sqrt{\frac{K}{b}}\ .$$

The K parameter's growth is characterized by a shift of the point of maximum to the right and decrease the maximum value, that is, "top" of the curve with increasing K slides right and down. Consequently, as closer to the axis q_1 isoprofits is, the higher level of profit it corresponds.

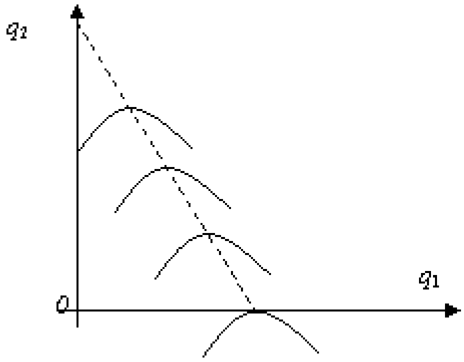

Fig. 2. Graphic interpretation of the model

Consider the situation in terms of conditional first duopolist. Fix the competitor production volume $q_2 = Const$ (Fig. 3). An infinite family of profit's isoprofit of the first duopolist crosses the corresponding horizontal line. The most favorable conditions for the given issue volume will set the abscissas isoprofits, below the others, but passing through the line $q_2 = Const$. Thus, the best answer duopolist for the selected level of rival's output q_2 will the level of output q_1 such that the point $(q_1;q_2)$ is the point of contact a line $q_2 = Const$ and some isoprofits. The totality of these points of contact, (corresponding to all possible values) form the line of response of first duopolist.

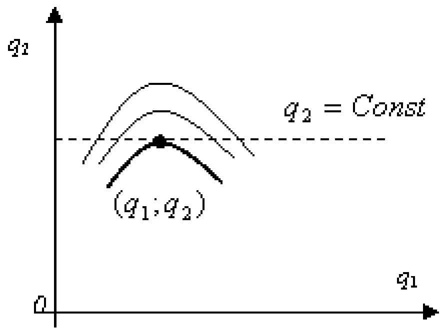

Fig. 3. Situation in terms of conditional first duopolist for fix the competitor production volume $q_2 = Const$.

The first duopolist will get greatest possible profit at zero production volume competitor, when the top of the respective isoprofits lies on the horizontal axis. When $q_2 = 0$ the sole volume release of the first company will:

$$q_m = \frac{a-c}{2b},$$

and when monopoly price

$$p_m = a - bq_m = a - b \cdot \frac{a-c}{2b} = \frac{a+c}{2}$$

its profit value will be equal

$$P_m = (a - bq_m)q_m - cq_m = \frac{(a-c)^2}{4b}.$$

Note that the equilibrium volume of output in Cournot duopoly is 2/3 of monopoly equilibrium volume, and the equilibrium profit of the company amounts 4/9 of the monopoly equilibrium profits [11].

3 Control of Cournot duopoly model

In this chapter, we study one of the Cournot duopoly models proposed by Kopel. He also investigated the complex adjustment dynamics in some Cournot duopoly models and obtained similar results.
General Kopel model described two-dimensional reaction functions

$$\begin{aligned} x(t+1) &= (1-\rho)x(t) + \rho\mu y(t)\big(1-y(t)\big), \\ y(t+1) &= (1-\rho)y(t) + \rho\mu x(t)\big(1-x(t)\big). \end{aligned} \quad (2)$$

For control we adding a state feedback controller, the dynamic process (2) turns to be

$$\begin{aligned} x(t+1) &= (1-\rho)x(t) + \rho\mu y(t)\big(1-y(t)\big) + u(t), \\ y(t+1) &= (1-\rho)y(t) + \rho\mu x(t)\big(1-x(t)\big), \end{aligned}$$

where $u(t) = k(y(t) - x(t))$ is the control law and k is the feedback gain.
Figure 4 shows the result of simulation for parameters ρ = 0.695; μ = 3.8; x(0) = 0.2; y(0) = 0.8; k = 0.8.

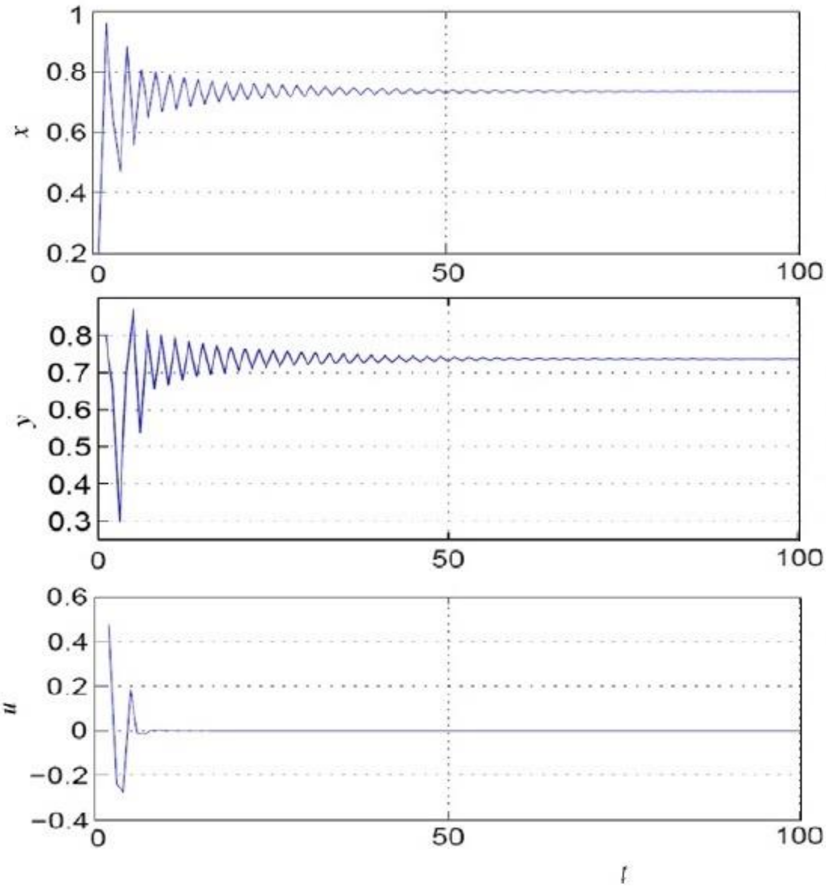

Fig. 4. Controlled trajectory of duopolists' productions x; y and the feedback controller u, with parameters ρ = 0.695; μ = 3.8; x(0) = 0.2; y(0) = 0.8; k = 0.8.

Conclusions

We have presented the complex nonlinear economic dynamics in a Cournot duopoly model proposed by Kopel. Detected conditions of the market equilibrium under which the income of both market participants – maximum. Added a state feedback controller shows possibility of control of the economic Cournot model.

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