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Predictive Chaos Control for the 1D-map of Action Potential Duration

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Abstract. In the present work, a nonlinear control method namely predictive control is investigated. The proposed method allows stabilizing unstable period-1 rhythm. Using mathematical analysis and computer simulations, we show that this method can be used to control chaotic behavior or pathological rhythms. As example, the results are illustrated in the case of the 1D-map action potential duration (APD_{i+1}) which modelizes the cardiac action potential duration as the function of the previous one (APD_i). .

Keywords: Action Potential Duration (APD), chaos, predictive control, equilibrium point, normal rhythm, irregular heart rhythm.

1 Introduction

Nonlinear dynamics and tools from chaos theory are used to understand and to characterize some cardiac pathologies [1,2,3]. These works are focused on the behavior of cardiac rhythm. From dynamic point of view, different states of the cardiac rhythm are qualified by the equilibrium points, the periodicity, the chaos [4]. Many works are dedicated to the suppression of abnormal rhythm and spatiotemporal chaos in cardiac tissue [5,6,7]. Control methods using external electrical stimulation are applied to alternances and irregular heart rhythms in order to recover normal rhythm [8]. Mathematically, the control is performed with small perturbations to system parameter in order to lead the periodic and chaotic behavior to the equilibrium point. In cardiac dynamics, the most accessible system parameters available for perturbation is usually the interval between successive stimuli or the timing of the next excitation, which can be advanced or (in some situations) delayed through low-magnitude current stimulation [9]. It is believed that ventricular fibrillation is characterized by chaotic dynamics of the heart [10,11]. This arrhythmia is generated by the loss of stability of the periodic rhythm, namely alternans, because the rapid pacing of the cardiac tissue [12, 13]. Chaotic attractors contain an infinite number of unstable periodic orbits of any desired period [14]. When the chaotic orbit

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approaches to the desired unstable periodic orbit, it can be attracted to and maintained on the orbit by applying small perturbations to the system [14,15]. The interest in the chaotic control systems has been largely initiated by the item E.Ott, C. Grebogi and J.York published in 1990 [16]. The key idea provided by the article, is a considerable change in the behavior of a chaotic system can be obtained through a very small change in one or more of its parameters. This known control method, namely OGY control method, was the first method of chaotic control systems. The latter is based on the feedback state control that uses the chaos in the dynamic system to stabilize an unstable periodic orbit. Several other methods have been developed for chaotic control systems [17, 18]. Among these methods, we can mention the predictive control method with the feedback state proposed by Pyragas [19]. This method is based on the feedback state control: the control law is calculated from the difference between the current state and the state with a time delay T, where T represents a period of the orbit to be stabilized. The state controlled by this technique converges to the desired orbit and not to an approximation, as is the case with the OGY method [20]. The advantage of this type of predictive control method lies in the fact that the approximations are not used in the state feedback. However it has been shown that when the specific number of real parts which greater than one is odd, the fixed point cannot be stabilized by this technique for discrete systems [21]. Ushio and Yamamoto [22] propose to stabilize the fixed points by a method based on the prediction of the states of the uncontrolled system. The OGY method was the first control technique applied to control cardiac rhythms. It is investigated to stabilize the aperiodic ventricular tachycardia dynamics of a rabbit. During chaos control a perturbation is applied to vary the interbeat intervals but the irregular chaotic dynamics was controlled and replaced by periodic rhythm, typically with a period-3 or period-4 rhythm. Hence, this way of perturbation prevented from having the period-1 desired rhythm [23]. Since the work of Garfinkel a number of theoretical and experimental studies were performed to control irregular cardiac rhythms and through diverse methods of nonlinear dynamics control [24]. A specific cardiac strategy called adaptive control of diastolic intervals (DI) to control the duration of action potential (APD) alternans is proposed by Jordan and Christini [25]. Specifically, they use a so-called restitution in the cardiac system: (APD) current as a function of previous (DI). This developed cardiac paradigm control is efficace to direct the periodic and aperiodic rhythm to the equilibruim point or normal rhythm and it can be applied to control both noisy and drifting rhythms [25]. The nonlinear control techniques used to show how to control certain irregular heart rhythms. However, since the only effective therapy for ventricular fibrillation remains high energy shocks, it would be essential to develop and obtain by the methods of nonlinear dynamics control protocols low energy to manage cardiac arrhythmias [26]. There is a long way to go before these methods are successful clinically. All these studies indicate that the combination of nonlinear methods of dynamic control of chaos, and defibrillation can give rise to new therapeutic strategies in the treatment of cardiac arrhythmias [26]. In the present paper, we give in Section 2, the dynamical properties of the one-dimensional map (APD) of an electric cardiac model describing the propagated cardiac action potential

[27]. In order to prevent chaotic behavior or to direct it into steady state, we apply in Section 3 the predictive control. Eventually, numerical results are presented to verify the efficiency of this paradigm control. Conclusion is given in Section 4.

2 The 1D-Map (APD)

The effects of a periodic stimulation on a strand of ventricular muscle have been investigated by Lewis and Guevara [27]. Electrical stimulations applied at a regular time intervals (t_s) generated an action potential. At arbitrary (t_s), the duration of any given action potential is controlled by its immediately preceding diastolic interval (DI) and this dependence is given by the APD restitution curve [27]:

$$APD_{i+1} = g(DI_{i+1}) = A - B_1 \exp\left(\frac{-DI_{i+1}}{\tau_1}\right) - B_2 \exp\left(\frac{-DI_{i+1}}{\tau_2}\right) \qquad (1)$$

Where APD_{i+1} is the APD of $(i+1)st$ action potential, DI_{i+1} is its associated diastolic interval ($DI_{i+1} \succ DI_{\min}$) and g is a double-exponential function describing the restitution curve. The constants A, B_1, B_2, τ_1, τ_2 are related to the heart electrophysiological constraints defined in [27]: $A = 270\ ms$, $B_1 = 2441\ ms$, $B_2 = 90.02\ ms$, $\tau_1 = 19.60\ ms$, $\tau_2 = 200.5\ ms$, and $DI_{\min} = 53.5\ ms$. Should one or more stimuli be blocked, then the relationship between the APD of the subsequent action potential and its diastolic interval is not affected by the presence of the subthreshold response(s) [27]. Then, the APD can be obtained as a function of the previous duration potential action

$$APD_{i+1} \simeq g(nt_s - APD_i) = f(APD_i) \qquad (2)$$

The eq (2) is obtained if $(n-1)$ blocked stimuli occur in the diastolic interval $DI_{(i+1)}$ (i.e., n is the smallest integer such that $nt_s - APD_i \geq DI_{min}$) [27]. One obtains:

$$APD_{i+1} = A - B_1 \exp\left(\frac{APD_i - nt_s}{\tau_1}\right) - B_2 \exp\left(\frac{APD_i - nt_s}{\tau_2}\right) \qquad (3)$$

The dynamics of the eq (3) is graphically studied in [27]. When the stimulation frequency t_s decreases from the value 400 ms to the value of 25.1 ms, with increment which equal to 0.1 and iteration starting from initial condition APD_1 which equal to 240 ms, one obtains the following sequence rhytms: [$1:1 \rightarrow 2:2$(alternans)$\rightarrow 2:1 \rightarrow 4:2 \rightarrow$chaos$\rightarrow 3:1 \rightarrow 6:2 \rightarrow$chaos$\rightarrow 4:1 \rightarrow 8:2 \rightarrow$chaos$\rightarrow 5:1 \rightarrow 10:2 \rightarrow$chaos$\rightarrow 6:1 \rightarrow 12:2 \rightarrow$total chaos]. The bifurcation diagram (see Fig. 1) shows the different dynamics of the system.

An $N:M$ rhythm ($N \geq 1$, $M \geq 0$) is periodic with Nt_s period [27], which contain the repeating $N:M$ cycles, each exhibiting N stimulus pulses and M action potential or M beat. Under low amplitude the response of cardiac tissu is similar with each stimulus [27]. This rhythm is noted $1:1$, it is the equilibruim point (or period-1orbit), where the first 1 indicates one stimulus

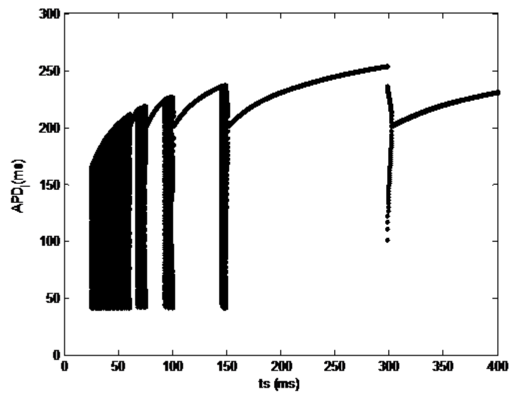

Fig. 1. Bifurcation diagram from [27].

and the second 1 indicates one beat or response. If the stimulation frequency is increased more than some critical value, the cardiac dynamic evolved into cycle (or period-2 orbit) noted that $2:2$. It is mentioned to as the alternans, which mean that the APD oscillates between tow values. In particular, beat-by-beat the long interval of APD is still followed by the short one. The rhythm $1:1$ became unstable via double period bifurcation. Still increasing the frequency, the $2:2$ rhythm loss it's stability, generating the $2:1$ rhythm.There are another periodic-2 orbit, noted as $2N:2$ rhythms $(N \succ 1)$. One will see two different actions potentials that alternate. But the size of the range of t_s over wich one sees any two different $2N:2$ rhythms $(N \succ 1)$ will be different. And the range of APD that one will encounter over these two ranges of t_s will be different. More than the value of t_s decreases more than the range of the $2N:2$ rhythms stability decreases. These cycles rapidly loses its stability since the dynamics will become very complexe [12]. It has been proved that [27]. The 1D-map APD is fundamental model to understand the evolution of regular cardiac rhythm into irregular one, mainly FV-type which lead to sudden cardiac arrest [27]. In the following sections, we studied the predictive control using 1D-map model APD to stabilize irregular rhythm into equilibrium points or (period-1 orbit), such as $N:1$ rhythm $(N \succeq 1)$ indicating healthy heart.

2.1 Determination of unstable fixed point

Our objective is to stabilize the periodic rhythms (or period-2 orbit) and irregular heart rhythm to the $N:1$ rhythm, $N \geq 1$ (or period-1 orbit). To apply the predictive control, it is necessary to know the unstable fixed point value when we have chaotic behavior.

For example, when t_s attained the value 146 ms the rhythm $2:2$ (or cycle) loss it's stability obtaining aperiodic orbit . The determination of the unstable fixed point value of $1:1$ rhythm noted x^* at t_s equal to 146 ms, is based on a numerical method. The fixed point of the system (3) satisfies the equation:

$$F(x^*) = f(x^*) - \quad x^* = 0 \tag{4}$$

We search the root α using *dichotomie* method which represents a fixed point for the iteration interval I. The goal is to try to isolate the root α on the iteration interval $I = [0, 270](a_0 = 0, b_0 = 270)$. $\alpha \in [a_k, b_k] \subset [a_0, b_0]$, $\alpha \simeq x_k$, the middle of the interval $[a_k, b_k]$,such that

$$|\alpha - x_k| \leq \left| \frac{b_k - a_k}{2^{k+1}} \right| < 10^{-5}$$

For $t_s = 146\ ms$, we found two unstable fixed points values corresponding to $1:1$ and $2:1$ rhythms:

The $1:1$ rhythm unstable (or period-1 orbit) value is $\alpha \simeq x_{12} = 86.42\ ms$.

The $2:1$ rhythm unstable (or period-1 orbit) value is $\alpha \simeq x_{12} = 196.00125$ ms

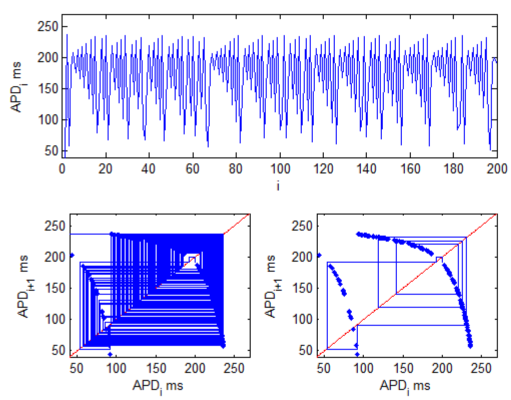

Fig. 2. Aperiodic rhythm at $t_s = 146\ ms$.

3 The predictive control

The predictive control based on feedback control strategy introduced for the control of chaotic systems [21]. This type of control system led to a regular attractor as a fixed point (or period-1 orbit) [19]. The predictive control provides a universally applicable, easy to implement the method that requires little knowledge of the system and has been successfully applied [28].

3.1 The mathematical approach

Let the nonlinear one-dimensional map defined by the flow f of class C^1. Considering I an invariant part of R with f , such as:

$$x_{i+1} = f(x_i) + u(i) \tag{5}$$

With x_i is a state of the phase space I. Let $u(i)$ is a real number applied to control the map (3). The control value equal to zero when the map generates chaos [29]. For any point x_i of the chaotic orbit which is far from fixed point neighborhood. The control $u(i)$ is activated when the controlled state x_n asymptotically converges to the target fixed point x^*.
In the predictive control method, the control $u(n)$ will be determined by the difference between the present state x_n and the uncontrolled predicted state $(x_n)_p$ [21].
To stabilize an unstable fixed point of the chaotic one-dimensional map , it is necessary to determine a gain K belongs an open real interval from the study of the asymptotic stability of the chaotic one-dimensional map (5) in the vicinity of the unstable fixed point x^* . The formula control $u(n)$ is given by [22]:

$$u(n) = K\left[(x_n)_p - x_n\right] \tag{6}$$

Using $f(x_n)$ as a prediction of the fixed point to be stabilized. The predicted state uncontrolled $(x_n)_p$ is given by:

$$(x_n)_p = f(x_n) = x_{n+1} \tag{7}$$

This control law is valid only when the system state is neigborhood of the desired unstable fixed point. The control zone is determined by the following relationship:

$$|x_{n+1} - x_n| \prec \varepsilon \tag{8}$$

Let $\Delta x(n) = |x_{n+1} - x_n|$ and assuming that ε is a small positive value ($\varepsilon \prec\prec 1$). Hence, to acheive the control it is necessary that the required maximum perturbation parameter γ is proportional to ε.

$$u(n) = K\Delta x(n) \tag{9}$$

The controlled chaotic one-dimensional map model will be given by:

$$x_{n+1} = f(x_n) + K\left(x_{n+1} - x_n\right) \tag{10}$$

3.2 Numerical results

In this part, we consider predictive control pradigm in 1D- map (APD). To determine the correction to the current state x_i of the chaotic system, the control law, which is defined from the equation (10) is calculated, we obtain:

$$u(i) = K\left(A - B_1 \exp\left(\frac{x_i - nt_s}{\tau_1}\right) - B_2 \exp\left(\frac{x_i - nt_s}{\tau_2}\right) - x_i\right) \tag{11}$$

Knowing that: $x_i = APD_i$

$$x_{i+1} = A - B_1 \exp\left(\frac{x_i - nt_s}{\tau_1}\right) - B_2 \exp\left(\frac{x_i - nt_s}{\tau_2}\right) + K\left(A - B_1 \exp\left(\frac{x_i - nt_s}{\tau_1}\right) - B_2 \exp\left(\frac{x_i - nt_s}{\tau_2}\right) - x_i\right) \tag{12}$$

The linearization of the equation (12) around the fixed point x^* is given by:

$$\delta x_{i+1} = J \delta x_i \tag{13}$$

$$J = \left[\frac{\delta x_{i+1}}{\delta x_i}\right]_{x^*} \tag{14}$$

Assuming that A_{x^*} is the Jacobien of the uncontrolled map (3) in the unstable fixed point x^*, then

$$J = K\left(A_{x^*} - 1\right) + A_{x^*} \tag{15}$$

The system stabilizes around a fixed point by the predictive control if the gain K value satisfies the following inequality control:

$$\left|K\left(A_{x^*} - 1\right) + A_{x^*}\right| \prec 1 \tag{16}$$

The predictive control of the map model into a fixed point x^* is defined as follows:

$$u(i) = \begin{cases} K\left(A - B_1 \exp\left(\frac{x_i - nt_s}{\tau_1}\right) - B_2 \exp\left(\frac{x_i - nt_s}{\tau_2}\right) - x_i\right) & \text{if} \quad |u(i)| \prec \gamma \\ 0 & \text{if} \quad |u(i)| \succ \gamma \end{cases} \tag{17}$$

To stabilize the system on the fixed point value $x^* = 86.42\ ms$ (or $1:1$ rhythm) at $t_s = 146\ ms$. The condition on K becomes:

$$\exists\ K \in \left]-1, -0.725\right[$$

To stabilize the system on the fixed point value $x^* = 196.00125\ ms$ (or $2:1$ rhythm) at $t_s = 146\ ms$. The condition on K becomes:

$$\exists\ K \in \left]-1, -0.094\right[$$

The result of control is shown in Fig.3 for stabilizing the rhythm $1:1$, using 20000 times of iterations, for the following suitable conditions:

$$\begin{cases} K = -0.9 \\ APD_1 = 240\ ms \\ \varepsilon = 0.1 \end{cases}$$

In Fig.3, when attained $n = 11540$, the state $x(11540)$, is in the vicinity of the the fixed point x^*(or $1:1$ rhythm), the control takes the value $u(n) =$

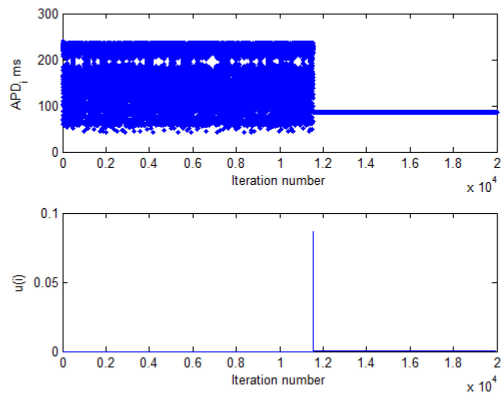

Fig. 3. Direct chaotic dynamics to the 1 : 1 rhythm or steady state under predictive control.

0.08655 ms, and to stabilize the system on the desired fixed point value $x^* = 86.41$ ms. And in Fig.4, we provide the cobweb plot for the same conditions to exhibit the evolution of iteration converges to 1 : 1 rhythm after activating of control.

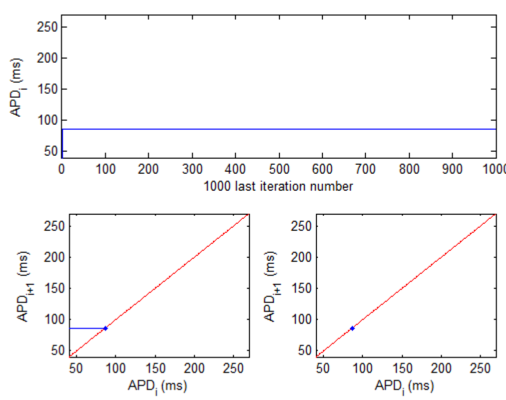

Fig. 4. Cobweb plot of the map (APD) under predictive contol at $t_s = 146$ ms.

The result of control is showing in Fig.5 for directing chaotic behavior to the 2 : 1 rhythm or steady state, we give the following suitable conditions:

$$\begin{cases} K = -0.3 \\ APD_1 = 240 \; ms \\ \varepsilon = 0.2 \end{cases}$$

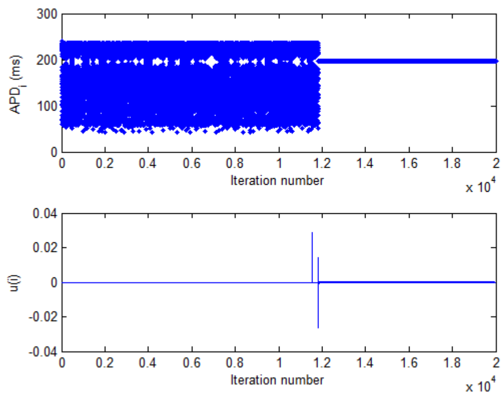

Fig. 5. Direct the chaotic dynamics to the 2 : 1 rhythm or syeady state under predictive control.

In Fig.5, when attained $n = 11840$, the state $x(11840)$, is in the vicinity of the fixed point x^* , the control takes the value $u(n) = -0.02653$ ms, and stabilizes the system on the fixed point value (or 2 : 1 rhythm) $x^* = 196.0073$ ms. And in Fig.6, we provide the cobweb plot for the same conditions to exhibit the evolution of iteration converges to 2 : 1 rhythm after activating of control

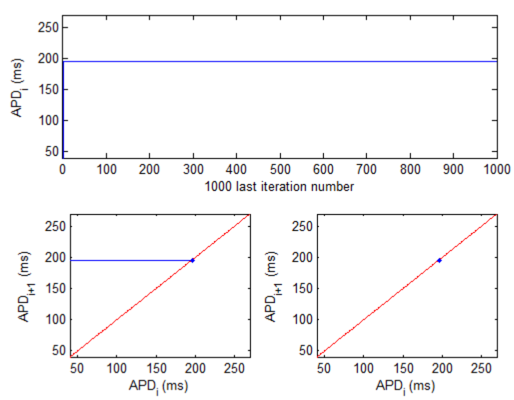

Fig. 6. Cobweb plot of 1D-map (APD) under predictive control at $t_s = 146$ ms, $K = -0.4$.

A majoration in the values of K is provided with 0.01 increment. As example, The range $(-1 \leq K \leq -0.73)$ is sufficient to direct chaotic dynamics to the fixed point for the first branch (or 1 : 1 rhythm) on the value 86.41 ms. Thus for example, using these following intervalls $(-0.72 \leq K \leq -0.56)$,

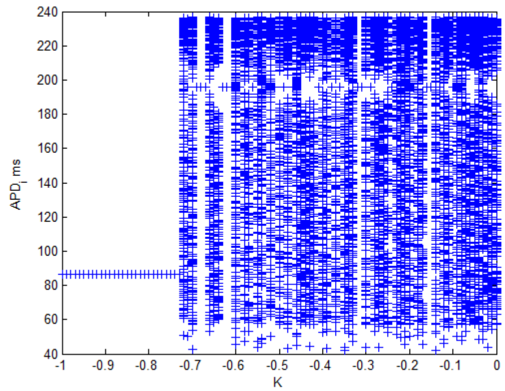

Fig. 7. Bifurcation diagram (APD_i $vs.$ K) for K between -1 et 0.

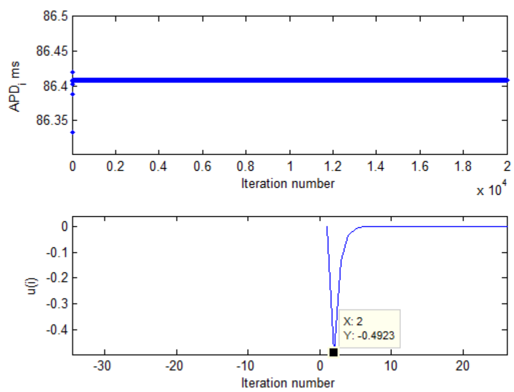

Fig. 8. Prevention of Chaotic behavior of the APD map (3) with predictive control for $t_s = 146$ ms, $APD_1 = APD^* = 86.42$ ms. $K = -0.9, \varepsilon = 0.8$. The dynamique stabilize from the next iteration at the fixed point with approximative value 86.41 ms. The map was iterated 20000 times.

$(-0.49 \leq K \leq -0.4)$ and $(-0.34 \leq K \leq -0.1)$, we obtain the stabilization to another fixed point value (or $2:1$ rhythm) which equal to 196.0073 ms. After applying the control, only the near iterates $x(n)$ to the fixed point which will converge to this stable state (See Fig.7).

Suitable values for the control of chaotic behavior when $t_s = 146$ ms of one-dimensional map (APD) model are presented on the bifurcation diagram, at each K the equation (10) was iterated 20000 times and the first 19800 iterates discarded to suppress transients due to initial conditions [27]. Increment in K was 0.01 (see Fig.7). To prevent the chaotic behavior with predictive methode, it is necessary may to take the fixed point value as an initial condition $x_1 = x^*$.

Hence, the predictive contol triggers in the next iteration and we direct the chaotic dynamics to the steady state since the first itration (See Fig.8).

4 Conclusion

It is regarded that ventricular fibrillation (VF) is chaotic rhythm. It is then obvious that it would be very valuable to be able to control the chaos once it has started up; thus is then reverting the heart back to its period-1 orbit "normal sinus rhythm". There have been many papers publised on "chaos control". Some of these are experimental, While others involve modelling (both differential equations and maps). In the purpose of reduction of irregular behaviors in cardiac dynamics. The predictive control is applied to stabilize the 1D-map (APD) modelizing stimulated strand of ventricular muscle . We have obtained very satisfactory numerical results to direct the chaotic behavior to a stable equilibrium points from the state equation. This proves the effectiveness of this proposed method to stabilize the irregular heart rhythm in the physiological normal sinus rhythm.

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