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Symplectic Synchronization of Five-dimensional Hyperchaotic Systems

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Abstract. In this paper, the problems on chaos control and symplectic synchronization of five-dimensional hyperchaotic systems are considered. First, two new five-dimensional chaotic systems are introduced. Second, the adaptive feedback controller for symplectic synchronization of the systems is investigated. Sufficient conditions for the guaranteed symplectic stability of the synchronized errors are provided. In addition, numerical studies are also performed to verify the effectiveness of presented schemes.
Keywords: chaos, symplectic synchronization, five-dimensional hyperchaotic systems, adaptive controller.

1 Instroduction

Synchronization phenomena in coupled chaotic systems has been an important topic in nonlinear areas since it was introduced in the first time by Pecora and Carroll [1]. It is found that chaos has wide applications in the fields of chemical reactions[2], biological systems[3], and secure communications[4], etc. As a result, it has been widely explored in various fields of related research. The idea of synchronization is to use the output of the drive system to control the response system, so that the output of the response system follows the output of the drive system asymptotically.

In the past two decades, a variety of types of synchronization in dynamical systems such as complete synchronization [5], phase synchronization [6], lag synchronization [7], anti-synchronization [8] and projective synchronization[9] have been proposed. They represent the difference in the degree of correlation between interacting systems. Among all kinds of synchronization schemes, generalized synchronization [10–12] which has been first reported by Rulkov is the most significant and interesting one. Generalized synchronization is defined as the presence of some functional relation between the states of the master system and those of the slave system. The generalized synchronization is noticeable because it is more applicable to secure communication than complete synchronization by introducing an additional function. More recently, the authors present the idea of symplectic synchronization [13], which can be seen as

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a extension of generalized synchronization. As a result, there also exists great potential of the application of the symplectic synchronization.

In this paper, the symplectic synchronization between five-dimensional hyperchaotic chaotic systems is investigated and an adaptive control law is proposed using Lyapunov stability theory and adaptive control theory. The proposed method is illustrated by applications to two nonidentical five-dimensional chaotic systems and the simulation results demonstrate the effectiveness of the proposed control method. The rest of this paper is organized as follows. The problem formulation and systems description are performed in section 2 and section 3, respectively. Numerical simulations are performed in section 4 to verify the effectiveness of the presented schemes, and concluding remarks are made in the final section.

2 Symplectic synchronization

Consider a drive system

$$\dot{x} = f(x) \tag{1}$$

and the controlled response system

$$\dot{y} = g(y) + u(x, y, t) \tag{2}$$

where $x = x_1, x_2, ..., x_n$ and $y = y_1, y_2, ..., y_n$ are the state vectors, and $f, g : R^n \to R^n$ are two continuous nonlinear vector functions, $u(x, y, t)$ is is the vector control input.

For symplectic synchronization, the error system is defined as

$$e(t) = y(t) - H(x, y, t) - F(t) \tag{3}$$

where $F(t)$ is a given function of time in different form, such as a regular or a chaotic function.

Our control goal is to design the controller u(t,x,y) for the response system (2), such that the error system (3) can be asymptotically stable at the zero equilibrium, i.e. $\lim_{t\to\infty} e(t) = 0$.

Note that when $H(x, y, t) + F(t) = x$, Eq. (3) reduces to the complete synchronization given in [5]. In addition, when $H(x, y, t) = x$, Eq. (3) reduces to the generalized synchronization given in [11]. Therefore both complete synchronization and generalized synchronization can be seen as a special case of symplectic synchronization.

3 Systems description

Hu [14] proposed a five-dimensional hyperchaotic Lorenz system by introducing two state feedback controllers to the classical three-dimensional Lorenz system, which is described by

$$\begin{cases} \dot{x_1} = a(x_2 - bx_1) + x_4 \\ \dot{x_2} = bx_1 - x_2 - x_1x_3 - x_5 \\ \dot{x_3} = -cx_2 + x_1x_2 \\ \dot{x_4} = dx_2 + x_1x_3 \\ \dot{x_5} = rx_2 \end{cases} \tag{4}$$

where $x_1, x_2, x_3, x_4,$ and x_5 are state variables, a, b, c, d and r are all positive real parameters. When the system parameters are $a = 10, b = 28, c = 8/3, d = 2, r = 3$, the system (4) is chaotic bursting. The five-dimensional hyperchaotic system with initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0), x_5(0)) = (-2, 1, 4, 2, -3)$ is depicted in Fig.1.

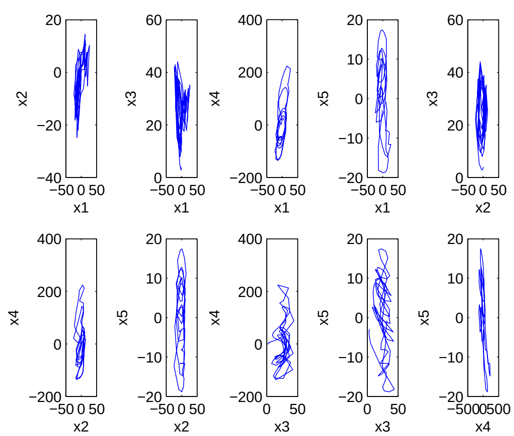

Fig. 1. Typical dynamical behaviors of five-dimensional hyperchaotic Lorenz system.

More recently, a new five-dimensional hyperchaotic system is given by [15]:

$$\begin{cases} \dot{y_1} = a_1(y_2 - by_1) + y_2y_3y_4y_5 \\ \dot{y_2} = b_1(y_2 + by_1) + y_1y_3y_4y_5 \\ \dot{y_3} = -y_3 + 0.1y_1^2 \\ \dot{y_4} = -c_1y_4 + y_1y_2y_3y_5 \\ \dot{y_5} = -d_1(y_5 - by_4) - r_1y_1 + y_1y_2y_3y_4 \end{cases} \tag{5}$$

where $y_1, y_2, y_3, y_4,$ and y_5 are state variables, $a_1, b_1, c_1, d_1,$ and r_1 are all positive real parameters. When we selected the parameters as $a_1 = 37, b_1 = 14.5, c_1 = 10.5, d_1 = 15$ and $r_1 = 9.5$, the system eyhibits a hyperchaotic behaviour, as shown in Fig.2.

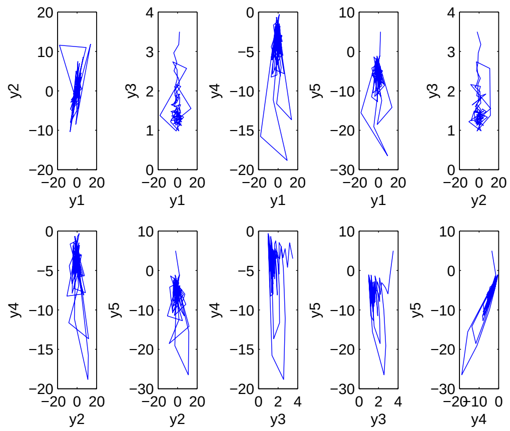

Fig. 2. Typical dynamical behaviors of five-dimensional hyperchaotic system (5).

4 Simulation examples

Suppose the master system is defined in (4) which drives the slave system given in the following form

$$\begin{cases} \dot{y_1} = a_1(y_2 - by_1) + y_2y_3y_4y_5 + u_1 \\ \dot{y_2} = b_1(y_2 + by_1) + y_1y_3y_4y_5 + u_2 \\ \dot{y_3} = -y_3 + 0.1y_1^2 + u_3 \\ \dot{y_4} = -c_1y_4 + y_1y_2y_3y_5 + u_4 \\ \dot{y_5} = -d_1(y_5 - by_4) - r_1y_1 + y_1y_2y_3y_4 + u_5 \end{cases} \tag{6}$$

where $y_i(i = 1, 2, 3, 4, 5)$ are state variables, and $u_i(i = 1, 2, 3, 4, 5)$ are external control inputs.

In this study, we take $F_1(t) = sin(x_5(t))$, $F_2(t) = sin(x_1(t))$, $F_3(t) = sin(x_2(t))$, $F_4(t) = sin(x_3(t))$, $F_5(t) = sin(x_4(t))$. They are chaotic functions of time. $H(x, y, t)$ is choose as $H_i(x, y, t) = -x_i^2 y_i, i = 1, 2, 3, 4, 5$. Then the error signals can be defined as

$$\begin{aligned} & e_i = y_i + x_i^2 y_i - sin(x_j) \\ & i = 1, 2, 3, 4, 5; j = \begin{cases} 5 & i = 1 \\ i - 1 & i \neq 1 \end{cases} \end{aligned} \tag{7}$$

From (7), we have

$$\begin{aligned} & \dot{e_i} = (1 + x_i^2)\dot{y_i} + 2x_i y_i \dot{x_i} - cos(x_j)\dot{x_j} \\ & i = 1, 2, 3, 4, 5; j = \begin{cases} 5 & i = 1 \\ i - 1 & i \neq 1 \end{cases} \end{aligned} \tag{8}$$

Then the detail error dynamics is as follows:

$$\begin{cases} \dot{e_1}(t) = (1+x_1^2)(a_1(y_2-by_1)+y_2y_3y_4y_5+u_1) \\ +2x_1y_1(a(x_2-bx_1)+x_4)-rcos(x_5)x_2 \\ \dot{e_2}(t) = (1+x_2^2)(b_1(y_2+by_1)+y_1y_3y_4y_5+u_2) \\ +2x_2y_2(bx_1-x_2-x_1x_3-x_5)-cos(x_1)(a(x_2-bx_1)+x_4) \\ \dot{e_3}(t) = (1+x_3^2)(-y_3+0.1y_1^2+u_3)+2x_3y_3(-cx_2+x_1x_2) \\ -cos(x_2)(bx_1-x_2-x_1x_3-x_5) \\ \dot{e_4}(t) = (1+x_4^2)(-c_1y_4+y_1y_2y_3y_5+u_4)+2x_4y_4(dx_2+x_1x_3) \\ -cos(x_3)(-cx_2+x_1x_2) \\ \dot{e_5}(t) = (1+x_5^2)(-d_1(y_5-by_4)-r_1y_1+y_1y_2y_3y_4+u_5)+2rx_5y_5x_2 \\ -cos(x_4)(dx_2+x_1x_3) \end{cases} \tag{9}$$

Choose a positive definite Lyapunov function as

$$V(e_1,e_2,e_3,e_4,e_5) = \frac{1}{2}(e_1^2+e_2^2+e_3^2+e_4^2+e_5^2) \tag{10}$$

The derivative of V along the trajectories of the system is given by

$$\begin{aligned} \dot{V} &= (e_1\dot{e_1}+e_2\dot{e_2}+e_3\dot{e_3}+e_4\dot{e_4}+e_5\dot{e_5}) \\ &= e_1((1+x_1^2)(a_1(y_2-by_1)+y_2y_3y_4y_5+u_1)+2x_1y_1(a(x_2-bx_1)+x_4) \\ &- rcos(x_5)x_2) \\ &+ e_2((1+x_2^2)(b_1(y_2+by_1)+y_1y_3y_4y_5+u_2) \\ &+ 2x_2y_2(bx_1-x_2-x_1x_3-x_5)-cos(x_1)(a(x_2-bx_1)+x_4) \\ &+ e_3((1+x_3^2)(-y_3+0.1y_1^2+u_3)+2x_3y_3(-cx_2+x_1x_2) \\ &- cos(x_2)(bx_1-x_2-x_1x_3-x_5)) \\ &+ e_4((1+x_4^2)(-c_1y_4+y_1y_2y_3y_5+u_4)+2x_4y_4(dx_2+x_1x_3) \\ &- cos(x_3)(-cx_2+x_1x_2)) \\ &+ e_5((1+x_5^2)(-d_1(y_5-by_4)-r_1y_1+y_1y_2y_3y_4+u_5)+2rx_5y_5x_2 \\ &- cos(x_4)(dx_2+x_1x_3)) \end{aligned} \tag{11}$$

To guarantee the error dynamical system converge to the origin asymptotically, we propose the following adaptive control law

$$\begin{cases} u_1 = \frac{-2x_1y_1(a(x_2-bx_1)+x_4)+rcos(x_5)x_2-e_1}{(1+x_1^2)} - (a_1(y_2-by_1)+y_2y_3y_4y_5) \\ u_2 = \frac{-2x_2y_2(bx_1-x_2-x_1x_3-x_5)+cos(x_1)(a(x_2-bx_1)+x_4)-e_2}{(1+x_2^2)} \\ -(b_1(y_2+by_1)+y_1y_3y_4y_5) \\ u_3 = \frac{-2x_3y_3(-cx_2+x_1x_2)+cos(x_2)(bx_1-x_2-x_1x_3-x_5)-e_3}{(1+x_3^2)} - (-y_3+0.1y_1^2) \\ u_4 = \frac{-2x_4y_4(dx_2+x_1x_3)+cos(x_3)(-cx_2+x_1x_2)-e_4}{(1+x_4^2)} - (-c_1y_4+y_1y_2y_3y_5) \\ u_5 = \frac{-2rx_5y_5x_2+cos(x_4)(dx_2+x_1x_3)-e_5}{(1+x_5^2)} - (-d_1(y_5-by_4)-r_1y_1+y_1y_2y_3y_4) \end{cases} \tag{12}$$

Thus

$$\dot{V} = -(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2) \leq 0 \tag{13}$$

Since $\dot{V} \leq 0$, we have $\lim_{t\to\infty} e_i(t) = 0, i = 1, 2, 3, 4, 5$.

In the following, some numerical simulations about the symplectic synchronization between the drive system (4) and the response system (6) are given to verify the effectiveness of the proposed method. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system. The system parameters are selected as $a = 10, b = 28, c = 8/3, d = 2, r = 3$ and $a_1 = 37, b_1 = 14.5, c_1 = 10.5, d_1 = 15, r_1 = 9.5$, such that the drive system and the response system are hyperchaotic with no control applied. The initial conditions are selected to be $x(0) = (-2, -3, 4, 2, 3)$ for the drive system and $y(0) = (2, 5, 4, -4, 8)$ for the response system, respectively.

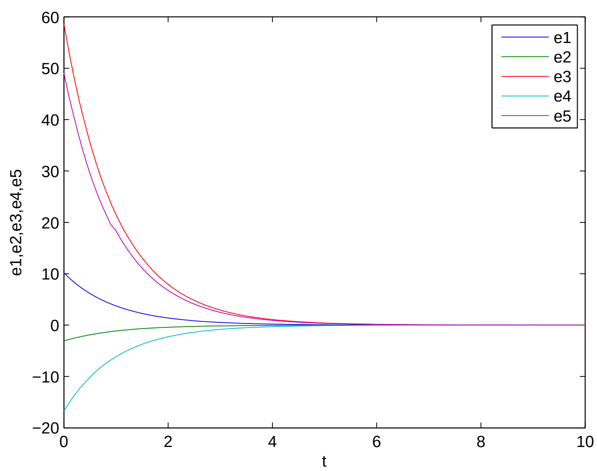

Fig. 3. Time response of the symplectic synchronization errors.

Fig.3 shows the time evolution of the symplectic synchronization errors, which displays that the errors tend to zero as $t \to \infty$. These results show that symplectic synchronization between two different five-dimensional hyperchaotic system has been achieved with our designed adaptive controllers (12).

5 Conclusions

In this paper, we have studied the robust adaptive symplectic synchronization between two different five-dimensional hyperchaotic systems based on adaptive control and stability theory. The effectiveness of the proposed approach have been verified by the numerical simulations. The presented control method can be applied in many other hyperchaotic systems and are valuable to be applied to the realization in engineering.

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