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Detecting Chaos Using the Strength of Extreme L Rule

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Abstract. For a time series we consider the quantity L, formally similar to angular momentum, and strength of a rule, named the extreme L rule, about actual value of L and two future time series elements. A four-dimensional vector is assigned to a scalar time series by the numerical titration with low level noise. Mean strength of the rule and standard deviation, for two levels of added noise, are the components of this vector. It is shown that the values of Lyapunov exponent are close if vectors of time series described by Feigenbaum map are close. Three sets of four-dimensional vectors are formed – Rg, St and Ch, for artificial regular, stochastic and chaotic time series respectively and their 2-norm distances are estimated. In such a manner we can distinguish chaos with small noise from pure noise, including colored noise. Chaotic time series are constructed using iterative maps (Feigenbaum, Henon, sine-square, ... map) and three-dimensional ODEs (Lorenz, Ueda, Rikitake, ... equations). For an experimental time series we find its four-dimensional vector and classify it, computing 2-norm distances to sets Rg, St and Ch. The proposed method is tested on a time series measured in the experiment with RLC circuit. Our result is in agreement with the results obtained by conventional methods.
Keywords: Time series, Chaos, Noise, Strength of rule, Numerical titration.

1 Introduction

In 2001 Poon and Barahona proposed a numerical titration procedure for detection of chaos [8]. Their method is analogous to neutralization of the acid with added base, for the purpose of determination of acid concentration. Poon and Barahona add noise of increasing standard deviation to time series until its nonlinearity goes undetected. Limiting value of standard deviation gives a relative measure of chaos intensity.
Hu and Raman are confirmed chaotic nature of AFM tip oscillations by Lyapunov exponent and noise titration calculations [3]. Chaotic human ventilation was identified in the same manner [10]. Freitas, Letellier and Aguirre are found that noise titration fails to distinguish colored noise from low-dimensional chaos [2]. Roulin, Freitas and Letellier propose usage of this method for detecting a

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nonlinear component in dynamics [9].
The genuine Poon-Barahona method is not applied here, but the level of added noise is restricted on two low values. Noise affects the strength of a rule, we formulate using a quantity formally similar to angular momentum. Then we introduce four-dimensional vectors describing time series. Characterization of a measured time series is possible by comparison of two vectors – vector of measured time series and vector of an artificial time series of known character. Our goal is to avoid difficulties in computing entropies, dimensions and Lyapunov exponents of a measured time series [5].

2 Extreme L Rule

For a time series A_1, A_2, ..., A_{2000} we compute

$$Z_k = (1-b)\frac{A_k}{A_{max}} + bG_k \tag{1}$$

where

$$A_{max} = max\{|A_k|; k = 1, 2, ..., 2000\} \tag{2}$$

and G_k is Gaussian noise. In numerical titration procedure we will take two levels of noise: $b = 10^{-6}$ and $b = 10^{-3}$.
The quantity formally similar to angular momentum is

$$L_j = X_j V_{yj} - Y_j V_{xj}, \quad j = 2, 3, ..., 999 \tag{3}$$

with

$$V_{xi} = X_i - X_{i-1}, \quad V_{yi} = Y_i - Y_{i-1} \tag{4}$$

and

$$X_k = Z_{2k+1}, \quad Y_k = Z_{2k} \tag{5}$$

We now formulate the extreme L rule. For N_α (N_β) different values of m

$$\begin{aligned} L_m > L_\alpha \quad (L_m < L_\beta) \quad \Rightarrow \\ sign(X_{m+1} - X_m) = const. \quad and \quad sign(Y_{m+1} - Y_m) = const. \end{aligned} \tag{6}$$

Strength of the rule is

$$N_\alpha + N_\beta - 2 \tag{7}$$

For example, we take a time series with the following rule

$$\begin{aligned} L_{80} > L_{400} > L_{300} > L_{658} > \cdots \\ L_{50} < L_{600} < L_{200} < L_{381} < \cdots \\ sign(X_{81} - X_{80}) = sign(X_{401} - X_{400}) = sign(X_{51} - X_{50}) = \\ sign(X_{601} - X_{600}) = sign(X_{201} - X_{200}) = sign1 \\ sign(Y_{81} - Y_{80}) = sign(Y_{401} - Y_{400}) = sign(Y_{51} - Y_{50}) = \\ sign(Y_{601} - Y_{600}) = sign(Y_{201} - Y_{200}) = sign2 \end{aligned} \tag{8}$$

where

$$sign(X_{301}-X_{300}) \neq sign1 \quad or \quad sign(Y_{301}-Y_{300}) \neq sign2$$
$$sign(X_{382}-X_{381}) \neq sign1 \quad or \quad sign(Y_{382}-Y_{381}) \neq sign2 \tag{9}$$

Then

$$L_\alpha = L_{300}, \quad L_\beta = L_{381}, \quad N_\alpha = 2, \quad N_\beta = 3, \quad N_\alpha + N_\beta - 2 = 3 \tag{10}$$

The strength of extreme L rule in this case is three.

3 Four-Dimensional Vectors Assigned to Time Series

If level of noise in titration procedure is $b = 10^{-6}$, we find strength of the rule $S_1 \pm \delta S_1$. If level of noise is $b = 10^{-3}$, the strength of the rule is $S_2 \pm \delta S_2$. Then we construct four-dimensional vector

$$< S_1, \delta S_1, S_2, \delta S_2 > \tag{11}$$

and assign it to considered scalar time series. In graphical representation of this vector, lengths of red, yellow, green and blue lines are equal to S_1, δS_1, S_2 and δS_2 (figure 1, figure 2).
Distance between two sets of four-dimensional vectors, $Set1$ and $Set2$, is

$$d(Set1, Set2) = min\{|| < P_{i1}, \delta P_{i1}, P_{i2}, \delta P_{i2} >$$
$$- < Q_{j1}, \delta Q_{j1}, Q_{j2}, \delta Q_{j2} > ||; \quad i = 1, 2, 3, ..., \quad j = 1, 2, 3, ...\} \tag{12}$$

where vectors $< P_{i1}, \delta P_{i1}, P_{i2}, \delta P_{i2} >$ belong to $Set1$, vectors $< Q_{j1}, \delta Q_{j1}, Q_{j2}, \delta Q_{j2} >$ belong to $Set2$ and $|| \; ||$ denotes 2-norm.

Considering damped oscillations

$$Ce^{-\beta t} \sin \omega t, \quad t = 0.01k \tag{13}$$

we can see that values of β and values of ω are close if corresponding four-dimensional vectors are close (figure 2). Therefore dynamics described by a time series and vector assigned to this time series are strongly connected.

For Feigenbaum map

$$A_i = 1 - qA_{i-1}^2 \tag{14}$$

values of the Lyapunov exponent

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} ln|2qA_{i-1}| \tag{15}$$

are approximately equal if vector components are approximately equal (figure 3).

We consider now time series $A_k = \xi(0.01k)$ constructed usin Lorenz equations

$$\frac{d\xi}{dt} = 10(\eta - \xi), \quad \frac{d\eta}{dt} = r\xi - \eta - \xi\zeta, \quad \frac{d\zeta}{dt} = \xi\eta - \frac{8}{3}\zeta \tag{16}$$

with

$$\xi(0) = 9.4, \quad \eta(0) = 8.8, \quad \zeta(0) = -7.8 \tag{17}$$

If vectors describing time series are close, then values of r are close (figure 4).

In many cases we have considered, if two vectors $< P_1, \delta P_1, P_2, \delta P_2 >$ and $< Q_1, \delta Q_1, Q_2, \delta Q_2 >$ are close, namely

$$\begin{aligned} &|| < P_1, \delta P_1, P_2, \delta P_2 > - < Q_1, \delta Q_1, Q_2, \delta Q_2 > || \\ << &|| < P_1, \delta P_1, P_2, \delta P_2 > ||, || < Q_1, \delta Q_1, Q_2, \delta Q_2 > || \end{aligned} \tag{18}$$

then characters of corresponding time series are very similar. Reversed statement is not valid. If characters of two time series (type of chaos or type of regularity for example) are very similar, their vectors can be very different.

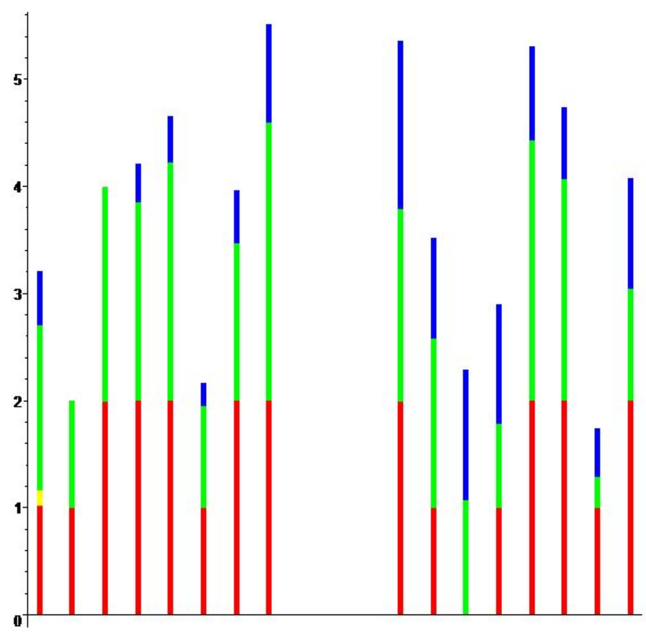

Fig. 1. First bundle contains vectors of eight stochastic time series. Lengths of red, yellow, green and blue lines are equal to S_1, δS_1, S_2 and δS_2. Eight vectors in the second bundle are assigned to eight regular time series (undamped periodic and quasi-periodic oscillations).

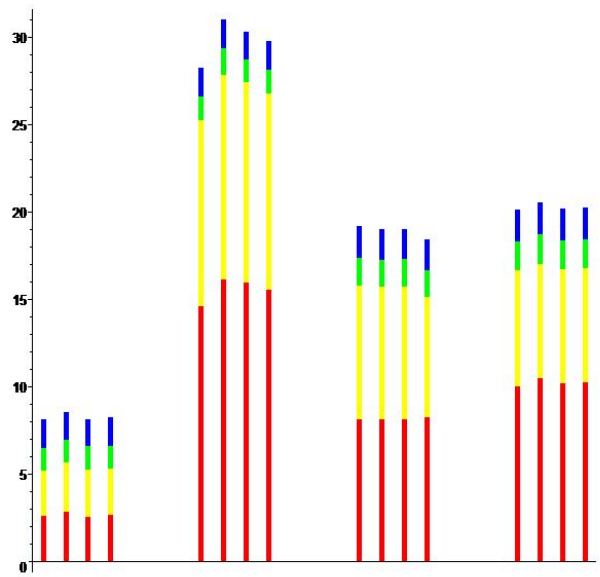

Fig. 2. Vectors of damped oscillations $Ce^{-\beta t}\sin\omega t$ $(t = 0.01k)$, where (1) $\omega = 1$ and $\beta = 0.001 - 0.0011$ (first bundle), (2) $\omega = 1$ and $\beta = 0.003 - 0.0031$ (second bundle), (3) $\beta = 0.001$ and $\omega = 4.0 - 4.005$ (third bundle), (4) $\omega = 3.005 - 3.0051$ and $\beta = 0.002 - 0.0021$ (fourth bundle).

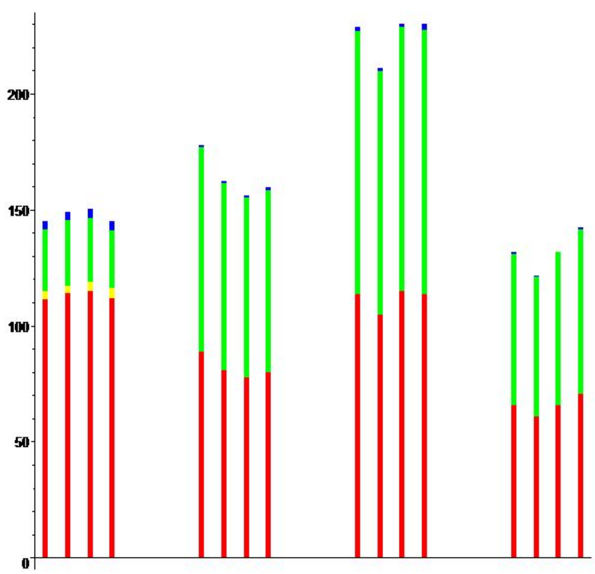

Fig. 3. Vectors of Feigenbaum map. Values of λ are: (1) from -0.0203 to -0.0191 (first bundle), (2) from 0.4064 to 0.4093 (second bundle), (3) from 0.5399 to 0.5407 (third bundle), (4) from 0.6425 to 0.6457 (fourth bundle).

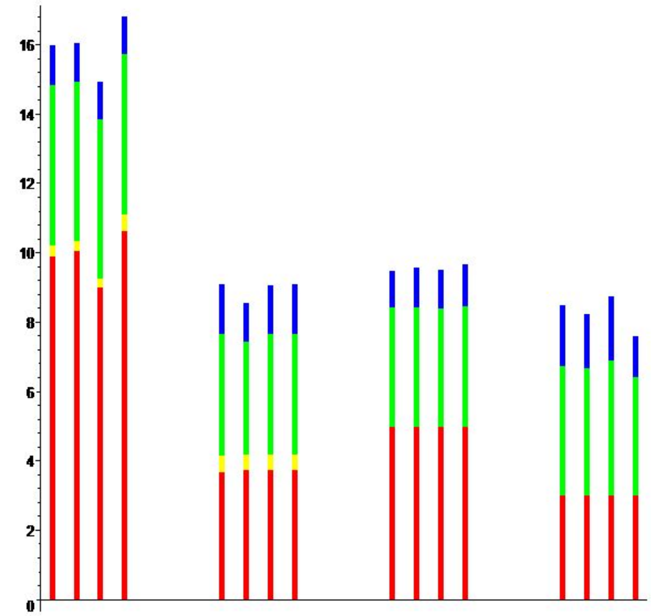

Fig. 4. Vectors of time series constructed using Lorenz equations. Four vectors in first bundle describe regular time series with $r = 10.2 - 10.4$. The following vectors describe chaotic time series with $r = 29.800002 - 29.800005$ (second bundle), $r = 30.100002 - 30.100005$ (third bundle), $r = 30.200002 - 30.200005$ (fourth bundle).

4 Sets of Vectors *Rg*, *St* and *Ch*

We now form three sets (Rg, St and Ch) containing four-dimensional vectors of artificial regular, stochastic and chaotic, with small noise, time series.
Set Rg contains vectors of regular time series (damped and undamped, periodic and quasi-periodic, oscillations and Feigenbaum map in regular regime). Form of the time series elements, in the case of undamped oscillations, is

$$A_k = \sum_i \left[C_1 \cos \omega_i k + C_2 \cos(\Omega_i k + \phi_i) \right] \tag{19}$$

Vectors of stochastic time series there are in set St. We have generated random numbers with uniform and Gaussian distributions (white and colored noise). Colored noise in St is generated by Bartosch algorithm [1].

Few hundred chaotic time series, described by vectors belonging to set Ch, are constructed using iterative maps (Feigenbaum, Henon, sine-square, etc) and three-dimensional ODEs (Lorenz, modified Lorenz, Rössler, Ueda, Rikitake, etc). The level of aded noise in these time series is from zero to 0.01%. This noise is included in A_j. The noise included in Z_k (eq. 1) is something else.

We are found out distances between sets Rg, St and Ch:

$$d(Rg, St) = 0.07, \quad d(Rg, Ch) = 0.15, \quad d(St, Ch) = 0.29 \tag{20}$$

5 Periodically Driven RLC Circuit

A time series K_i of length 25000 is measured in the experiment with periodically driven RLC circuit, performed by Kodba, Perc and Marhl [7]. K_i is the output voltage with a sampling rate of 500 measurements per second. Chaos is detected using basic methods – determinism test, attractor reconstruction and calculation of the largest Lyapunov exponent. The mutual information method and the false neighbor method yield the proper embedding delay and the proper embedding dimension [6].
We analyze subseries

$$A_i = K_{i+p} \quad (i = 1, 2, ..., 2000) \tag{21}$$

and confirm presence of chaos (table 1).

p	$vector$	$\frac{d(vector,Rg)}{\|\|vector\|\|}$	$\frac{d(vector,St)}{\|\|vector\|\|}$	$\frac{d(vector,Ch)}{\|\|vector\|\|}$
0	$< 1, 0, 1.55, 0.88 >$	0.033	0.198	0.246
32	$< 1, 0, 1.46, 0.96 >$	0.060	0.235	0.204
65	$< 0, 0, 0.54, 0.73 >$	0.086	0.259	1.262
82	$< 1, 0, 1.54, 1.03 >$	0.047	0.261	0.168
83	$< 0, 0, 0.37, 0.60 >$	0.083	0.158	1.928
582	$< 0, 0, 0.11, 0.31 >$	1.011	0.068	5.284
624	$< 0, 0, 0.12, 0.33 >$	0.351	0	4.893
718	$< 0, 0, 0.16, 0.37 >$	0.631	0.070	4.124
824	$< 0, 0, 0.16, 0.39 >$	0.571	0.047	3.916
970	$< 0, 0, 0.18, 0.39 >$	0.527	0	3.805
923	$< 2, 0, 2.40, 1.39 >$	0.149	0.149	0.044
941	$< 2, 0, 2.60, 1.46 >$	0.168	0.150	0.034
969	$< 2, 0, 2.29, 1.49 >$	0.150	0.192	0.004
997	$< 2, 0, 2.11, 1.04 >$	0.148	0.175	0.048
999	$< 2, 0, 2.26, 1.19 >$	0.109	0.134	0.017
1001	$< 2, 0, 2.21, 1.10 >$	0.098	0.135	0.016

Table 1. Considering time series measured by Kodba, Perc and Marhl we find regular subseries ($p = 0, 32, 65, 82, 83$), stochastic subseries ($p = 582, 624, 718, 824, 970$) and chaotic subseries ($p = 923, 941, 969, 997, 999, 1001$).

6 Kobe Earthquake

We have computed vectors for subseries of the recorded Kobe earthquake time series [4]. Most often we find

$$d(vector, Rg) > 0, \quad d(vector, Ch) > 0, \quad d(vector, St) = 0 \tag{22}$$

with

$$vector =< 1, 0, 1, 0 > \ or \ < 2, 0, 2, 0 > \ or \ < 3, 0, 3, 0 > \quad (23)$$

For other vectors, the distances often satisfy

$$d(vector, Ch) > d(vector, Rg) >> d(vector, St) > 0 \quad (24)$$

and rarely much greater is replaced by greater

$$d(vector, Ch) > d(vector, Rg) > d(vector, St) > 0 \quad (25)$$

We can conclude that analyzed time series is stochastic one.

7 EEG Time Series

We consider here EEG time series E_k $(k = 1, 2, ..., 3595)$ recorded on a patient undergoing ECT therapy for clinical depression [11]. A vector is assigned to

$$A_j = E_{j+p} \quad (j = 1, 2, ..., 2000; \, p = 1, 2, ..., 1595) \quad (26)$$

with certain p. Then we compute distance from the vector to sets Rg, St and Ch. When p is increasing, the vector oscillates between St and Ch, or between Rg and Ch (table 2).

p	$vector$	$\frac{d(vector,Rg)}{\|\|vector\|\|}$	$\frac{d(vector,St)}{\|\|vector\|\|}$	$\frac{d(vector,Ch)}{\|\|vector\|\|}$
1	$< 1, 0, 1.27, 1.54 >$	0.27	0.38	0.06
2	$< 13, 0, 13.15, 0.39 >$	0.07	0.02	0.43
3	$< 1, 0, 1.53, 2.12 >$	0.31	0.37	0.02
4	$< 13, 0, 13.07, 0.33 >$	0.07	0.02	0.42
5	$< 1, 0, 1.17, 1.20 >$	0.09	0.38	0.06
6	$< 13, 0, 13.13, 0.34 >$	0.07	0.02	0.43
7	$< 1, 0, 1.55, 2.19 >$	0.33	0.37	0.04
8	$< 13, 0, 13.13, 0.34 >$	0.07	0.02	0.43
751	$< 1, 0, 1.40, 1.97 >$	0.27	0.38	0.08
752	$< 17, 0, 17.03, 0.17 >$	0.08	0.12	0.55
753	$< 1, 0, 1.77, 2.63 >$	0.43	0.39	0.18
754	$< 17, 0, 17.01, 0.10 >$	0.08	0.12	0.55
755	$< 1, 0, 1.60, 2.39 >$	0.38	0.38	0.11
756	$< 17, 0, 17.01, 0.10 >$	0.08	0.12	0.55
757	$< 1, 0, 1.39, 1.92 >$	0.26	0.39	0.09
758	$< 17, 0, 17, 0 >$	0.09	0.12	0.55

Table 2. Results we are obtained investigating subseries of EEG time series. For p from 1 to 8, the vector approaches to Ch, then to St, again to Ch, and so on. For p from 751 to 758 similar oscillations happen, but St is replaced by Rg.

8 Conclusion

A new method for time series analyze is proposed here. The extreme L rule and four-dimensional vectors assigned to time series are in the basis of this method. In plenty of examples, closeness of vectors leads to similar characters of time series described by these vectors. We compute distances from vector of a measured time series to sets Rg, St and Ch, containing vectors of artificial regular, stochastic and chaotic time series. If minimal distance is significantly smaller than other distances, we assume that the character of time series is determined correctly with high probability.
In our further investigations we can add vectors of other artificial time series to sets Rg, St and Ch. It is also possible to replace Ch with a few sets corresponding to different types of chaos.

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