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Adaptive Control of Mixed-Interlaced forms

Kyriakos G. Vamvoudakis[1], Manolis A. Christodoulou[2]

[1] Automation and Robotics Research Institute, The University of Texas at Arlington, 7300 Jack Newell Blvd. S., Ft. Worth, TX 76118
[2] Technical University of Crete, Chania, Crete, Greece
(E-mail: kyriakos@arri.uta,edu , manolis@isc.tuc.gr)

Abstract: In this paper we combine forwarding and backstepping techniques to stabilize mixed interlaced systems. All the signals in the close loop remain semiglobally ultimately bounded the output signal y follows a desired trajectory signal y_d, with bounded derivatives up to m^{th} order. We also present simulation examples that prove the adaptation of mixed interlaced forms, using a backstepping controller.

1 Introduction

Recent technological developments have forced control engineers to deal with extremely complex systems that include uncertain and possibly unknown nonlinearities, operating in highly uncertain environments. Man has two principal objectives in the scientific study of his environment: he wants to understand and to control. The two goals reinforce each other, since deeper understanding permits firmer control, and, on the other hand, systematic application of scientific theories inevitably generates new problems which require further investigation, and so on. Nonlinear control includes two basic forms of systems, the feedforward systems and the feedback systems.

The strict feedback systems can be controlled using the well known backstepping technique. The purpose of backstepping is the recursive design of a controller for the system by selecting appropriate virtual controllers. Separate virtual controllers are used in order to stabilize every equation of the system. In every step we select appropriate update laws. The strict feedforward systems can be controlled using the forwarding technique that is something like backstepping but in reverse order. Other cases of systems that can be converted to the previous forms are part of a larger class of systems that are called interlaced systems as described by [9], and [3]. In these systems we combine backstepping and forwarding techniques together in order to recursively design feedback control laws. Interlaced systems are not in feedback form, nor in feedforward form. These systems have a specific methodology that differs from backstepping and forwarding. We don't start from the top equation, neither from the bottom.

Other special cases of systems are part of other forms that we call mixed interlaced and we introduce their study in the present paper. The methodology is based on classical interlaced systems and is developed by the authors. We want

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to make the systems solvable by one of the well known backstepping and forwarding methods. This can be reached after some specific steps that convert the system into a known form. We start from the middle equation and we continue with the top. The previous method is based on classical interlaced forms that are introduced by [9] and [3] and can be extended to more complicated systems.

A lot of researchers developed a series of results that generalized and explained the basic idea of nonlinear control. Teel [10] in his dissertation introduced the idea of nested saturations with careful selection of their parameters to achieve robustness for nonlinear controllers. After Teel , Sepulchre, Jankovic and Kokotovic [9] proposed a new solution to the problem of forwarding that is based on a different Lyapunov solution.

The paper consists of four sections including the current one. The next section introduces the meanings of Adaptive Control, Backstepping and Forwarding techniques. In Section 3, the main body of this paper, the mixed interlaced forms are analyzed. Finally section 4 draws some concluding remarks.

2 Background in Adaptive Control

The history of adaptive control began from the early 1950's. With the passing of the years a lot of papers and books have been published. These research activities have proposed solutions for basic problems and for broader classes of systems. Especially the interest for nonlinear adaptive control began from the mid-1980's. A lot of great scientists, such as Kokotovic et al [2], Lewis et al [4], Ioannou and Sun [7], Christodoulou and Rovithakis [5] have studied adaptive control and its applications extensively.

Adaptive control is a powerful tool that deals with modeling uncertainties in nonlinear (and linear) systems by on line tuning of parameters. Very important research activities include on-line identification and pattern recognition inside the feedback control loop.

Through time, adaptive control has existed big development (Sepulchre et al [9]) in order to control plants with unknown dynamics that appear linearly. Adaptive control is based on Lyapunov design.

In order to make it clear, a short example will be reported. Let us consider the nonlinear plant:

$$\dot{x} = u + \theta x^2 \tag{1}$$

And select the control law as:

$$u = -qx - \hat{\theta} x^2 \tag{2}$$

which, if the estimated θ ($\hat{\theta}$) is equal to real θ such that $\hat{\theta} \equiv \theta$, then the result is a close loop system of the form:

$$\dot{x} = -qx \tag{3}$$

The filtered version of the signals x is:

$$x_f = \frac{1}{s+1} x^2 \tag{4}$$

The prediction error e is:

$$e = x - \hat{x} = (\theta - \hat{\theta}) x_f = \hat{\theta} x_f \tag{5}$$

We use the commonly normalized update law:

$$\dot{\hat{\theta}} = -\frac{\gamma}{1 + x_f^{\,2}} x_f^{\,2} \tilde{\theta} \tag{6}$$

The previous update law is linear. It can be proved that $\tilde{\theta}$ does not converge to zero faster than exponentially and the easiest case is:

$$\tilde{\theta} = e^{-\gamma t} \tilde{\theta}(0) \tag{7}$$

Finally the close loop system has the following form:

$$\dot{x} = -x + \tilde{\theta} x^2 \tag{8}$$

where for simplicity q substituted with 1 and by substituting $\tilde{\theta}$ from the previous equation is obtained:

$$\dot{x} = -x + e^{-t} \tilde{\theta}(0) x^2 \tag{9}$$

where for simplicity γ substituted with 1.
It is easy to see that the explicit solution of the previous is determined by the following equation:

$$x = \frac{2x(0)}{x(0)\tilde{\theta}(0)e^{-t} + [2 - x(0)\tilde{\theta}(0)]e^{-t}} \tag{10}$$

From the previous it is clear that if $x(0)\tilde{\theta}(0) < 2$ then it is obvious that x converge to zero as $t \to \infty$. At the case that $x(0)\tilde{\theta}(0) > 2$, at the time:

$$t_{esc} = \frac{1}{2} \ln \frac{x(0)\tilde{\theta}(0)}{x(0)\tilde{\theta}(0) - 2}$$

the difference of the two terms of the exponential in the denominator becomes zero, that is:

$$|x(t)| \to \infty \text{ as } t \to t_{esc}$$

The previous model is unstable (x goes to infinity at t_{esc}) and Lyapunov design models must be specified in order to achieve stabilization.
Let choose the following Lyapunov function:

$$V = \frac{1}{2}x^2 + \frac{1}{2}(\hat{\theta} - \theta)^2 \tag{11}$$

The derivative of the Lyapunov function for our nonlinear plant is:

$$\dot{V} = x(u + \theta x^2) + (\hat{\theta} - \theta)^2 \dot{\hat{\theta}}$$

In order to find a control and an update law we must specify:

$$\dot{V} \le -x^2 \Rightarrow x(u + \theta x^2) + (\hat{\theta} - \theta)^2 \dot{\hat{\theta}} \le -x^2 \tag{12}$$

From the previous equation in order to remove the unknown θ we use the update law:

$$\dot{\hat{\theta}} = x^3$$

And the control law is:

$$u = -x - \hat{\theta}x^2$$

Both control law and update law yield $\dot{V} \le -x^2$ such that stability maintains in opposition to the previous approach without Lyapunov.

Adaptive control in most cases has tracking error that converges to zero.

i) Adaptive Backstepping Design

Backstepping ([1], [2], [4], [7]) is a recursive design for systems of the form:

$$\begin{aligned}
\dot{x}_1 &= x_2 + \varphi_1^{\mathrm{T}}(x_1, x_2)\theta \\
\dot{x}_2 &= x_3 + \varphi_2^{\mathrm{T}}(x_1, x_2, x_3)\theta \\
\dot{x}_3 &= u \ + \varphi_3^{\mathrm{T}}(x_1, x_2, x_3)\theta
\end{aligned}$$

with state $x=[x_1^T, x_2^T, x_3^T]$ and control input u. The value θ is a p x 1 vector which is constant and unknown. The function φ_1 depends only to x_1, x_2 function φ_2, φ_3 depends only to x_1, x_2, x_3.

The purpose of backstepping is the recursive design of a controller for the previous system by selecting appropriate virtual controllers. The virtual controller for the first equation of the system is x_2 and is used to stabilize the first equations, the virtual controller for the middle equation is x_3 and is used to stabilize the first two equations, and finally the controller for the last is u. We use separate virtual controllers in order to stabilize every equation of the system. In every step we select appropriate update laws.

In classical backstepping, the output is selected as the state x_1 and the purpose of adaptive control is to make this state to follow a desired trajectory x_{1d}.

Adaptive backstepping design is a Lyapunov based design [4]. The previous procedure can be applied only to systems that have (or transformed to) the previous form (strict feedback).

ii) Adaptive Forwarding Design

Forwarding ([9]) is something like backstepping but for strict feedforward systems. Let us introduce forwarding technique with an example such as:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_3^2 + x_2 u \\ \dot{x}_2 &= x_3 - x_3^2 u \\ \dot{x}_3 &= u \end{aligned}$$

In the previous example we do not have feedback paths.

Firstly we stabilize the last equation ($\dot{x}_3 = u$). We take the following Lyapunov function:

$V_3 = \frac{1}{2} x_3^2$ and a feedback to stabilize the system is $u = -x_3$. With the previous we augment $\dot{x}_3 = -x_3$ by the middle equation, and write our system in the cascade form:

$$\begin{aligned} \dot{x}_2 &= \varphi_2(x_3) \\ \dot{x}_3 &= -x_3 \end{aligned}$$

where $\varphi_2(x_3) = x_3 - x_3^3$ is the interconnection term. $\dot{x}_2 = 0$ is stable and $\dot{x}_3 = -x_3$ is GAS and LES. The next step is to construct Lyapunov function V_2 for the augmented system when V_3 is given.

After some specific steps we reach the following control law:

$$u = -x_3 - (x_2 + x_3 + \frac{x_3^{\ 3}}{3})(1 + x_3^{\ 2}) \tag{13}$$

3 Mixed Interlaced Forms

a. Introduction and Linearization Method

To begin with we consider the following third order mixed interlaced system and via an example we will introduce mixed interlaced forms [12]:

$$\begin{aligned}
\dot{x}_1 &= -\beta_3 x_1 + a_{32}(c_3 - x_1)x_2 + a_{31}(c_3 - x_1)x_3 \\
\dot{x}_2 &= -\beta_2 x_2 + a_{23}(c_2 - x_2)x_1 + a_{21}(c_2 - x_2)x_3 \\
\dot{x}_3 &= -\beta_1 x_3 + (c_1 - x_3)u(t)
\end{aligned} \tag{14}$$

The previous system is not in feedback nor is it in feedforward form because of specific terms such as x_1x_2, x_1x_3, x_2x_3. The Jacobi linearization of the previous system is a chain of integrators.

Instead from starting on top, we start from the middle equation and treat x_3 as virtual control and we want $\dot{x}_2 = -x_2$ for stability. There exists a Lyapunov function of the form $V_1 = \frac{1}{2}x_2^{\ 2}$ and a stabilizing feedback is $x_3 = \frac{-\beta_2 x_2 + a_{23}c_2 x_1 - a_{23}x_2 x_1 + x_2}{a_{21}x_2 - a_{21}c_2}$ which is $x_3 = a(x_1, x_2)$. We employ one step of backstepping to stabilize the middle equation augmented by the top equation of our system:

$$\begin{aligned}
\dot{x}_1 &= -\beta_3 x_1 + a_{32}(c_3 - x_1)x_2 + \\
&+ a_{31}(c_3 - x_1)\left(\frac{-\beta_2 x_2 + a_{23}c_2 x_1 - a_{23}x_2 x_1 + x_2}{a_{21}x_2 - a_{21}c_2}\right) + \\
&+ a_{31}(c_3 - x_1)v \\
\dot{x}_2 &= -x_2 + v
\end{aligned} \tag{15}$$

where the control x_3 has been augmented to $x_3 = a(x_1, x_2) + v$. With v=0, the equilibrium $(x_1, x_2) = (0, 0)$ is globally stable and forwarding yields the following Lyapunov function:

$$V_2 = V_1 + \lim \tilde{x}_1(s)$$
$$= \frac{1}{2}x_2^{\,2} + \frac{1}{2}\xi_1^{\,2}, \qquad (16)$$
$$\xi_1 = x_1 + x_2 - \frac{1}{2}x_2^{\,2} - \frac{1}{3}x_2^{\,3}$$

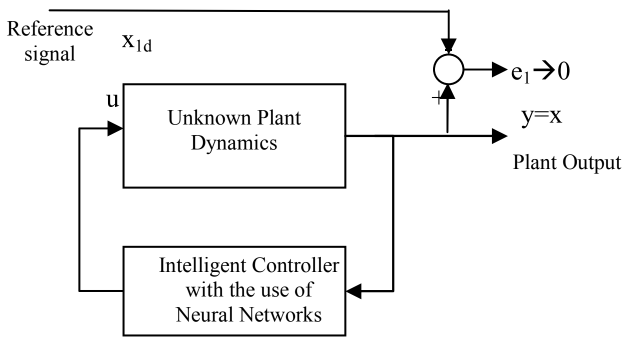
The feedback law: $v = -(1-x_2^{\,2})\xi_1$ maintains the system globally stable and the augmented control is

$$x_3 = a_1(x_1, x_2) \ + v = \frac{-\beta_2 x_2 + a_{23}c_2 x_1 - a_{23}x_2 x_1 + x_2}{a_{21}x_2 - a_{21}c_2} \qquad (17)$$
$$-(1-x_2^{\,2})\xi_1 \ = a_2(x_1, x_2, \xi_1)$$

In order to stabilize our system we apply the backstepping technique.

b. Mixed Interlaced Forms, Adaptive Control and Simulations

Adaptive Control of dynamical systems has been an active area of research since the 1960's. The system is described by the following figure:



Because we have 3 states our controller design is described with Kaynak et al [1] controller in 3 steps.

Step1: In this step we want to make the error between x_1 and x_{1d} $(=y_d)$ as small as possible.

The previous is described by the following equation:

$$e_1 = x_1 - x_{1d} \qquad (18)$$

We take the derivative of e_1. After that we have:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} \Rightarrow \dot{e}_1 = f_1(x_1) + g_1(x_1)x_2 - \dot{x}_{1d} \quad (19)$$

by using x_2 as the virtual control input. The previous equation can be changed by multiplication and division with $g_1(x_1)$ to the following form:

$$\dot{e}_1 = g_1(x_1)[g_1^{-1}(x_1)f_1(x_1) + x_2 - g_1^{-1}(x_1)\dot{x}_{1d}] \quad (20)$$

We choose the virtual controller as:

$$x_{2d} = x_2 = -g_1^{-1}(x_1)f_1(x_1) + g_1^{-1}(x_1)\dot{x}_{1d} - k_1 e_1 \quad (21)$$

where k_1 is a positive constant. In order to approximate the unknown nonlinearities (functions $f_1(x_1)$ and $g_1(x_1)$) we use RBF Neural Networks ([11]). A Neural Network based virtual controller is used as follows:

$$x_{2d} = -\theta_1^{\mathrm{T}}\xi_1(x_1) + \delta_1^{\mathrm{T}}n_1(x_1)\dot{x}_{1d} - k_1 e_1 \quad (22)$$

where we have substituted the unknown nonlinearities $g_1(x_1)^{-1}f_1(x_1)$ and $g_1(x_1)^{-1}$ with the RBF Neural Networks $\theta_1^{\mathrm{T}}\xi_1(x_1)$ and $\delta_1^{\mathrm{T}}n_1(x_1)$ respectively based on Lyapunov stability ([6], [8]).

We take the following adaptation laws (σ-modification) in order to avoid large values of the weights:

$$\dot{\theta}_1 = \Gamma_{11}[e_1\xi_1(x_1) - \sigma_1\theta_1] \quad (23)$$
$$\dot{\delta}_1 = \Gamma_{12}[-e_1 n_1(x_1)\dot{x}_{1d} - \gamma_1\delta_1] \quad (24)$$

with σ_1, γ_1 small and positive constants and $\Gamma_{11}=\Gamma_{11}^T>0$, $\Gamma_{12}=\Gamma_{12}^T>0$ are the adaptive gain matrices.

Step 2: In this step we make the error between x_2 and x_{2d} as small as possible. The previous is described by the following equation:

$$e_2 = x_2 - x_{2d} \quad (25)$$

We take the derivative of e_2. After that we have:

$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{x}_{2d} = f_2(\bar{x}_2) + g_2(\bar{x}_2)x_3 - \dot{x}_{2d} \\ &= g_2(\bar{x}_2)[g_2(\bar{x}_2)^{-1} f_2(\bar{x}_2) + x_3 - g_2(\bar{x}_2)^{-1}\dot{x}_{2d}] \end{aligned} \tag{26}$$

By taking the x_{3d} as a virtual control input and by substituting the unknown nonlinearities $g_2(\bar{x}_2)^{-1} f_2(\bar{x}_2)$ and $g_2(\bar{x}_2)^{-1}$ with the RBF Neural Networks $\theta_2^{\mathrm{T}}\xi_2(\bar{x}_2)$ and $\delta_2^{\mathrm{T}} n_2(\bar{x}_2)$ respectively based on Lyapunov stability ([6], [8]), we have:

$$x_{3d} = -e_1 - \theta_2^{\mathrm{T}}\xi_2(\bar{x}_2) + \delta_2^{\mathrm{T}} n_2(\bar{x}_2)\dot{x}_{2d} - k_2 e_2 \tag{27}$$

We take the following adaptation laws (σ-modification) in order to avoid large values of the weights:

$$\begin{aligned} \dot{\theta}_2 &= \Gamma_{21}[e_2\xi_2(\bar{x}_2) - \sigma_2\theta_2] \\ \dot{\delta}_2 &= \Gamma_{22}[-e_2 n_2(\bar{x}_2)\dot{x}_{2d} - \gamma_2\delta_2] \end{aligned} \tag{28}$$

with σ_2, γ_2 small and positive constants and $\Gamma_{21}=\Gamma_{21}^T>0$, $\Gamma_{22}=\Gamma_{22}^T>0$ are the adaptive gain matrices.

Step 3(Final): In this step we make the error between x_3 and x_{3d} as small as possible.
The previous is described by the following equation:

$$e_3 = x_3 - x_{3d} \tag{29}$$

We take the derivative of e_3. After that we have:

$$\begin{aligned} \dot{e}_3 &= \dot{x}_3 - \dot{x}_{3d} = f_3(\bar{x}_3) + g_3(\bar{x}_3)u - \dot{x}_{3d} \\ &= g_3(\bar{x}_3)[g_3(\bar{x}_3)^{-1} f_3(\bar{x}_3) + u - g_3(\bar{x}_3)^{-1}\dot{x}_{3d}] \end{aligned} \tag{30}$$

Where u is the control input and by substituting the unknown nonlinearities $g_3(\bar{x}_3)^{-1} f_3(\bar{x}_3)$ and $g_3(\bar{x}_3)^{-1}$ with the RBF Neural Networks $\theta_3^{\mathrm{T}}\xi_3(\bar{x}_3)$ and $\delta_3^{\mathrm{T}} n_3(\bar{x}_3)$ respectively, we have:

$$u = -e_2 - \theta_3^{\mathrm{T}}\xi_3(\bar{x}_3) + \delta_3^{\mathrm{T}} n_3(\bar{x}_3)\dot{x}_{3d} - k_3 e_3 \tag{31}$$

We take the following adaptation laws (σ-modification) in order to avoid large values of the weights:

$$\dot{\theta}_3 = \Gamma_{31}[e_3\xi_3(\bar{x}_3) - \sigma_3\theta_3]$$
$$\dot{\delta}_3 = \Gamma_{32}[-e_3 n_3(\bar{x}_3)\dot{x}_{3d} - \gamma_3\delta_3] \qquad (32)$$

with σ_3, γ_3 small and positive constants and $\Gamma_{31}=\Gamma_{31}^T>0$, $\Gamma_{32}=\Gamma_{32}^T>0$ are the adaptive gain matrices.

In order to prove the stabilization of mixed interlaced systems we apply the previous described by [1] and we perform the following simulations:

We make the assumption that $c_1>>x_1$, $c_2>>x_2$, $c_3>>x_3$ and $a_{21}=a_{32}=\beta_1=\beta_2=\beta_3=1$, $c_1=9.99$, $c_2=6.66$, $c_3=3.33$. Also we want our desired output to be $y_d=sin(t)$.

Figs. 1-6 show the simulation results of applying the controller for tracking the desired signal y_d. From figure 1 we can see that good tracking performance is obtained. Figure 2 shows the trajectory of the controller. Figure 3 shows the phase plane of the system. Figure 4 shows the error e_1, Figure 5 shows the error e_2 and finally Figure 6 shows the error e_3.

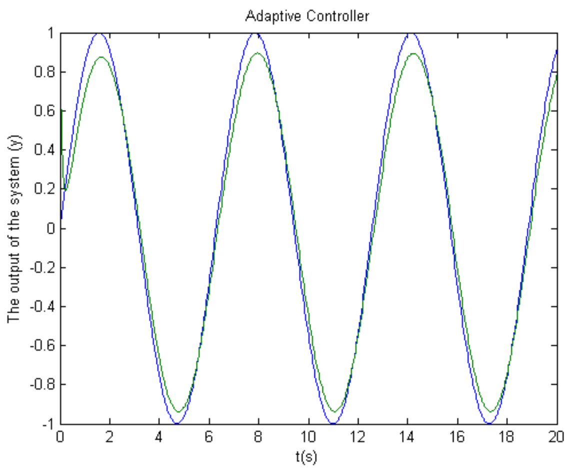


Fig. 1: The output of the system under adaptive controller.

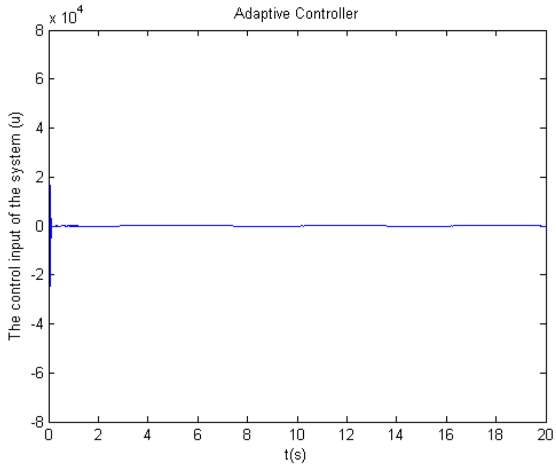


Fig. 2: The trajectory of the adaptive controller.

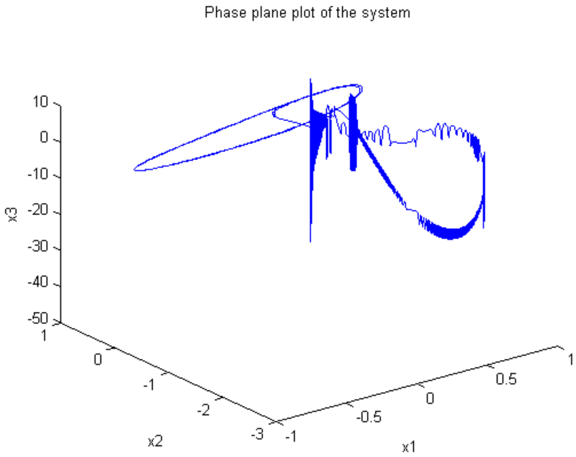


Fig. 3: The phase plane plot of the system.

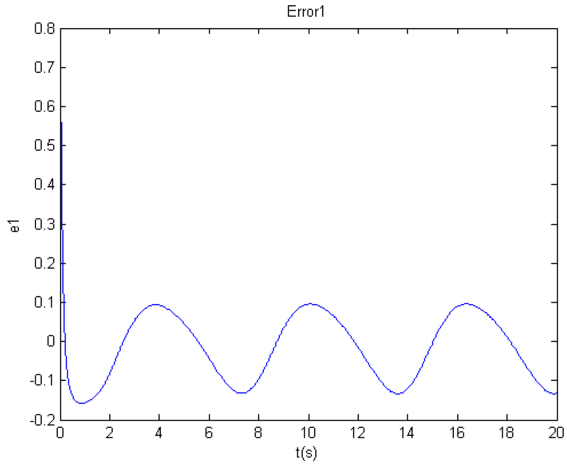


Fig. 4: Error e_1

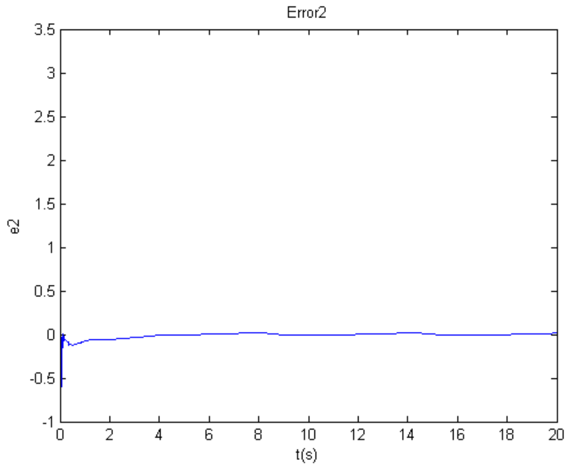


Fig. 5: Error e_2

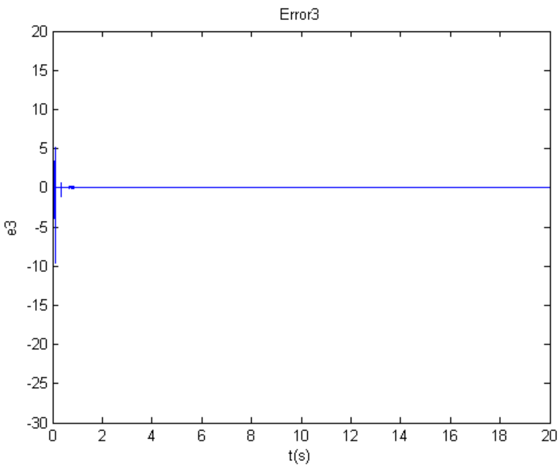


Fig. 6: Error e_3

4 Conclusion

In this paper, we recognize a new form of systems that we call mixed interlaced form. We apply the well known backstepping and forwarding techniques via specific steps. Also Lyapunov functions can be selected to approve convergence and stability. A lot of systems have the mixed interlaced form. For example we can think systems in biological models that have many terms from different states. After the appropriate selection of the controller we can apply adaptive control to make the systems follow a desired trajectory.

The tracking error is bounded and is established on the basis of the Lyapunov approach. Finally, only the states of the unknown plant which are related to the reduced order model are assumed to be available for measurement.

The authors hope that the proposed approach would serve as a promising tool to analyze more complex systems.

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