The Chaotic Analysis of Financial Time Series: Classification of Foreign Exchange Rates Series via Their Exponential Divergence Curves

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Abstract. In this work we evaluate the notable results of four interrelated successive works ([2–5]) dealing with the classification properties and temporal evolution of foreign exchange rates series (ForEX). The main idea in these works can be conceptualized through the behavior of the exponential divergence curves of financial time series that make a clear distinction for both spatial (between countries) and temporal (between different time segments of ForEX series) patterns. Despite being a well known concept, the use of exponential divergence curves for the classification of ForEX series is a relatively new concept. The classification procedure discussed here is based on the surrogate testing procedure where the statistics gathered from the original system is compared to the ones that are gathered from a completely randomized system. Our new researches on the data during the period of present economic recession (January 2008-October 2009) by calculating the largest Lyapunov exponent (LLE) has shown that the earlier classification of countries based on LLE's holds true. By a similar approach, we have investigated the temporal evolution of the exponential divergence distance metrics where we have developed a computationally consistent procedure to obtain the metrics for various ForEX series. Finally we obtained strong indicators for the distinction of the temporal evolution of ForEX series for developed and developing countries. We discuss possible reasons for the existing separation of temporal structures.
Keywords: Lyapunov Exponents, surrogate test, randomness, fluctuation, nonlinear classification, foreign exchange rates.

1 Introduction

In 1966, Benoit Mandelbrot introduced the basic principle of a Martingale in finance theory as to describe the efficient market hypothesis (EMH). The idea

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can be written as follows: the random process $z(t)$ is produced by a fair game if,

$$\langle z(t+\Delta t)\rangle_{\Phi} = z(t) \tag{1}$$

where the average is calculated via an information source Φ. Typically such an information may be evaluated through the historical observation of $z(t)$ as $\{z(t-\Delta t), z(t-2\Delta t), \ldots\}$. If $z(t)$ is a martingale, then the historical observations are irrelevant in predicting the future prices, $\langle z(t+\Delta t)\rangle \approx z(t) + R$ for some return ratio R [1]. In this work, our approach towards financial series is different, such that historical observations of financial time series effects to the current conditions where nonlinear statistics obtained from the series follow distinguishably different patterns comparing to a random series. Assumptions of classical financial time series analysis about the source of local or global fluctuations strongly dissociate from the ones belonging to the nonlinear deterministic analysis. Hsieh [6] and Scheinkman [7] emphasize the two essential sources of the fluctuations that originate from the business cycles: (1) according to the Box-Jenkins formalism, the overall economy is stable however it is perturbed by permanent external shocks (such as whether, wars). In this case the fluctuating behavior of the system is a result of external effects. (2) In chaotic growth models, the system comes from a nonlinear dynamic one and has an intrinsic self-generating structure where the system is supposed to behave randomly. Although it is still possible to make short-term predictions, such effort is bounded by the information loss (kolmogorov entropy) due to the exponential divergence in phase space.

Few works investigate the similarity of nonlinear measures between original dynamics and surrogate data while measuring nonlinear similarity of time series. The independent works of Schreiber and Schmidtz [12] and Cellucci et al. [8] can be given as the two important contributions coming into prominence. The nonlinear similarity measures based on the exponential divergence of nearby trajectories was investigated by Cellucci et al. [8] where they have proposed an efficient procedure for estimating or to what extent a time series is noise corrupted. From their point of view the distance between exponential divergence curves ([13]) and the curves that were created from the phase-shuffled surrogates reduces while signal to noise ratio decreases. Originally they introduced the link between randomness and its effects on the statistics obtained from phase flow of the dynamics. If the the governing dynamics are known, then this effect may also be used to estimate the randomness of a time series [9]. One of the interpretation of this observation as verified by Schreiber and Schmidtz [12] is that the power of surrogate testing depends on the randomness level of the investigated time series. Thus, when the noise level of a time series is increased, then the nonlinear statistics gathered from the original series gets closer to the ones obtained from the surrogate ones.

Nonlinear phenomenon is widely discussed in financial studies due to its inevitable effects towards the evaluation of the nature of the considered system. When a system is governed by a deterministic law, the characteristics of the structure often show globally definable invariant measures such as fractal dimension and Lyapunov exponents as an extension of sensitive dependence to initial conditions. In this work, we try to observe the behavior of separation between ForEX series depending on the Lyapunov exponents as the nonlinear statistics. The sections are arranged as follows: in Section 2 we give the brief results of the literature dealing with the existence of nonlinear dynamics in financial systems. In Section 3 we give the basic results of the nonlinear similarity tests including the crisis period. To analyze the temporal patterns, we also investigate the historical reactions of the ForEX system towards time depended random fluctuation under the assumption of a *stable nonlinear dynamic* structure. Our conclusions and future perspectives are given in Section 4.

2 Deterministic Flow of Exchange Rates

The first studies related to the determination of nonlinearity in financial time series approach the problem via statistical frameworks like BDS test for independence or bispectrum test for statistical nonlinearity. In spite of the difficulties in data quality concept, studies related to financial data have found evidence for nonlinear or chaotic relationships. Due to the importance of a key variable for macroeconomic policy, the exchange rates data have been researched widely in financial analysis. Through all these, results of the studies covering exchange rate dynamics are summarized by [10] as follows:

- The tests based on correlation dimension and some others confirmed nonlinear structure in exchage rates.
- Limited evidence for chaos.
- Residuals gathered from suitable statistical models (ARCH and GARCH) do not show nonlinear characteristics, so the mentioned models are adequate.

On the other hand studies that tries to quantify the geometrical peculiarities of phase flow assume a self-similar structure which is a result of sensitive dependence to initial conditions. For example Çoban and Büyüklü [3] analyzed the time series of New Turkish Lira with respect to US Dollars in between August 2001 and February 2007. The applied locally projective filtering methodology removed most of the noise contaminant which irregularly shadows the original phase flow (see Figure.(1)). The filtered series has a correlation dimension of $D_2 = 4.5$ and Lyapunov exponent of $\lambda = 0.05$. We state that, the phase flow properties of daily ForeX return series exhibit continuous flow characteristics rather than a discrete chaotic map or high dimensional random walk (Figure.(1)). Obviously, for such flow properties, the exponential divergence metrics has sense in terms of nonlinear dynamic analysis. Naturally, the surrogate testing procedure is able to show whether the nature of the governing dynamics of considered series has a significantly different behavior from a stochastic system.

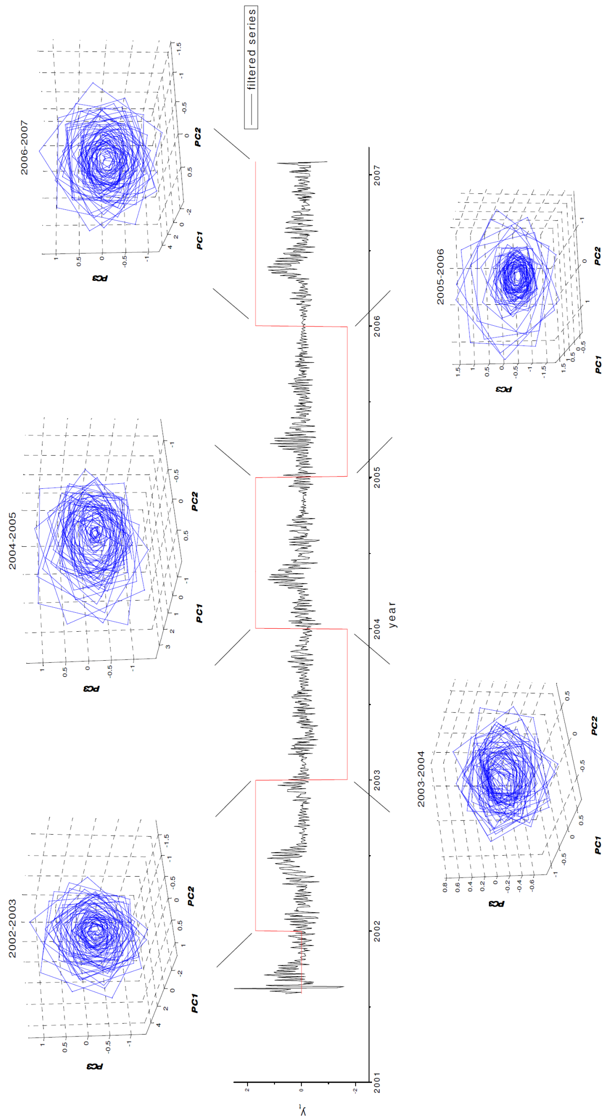

Fig. 1. The yearly phase flow representation of a highly noise reduced version of Turkish Lira-Dollar daily logarithmic return series in between 2001-2007. $PC_{1,2,3}$ stands for the first three principal directions used for projection.

3 Classification via Surrogate Testing

While working with real world observations generated by unknown dynamics some distributive properties should be acquired for the system to be analyzed. This may be realized by constructing random surrogate data sets that originate from the real observations. Surrogate data are designed to mimic constrained properties of original data to allow a comparison between assumed dynamics and some other stochastic structure which have similar properties. The two basic versions for the surrogate data are produced via shuffling observations (Random Surrogate) in random order and randomization of complex phase parameter of 'Fast Fourier Transformed' series (FFT). Any random shuffle of the original data destructs all linear and nonlinear dependencies between the observations and preserves only the empirical distribution. Thus, the observations became completely independent by construction which could be evaluated as an extreme case among surrogate methods. In FFT procedure, all linear properties of original sequence remains constant since the power spectrum is preserved. Thus, comparing invariant properties are gathered from the original signal and FFT sequences by point of statistical assumptions under defined significance level yield an interpretation about nonlinearity ([14]).

What makes the exchange rate changes in the recession era much more complicated is the intervention from the respective governments during recession period, in addition to the news in fundamentals as detailed in [2]. Apart from usual complex rules governing the exchange rate, during the recession, most of the countries tried to resist it by direct interventions. Sometimes it could be change in policy or some times pumping huge amount of money into financial market or imposing restrictions on pay or interest rates, huge economic stimulus package etc. One can for example refer to Euro News for country-wise details (for both EU and non-EU countries) [11]. For the present purpose, we like to see if all these initiatives have effect on the exchange rate in the said time period.

Based on the surrogate analysis, the work of Das & Das [2] has shown that the direct distance of divergence curves between original and surrogate data series is a good qualitative parameter for clustering exchange rates series. Here and in the following, we use the term *distance to a surrogate* as a nonlinear *similarity measure* which is the distance between exponential divergence curves and their surrogate ones. The analysis in [2] have shown that ForEX series can be classified in three categories via their distances to surrogates:

- Group A: For some countries (India, China, Sri Lanka) the distance is too high.
- Group B: For some countries (Australia, Malaysia, Thailand) the distance is moderate.
- Group C: For some countries (Canada, Japan, Singapore, Sweden, Switzerland, UK) the distance is small.

It is also conjectured that the behavior of macroeconomic indicators (for example, the balance of trade) are highly related to the mentioned nonlinear similarity in between the clusters.

In line with the work [2], nonlinear data analysis of the data during which the economic recession had started. In that work, we considered daily data for twelve countries, over the span of nearly 36 years. Now we investigate data from the same 12 countries for the periods of January 2008 to October 2009, as the present recession had started around July 2008. We have again calculated the largest Lyapunov exponent (LLE) and compared the LLE values calculated in previous work to the present values- that is LLE values previous and during recession.

During recession time, we find again that for countries whose LLE change is positive are China, India and Sri Lanka. They exactly correspond to earlier result of Group A. So we can say that countries with more the nonlinear structure in its ForExRate data, LLE change is positive. For other countries, we divide the change in two groups:

- When change is high- nearly -50% or more: Australia, Malaysia, Thailand and UK
- When change is moderate: Canada, Japan, Singapore, Sweden, Switzerland

Again, we see that the countries falling in group B (Except UK) has suffered high change. And finally, countries showing moderate change correspond to Group C. So, on the basis of change in LLE value during recession, we can conclude that the more nonlinear structure its foreign exchange rate shows the more its LLE changes.

3.1 Temporal Analysis of Surrogate Measures for ForEX Series

Do nonlinear similarity statistics exhibit a high frequency random scattering over long time intervals or they represent global fluctuations with wide wavelengths? In our simulations we show that the structure of the fluctuations of nonlinear statistics over long time intervals can be used to classify the ForEX series through their historical behavior. That means, for the last 3-4 decades financial time series have shown different evolutionary patterns. In this section our main aim is to give a consistent computational procedure depending on the concepts in [8] to obtain the conjunctural fluctuation of the distance to surrogate measures for various ForEX series. Here we use the term *conjunctural* since the basic consideration is on the observation of similarity variation for long time intervals (decades). For the simulations, we adopt the idea of Celluci et.al. [8] for our similarity measures. Here we will not explain the implementation of the algorithm in detail, instead we refer the reader to [5].

We start with the second formula of [8] defining the D statistics given in Equation.(2)

$$D = \frac{1}{N_k \, N_{surr}} \sum_{j=1}^{N_{surr}} \sum_{i=1}^{N_k} |\Lambda_{orig_j}(k_i) - \Lambda_{surr_j}(k_i)|. \tag{2}$$

where

$$\Lambda(k) = \frac{1}{N_{ref}} \sum_{n_0=1}^{N_{ref}} \log_2 \left(\frac{1}{|\mathcal{U}_\epsilon(\mathbf{s}_{n_0})|} \sum_{\mathbf{s}_n \in \mathcal{U}_\epsilon(\mathbf{s}_{n_0})} \frac{\|\mathbf{s}_{n_0+\delta k} - \mathbf{s}_{n+\delta k}\|}{\|\mathbf{s}_{n_0} - \mathbf{s}_n\|} \right) \tag{3}$$

In Equation.(3), $\| \cdot \| = \| \cdot \|_{L_2}$ and $\mathbf{s}_{n_0}$ are embedding vectors satisfying $\mathbf{s}_i = (y_i, y_{i+\tau}, \ldots, y_{i+(m-1)\tau})$ that are generated by the time-delay reconstruction of the time series sequence $\{y_i\}_{i=0}^{N}$ with delay time (τ) and embedding

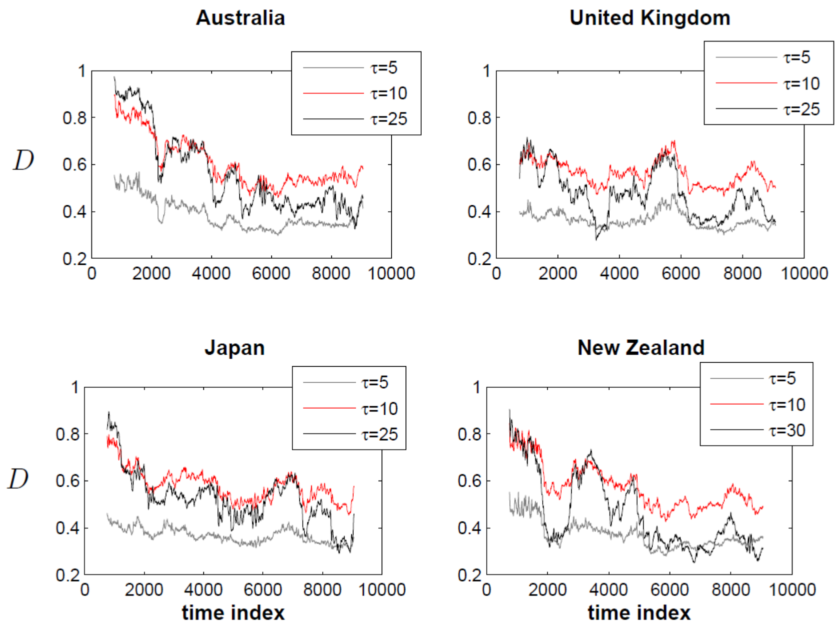

Fig. 2. D curves for the investigated ForEX series (subsegments set ${}_pD_{1500,5}$) where $m = 6$, $N_{ref} = 500$, $N_\epsilon = 20$, $w = 80$, $N_{surr} = 100$.

dimension (m). k is the time evolution parameter where δk is the time span of the reference trajectory to calculate Λ statistics. $\mathcal{U}_\epsilon(\mathbf{s}_{n_0})$ is the temporally uncorrelated neighborhood of $\mathbf{s}_{n_0}$ where at least N_ϵ vectors are found.

D is a measure of direct distance between Λ_{orig} and Λ_{surr} scores which can be evaluated as a similarity measure that defines the rate of 'geometrical equivalence' of Λ curves for original and random shuffled surrogates. Obviously D depends on many parameters which lead to the form for any p given in Equation(4).

$$_{(p)}D_{n,l}(\cdot) =_{(p)} D_{n,l}(m, \tau, N, N_k, N_{surr}, N_\epsilon, N_{ref}, w) \tag{4}$$

m embedding dimension
τ delay time
N length of time series sequence
N_k time span of the reference trajectory vectors
N_{surr} number of surrogates
N_ϵ neighbor trajectory vectors
N_{ref} number of reference trajectory vectors
w minimum window length for temporal correlation

The notation ${}_{(p)}\mathcal{D}_{n,l}$ is used to describe the p^{th} subinterval of the original series $\{y\}_{i=1}^N$ of length n with a shifting parameter l. Then the absolute distance

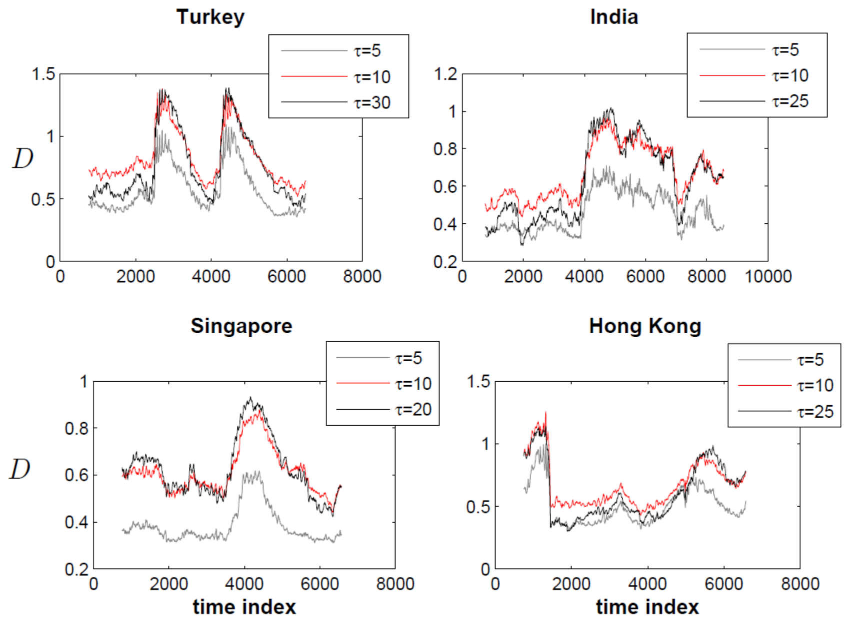

Fig. 3. D curves for the investigated ForEX series (subsegments set ${}_pD_{1500,5}$) where $m = 6$, $N_{ref} = 500$, $N_\epsilon = 20$, $w = 80$, $N_{surr} = 100$.

between two subsegments is l. The D statistics are calculated via reconstructing the phase space of these distinct subsegments.

Our findings on the ForEX rates let us to group the series in to two main categories. The first group (I) includes Australia, UK, Japan and New Zealand where their D statistics exhibit long term decreasing trend for the last 40 years (see Figure.(2)). The mentioned trend is obvious for Australia and Japan. The second group (II) includes Turkey, India, Singapore and Hong-Kong where each series has their own characteristic fluctuations, probably depending on the financial events in their history (see Figure.(3)). The fluctuations in the second group is conjunctural including the jumps across different global phases. The main difference of the sets is the decay pattern of fluctuations. Obviously the series in group I are all developed economies whereas the second group includes developing countries which brings the possibility of financial stability issues effecting nonlinear statistics. Such stability issues are discussed in [2] where the balance of trade data indicating the stability are consistent with their classification.

4 Results and Discussion

In this short work, we tried to analyze the classification of ForEX series based on Lyapunov exponents as the nonlinear statistics. We support that such a classification of financial time series has ability to put forward bright understanding towards the nature of the system. In [15] Thomas Schreiber argue the validity of nonlinear statistics whether they are supposed to diverge from the original value. He states that, "*we do not have to worry too much about the theoretical basis of the quantities. The results are validated by the statistical significance for the discriminative power. The classification of states can give valuable insights into the structure of a problem*". From this point of view, we give results of related works which make a clear distinction for both spatial (between countries) and temporal (between different time segments of ForEX series) patterns. Our next investigation will be based on the statistical clustering applications of nonlinear measures through a possible link between daily return risk and nonlinear statistics from a temporal perspective.

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