

## **Simulation and Control of Highly Maneuverable Aircraft under Turbulent Atmosphere using Nonlinear Dynamics Inversion Technique**

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### **Nomenclature**

V	Net Velocity of flight in body frame
$V_z$	Component of net velocity along body Z axis
$\alpha$	Angle of attack
$\beta$	Sideslip angle
$\mu$	Velocity vector roll angle
$\gamma$	Flight path angle
$\phi, \theta, \psi$	Body axis Roll, Pitch, Yaw angle respectively (Euler angles)
$\Omega=[p, q, r]$	Body axis Roll, Pitch, Yaw rate respectively (Angular velocities)
$\delta_a, \delta_e, \delta_r$	Aileron, Elevator, Rudder deflection respectively
T	Thrust command

**Abstract :** This paper presents a robust technique to design the flight controllers for the aircraft to fly under turbulent atmosphere as well as to perform maneuvers incorporating the whole highly nonlinear dynamics of the aircraft system. Aircrafts have 6 degrees of freedom (DOF) and so translational as well as rotational motion can be performed by the aircrafts in all those directions of freedom. Aircraft flight controller is required for the aircraft to undergo various flight conditions and to perform various types of maneuvers in a desired and controlled manner. In this study, completely nonlinear set of equations defining whole dynamics of the aircraft have been used for simulation and Nonlinear Dynamics Inversion (NDI) control technique has been used to design the controller of the flight vehicle. NDI control technique is a highly emerging time domain control methodology used to design the controllers for various types of highly nonlinear systems.

**Keywords :** Nonlinear dynamics inversion (NDI), Aircraft flight controller, Flight envelope.

### **1. Introduction**

In the field of aerospace vehicles, flight vehicle control law design methods have gained a lot of attention due to advancements in the theoretical concepts as



well as exponential improvements in the hardware technologies over past decades. Any sort of flight vehicle designed i.e aircraft, rocket, missile is required to perform its intended task and along with that, it is an essential requirement for the vehicle to perform the task in a well controlled and desired manner. To implement that, there is requirement of a controller which would ensure that the desired task is done in a controlled manner, even if there is sudden turbulence caused by wind gusts.

It is extremely important that these vehicles undergo any of the flight condition in a controlled manner to avoid failure of missions and lethal accidents. Several attempts have been made to design the controllers for all sort of flight vehicles. There are a number of control techniques to design controllers for nonlinear dynamics systems like aircrafts. In this study, NDI technique is discussed and implemented to design the controller of an aircraft.

The advantage of preferring NDI control technique over other linear control methods is that the linear control methods linearize the nonlinear system about the equilibrium points to approximate it into a linear system and then design a control law; In this manner the approximated linearized equations can predict the actual system performance only in a very small flight envelope i.e. in a small range of operations and if the system goes beyond that range then these equations do not simulate the actual behavior of the system and so the linear controllers stay no more effective; whereas the NDI control technique does not linearize the system about any equilibrium point, rather it incorporates all the system nonlinearities while developing the control law and so NDI controllers stay quite efficient over a wide flight envelope. Thus NDI is a very efficient control technique to design controllers for the nonlinear systems.

In the field of control of aerospace vehicles, NDI control technique has gained a lot of attention and it has been applied to many of aircraft applications, such as F-16[1], F-18 HARV [2], F-117 [3] for designing the controller.

## 2. The Aircraft Model

The modeled aircraft used in this study is McDonnell Douglas F-4 which is a highly maneuverable fighter aircraft. An attempt has been made to control the various flight conditions of the aircraft using NDI. The aircraft 6 DOF equations of motion are given by the following set of differential equations which explain the translational and rotational dynamics of the aircraft model [4,5].

$$\dot{V} = (f_x \cos \alpha \cos \beta + f_y \sin \beta + f_z \sin \alpha \cos \beta) / m$$

$$\dot{\alpha} = [(f_z \cos \alpha - f_x \sin \alpha) / (mV \cos \beta)] - p \cos \alpha \tan \beta + q - r \sin \alpha \tan \beta$$

$$\dot{\beta} = [(f_y \cos \beta - \sin \beta (f_x \cos \alpha + f_z \sin \alpha)) / (mV)] + p \sin \alpha - r \cos \alpha$$

$$\dot{\Omega} = [I]^{-1}[M - \Omega * ([I]\Omega)]$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta \sin\phi & \tan\theta \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi \sec\theta & \cos\phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\dot{\mu} = p_w + (q_w \sin\mu \tan\gamma) + (r_w \cos\mu \tan\gamma)$$

$$\dot{\gamma} = (q_w \cos\mu) - (r_w \sin\mu)$$

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \end{bmatrix} = C_1(\phi) C_2(\theta) C_3(\psi) \begin{bmatrix} V \cos\alpha \cos\beta \\ V \sin\beta \\ V \sin\alpha \cos\beta \end{bmatrix}$$

$$p_w = (p \cos\alpha + r \sin\alpha)(\cos\beta + \tan\beta \sin\beta) + \left( \frac{f_x \sin\alpha - f_z \cos\alpha}{mV} \right) \tan\beta$$

$$q_w = \left( \frac{f_x \sin\alpha - f_z \cos\alpha}{mV} \right)$$

$$r_w = [f_y \cos\beta - \sin\beta(f_x \cos\alpha + f_z \sin\alpha)] / (mV) \quad \text{--Eq.Set (1)}$$

Here  $f_x, f_y, f_z$  represent the net forces along X,Y,Z axes of the aircraft. Matrix [I] represent the moment of inertia matrix and M consists of the rotational moments about X,Y,Z axes of the aircraft and  $x_e, y_e, z_e$  represent the spatial position of the aircraft with respect to the earth axis system.  $C_1, C_2, C_3$  represent the transformation matrices about roll, pitch, yaw axis respectively.

### 3. NDI Control Law

In the implementation of NDI control law, the control commands are generated based upon the error signal generated from the desired state and current state values received from the sensors via feedback path. In the NDI technique, generally a robust 2-scale separation method is used which allows the order of the controller to be smaller [6,7]. The NDI law used in this study uses time scale separation between slow variables and fast variables and correspondingly generates the control commands. Any aircraft system can be represented by the following nonlinear vector form dynamics equation

$$\dot{x} = f(x) + g(x)u \quad \text{--Eq.(2)}$$

$x$  represents the vector representing state variables,  $f(x)$  represent nonlinear state dynamic function and  $g(x)$  represent the control distribution function. NDI

control law inverts the dynamics equation and then replaces the inherent rate of change of state variable by the desired rate of change of that variable to generate the required command which is fed to the system. Inverting eq. (2) we get

$$\mathbf{u} = \mathbf{g}(\mathbf{x})^{-1}[\dot{\mathbf{x}} - \mathbf{f}(\mathbf{x})] \quad \text{--Eq.(3)}$$

Applying NDI control logic, above equation is converted into a form as

$$\mathbf{u}_d = \mathbf{g}(\mathbf{x})^{-1}[\dot{\mathbf{x}}_d - \mathbf{f}(\mathbf{x})] \quad \text{--Eq.(4)}$$

where,  $\dot{\mathbf{x}}_d = \mathbf{k}(\mathbf{x}_d - \mathbf{x}) \quad \text{--Eq.(5)}$

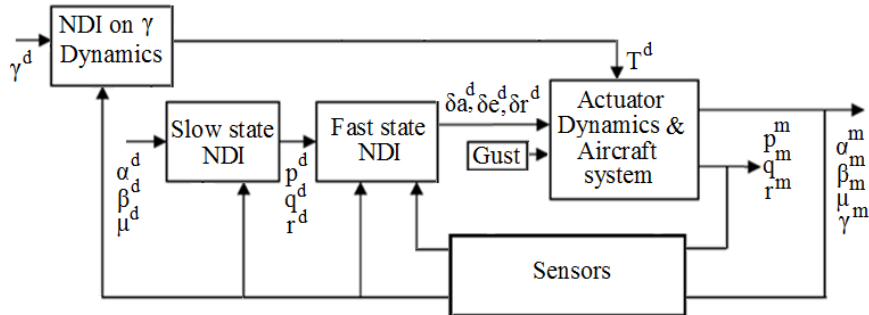
$\mathbf{x}_d$  in eq.(5) represents the vector consisting of the desired values of state variables and  $\mathbf{x}$  represents the vector consisting of the measured values of corresponding state variables obtained via feedback path.  $\mathbf{k}$  represents state gain matrix whose elements are the tuning parameters of the controller and  $\mathbf{u}_d$  represents the vector consisting of the control commands generated i.e. elevator, aileron, rudder deflections and thrust command which are to be fed to the aircraft system as control input.

#### 4. Application of NDI under various flight conditions

The purpose of this study is to control the various parameters of the aircraft for different flight conditions like cruise flight, sideslip flight, coordinated turn, pull-up maneuver, velocity vector roll maneuver etc. Table 1 shows all the flight conditions studied in this paper and shows the corresponding variables to be controlled in each flight condition so that the flight vehicle performs in the desired manner. For each case, NDI control law is implemented on the concerned set of governing nonlinear equations of the aircraft system and control commands corresponding to the desired states are generated.

**Table 1. Various flight conditions and corresponding control variables**

Flight conditions	Control variables
Cruise flight	$\alpha, \beta, \mu, \gamma$
Steady sideslip flight	$\alpha, \beta, \Psi, \gamma$
Coordinated turn	$\alpha, \beta, \gamma, \dot{\Psi}$
Pull-up maneuver	$p, q, r, V$
Velocity vector roll maneuver	$\alpha, \beta, \gamma, \dot{\mu}$



**Figure 1. NDI control approach for control variables  $\alpha, \beta, \mu, \gamma$**

Figure 1 explains the implementation of NDI control law for the cases in which control variables are  $\alpha, \beta, \mu, \gamma$ . The desired states are represented by  $(\alpha^d, \beta^d, \mu^d, \gamma^d)$ . Similarly other variables can be controlled in the similar fashion for other cases. In present case,  $(\alpha, \beta, \mu, \gamma)$  act as slow state variables whereas  $(p, q, r)$  act as fast state variables. NDI is applied on slow state variables as well as fast state variables as explained in equations (2)-(5) and control surfaces deflection commands  $(\delta_a^d, \delta_c^d, \delta_r^d)$  are generated.

These command values are passed through the actuator dynamics system so as to ensure that the commands generated are well within the control surfaces deflection limits as well as within the maximum rate of deflection of control surfaces. Thrust command ( $T^d$ ) is generated by applying NDI on  $\gamma$  dynamics equation in case of various flight conditions except pull-up and pull-down maneuvers as in these maneuvers, the thrust command is generated by applying NDI on dynamics equation of velocity.

### 5. Simulation, Control and Results

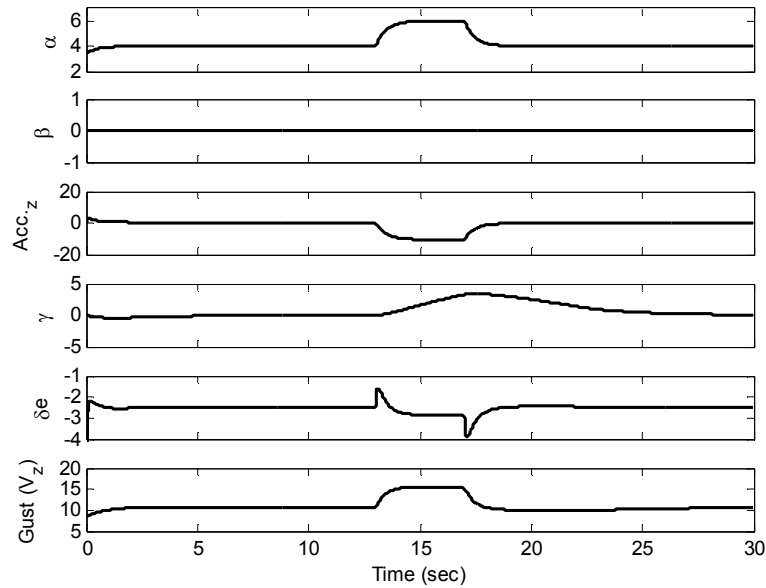
The 6 DOF equations of motion of the aircraft explain its translational and rotational dynamics. The equations were simulated using numerical method Runge-Kutta-4 (RK-4) algorithm. For simulation, completely nonlinear set of aerodynamic data of McDonnell Douglas F-4 aircraft has been used [8]. Results have been shown for different flight conditions.

**Note :** For all the following figures of results, the values of all the angles are in degree, distances are in meter, time is in second, angular velocities  $(p, q, r)$  are in radian/sec, velocities are in meter/sec and acceleration is in meter/sec<sup>2</sup>.

#### Case 1. Cruise flight control under effect of wind gusts:

In this case, Cruise flight is controlled under turbulent atmosphere as sudden gust comes and aircraft trim condition is disturbed and the controller has to

control the aircraft and bring it back to the trim condition. As shown in figure 2, Aircraft is cruising at  $\alpha = 4$  deg and a sudden gust comes to disturb the trim condition of the aircraft and the controller acts to bring the aircraft back to the trim condition.



**Figure 2. Cruise flight control under Turbulent Atmosphere**

### Case 2. Steady sideslip flight under effect of wind gusts

In this case, aircraft is undergoing steady sideslip flight and suddenly a vertical wind gust is introduced to disturb the aircraft states and the controller has to control and bring the aircraft states back to the desired values.

As shown in figure 3, aircraft is flying at  $\alpha = 4$  deg,  $\beta = 2$  deg and the aircraft is holding  $\Psi = -2$  deg for proper steady sideslip and then a sudden gust is introduced but the aircraft controller still performs in the desired manner.

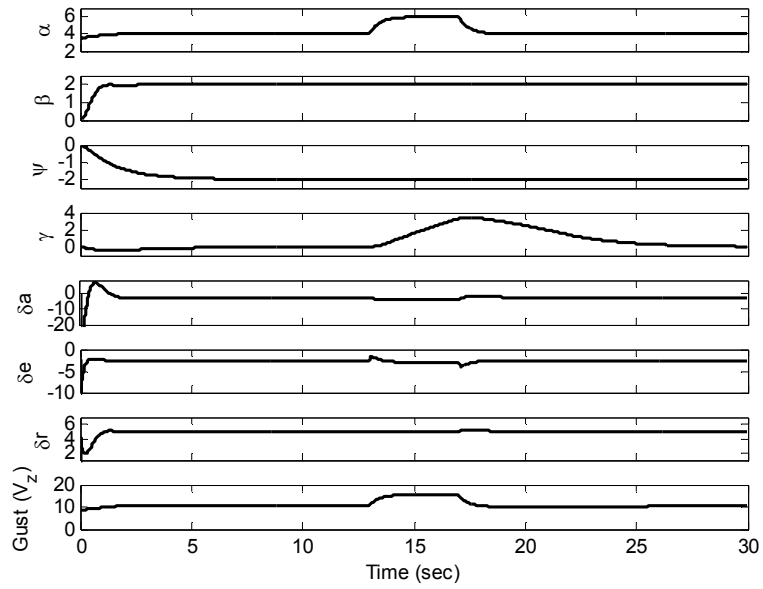


Figure 3. Steady sideslip flight control under Turbulent Atmosphere

Case 3. Steady Coordinated turn

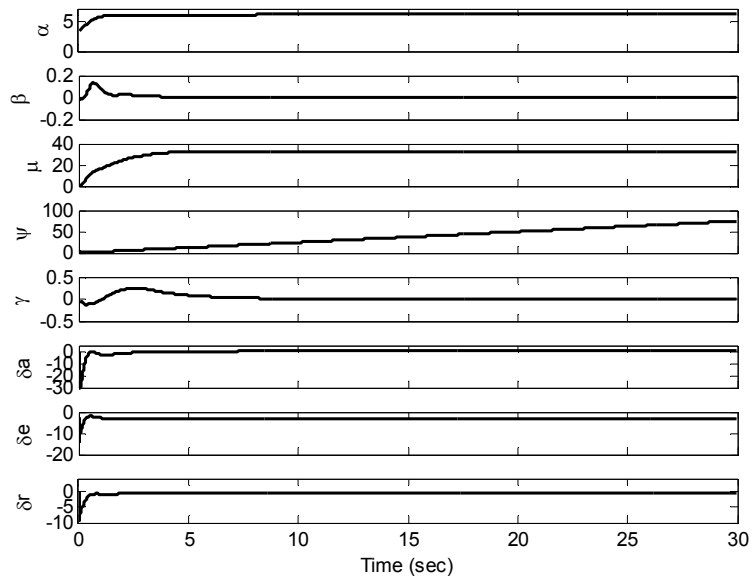
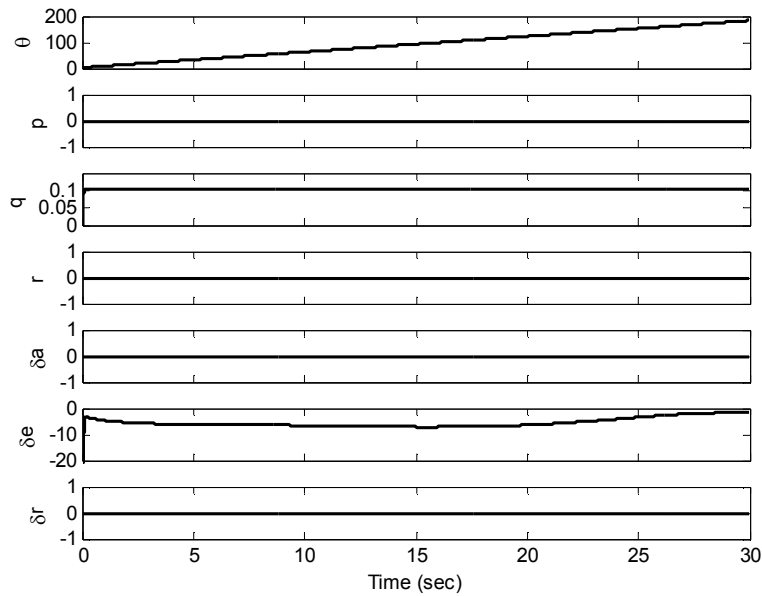


Figure 4. Aircraft states and control commands for Coordinated turn

In case 3, as explained by figure 4, the aircraft has to undergo steady coordinated turn i.e. the sideslip angle should be zero during the turn. In this case, aircraft is turning at the rate of change of  $\Psi$  as 2.5 deg/sec at  $\alpha= 6$  deg while maintaining sideslip to zero value.

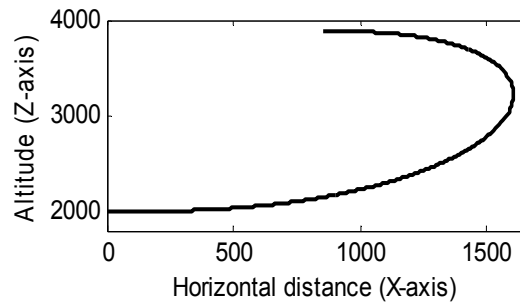
**Case 4. Pull-up Maneuver**

In this case, aircraft performs a continuous pull-up maneuver in vertical XZ plane. In this case, maneuver is done at pitch rate of 0.1 rad/sec as shown by figure 5 which also shows that for vertical pull-up, the roll angle, roll rate, yaw rate should be zero.



**Figure 5. Aircraft states and control commands for Pull-up Maneuver**

Figure 6 shows the aircraft trajectory in XZ plane during this flight condition.



**Figure 6. Aircraft trajectory in XZ plane during Pull-up Maneuver**



### Case 5. Velocity vector roll Maneuver

Aircraft performs a continuous roll maneuver about the velocity axis at high  $\alpha$ . During this maneuver, aircraft should not lose altitude as well as should not go under sideslip motion. In this case, aircraft performs this maneuver at  $\alpha=12$  deg as shown in figure 7.

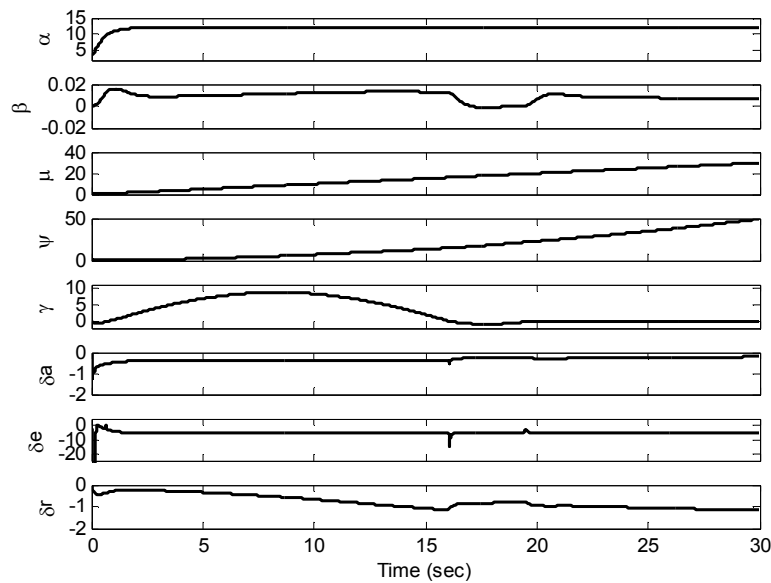


Figure 7. Velocity vector roll Maneuver at  $\alpha = 12$  degree

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