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Search for Deterministic Nonlinearity in the Light Curves of the Black Hole System GRS 1915+105

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Abstract. GRS 1915+105 is prominent black hole system exhibiting variability over a wide range of time scales and the light curves from the source have been classified into 12 temporal states. Here we undertake an analysis of the light curves from all the states using three important quantifiers from nonlinear time series analysis, namely, the correlation dimension (D_2), the correlation entropy (K_2) and singular value decomposition (SVD). An important aspect of our analysis is that, for estimating these quantifiers, we use algorithmic schemes which we have proposed recently and tested successfully on synthetic as well as practical time series from various fields. We show that nearly half of the 12 temporal states exhibit deviation from randomness and their complex temporal behavior can be approximated by a few (3 or 4) coupled ordinary differential equations. Based on our results, the 12 states can be broadly classified into three from a dynamical perspective: purely stochastic with D2 tending to infinity, affected by colored noise and those which are potential candidates for deterministic non linearity with $D2 \leq 4$. Our results could be important for a better understanding of the processes that generate the light curves and hence for modeling the temporal behavior of such complex systems. A more detailed analysis and results are presented elsewhere [1].
Keywords: Time Series Analysis, Applied Chaos, Black Hole Binaries.

1 Introduction

Most of the systems in Nature are described by models which are inherently nonlinear. Since the discovery of *deterministic chaos* a few decades back and the development of various techniques in subsequent years, there remained the exciting prospect of a better understanding of the complex behavior shown by various natural systems in terms of simple nonlinear models. A large number of techniques from nonlinear dynamics are routinely being employed for this

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purpose. For example, see Hilborn [2] and Lakshmanan & Rajasekhar [3] for details.

Astrophysical objects are among the most interesting real world syatems where methods from nonlinear dynamics have been attempted right from the development of chaos theory. Important examples include the analysis of variable stars [4] to understand the nature of variability, the study of the temporal variations in the sun spot activities [5] and to develop possible measures to differentiate between AGNs and black holes [6]. One major limitation regarding the analysis of astrophysical objects is that the only available information regarding the source is the light intensity variations emitted by it, called the *light curve*, over which one has no control. It is a single scalar variable recorded as a function of time, namely, a *time series*. Thus the main task in the analysis is to understand the nature of variability and to reconstruct the underlying model using the methods of time series analysis.

A number of computational schemes and measures are used for the nonlinear time series analysis as discussed by many authors [7,8]. The most important quantifiers among these are the correlation dimension (D_2) and the correlation entropy (K_2). We have recently proposed automated algorithmic schemes [9,10] for computation of D_2 and K_2 from time series based on the delay embedding technique and applied them successfully to various types of time series data. In this work, we apply these computational schemes to analyse the X-ray light curves from a very prominent black hole binary, GRS 1915+105.

2 Analysis of the Light Curves

Among the most important nonlinearity measures used for the analysis of time series data are D_2 and K_2. D_2 is often used as a discriminating measure for hypothesis testing to detect nontrivial structures in the time series. However, if the time series involves colored noise, a better discriminating measure is considered to be K_2 [11]. We employ surrogate analysis using both D_2 and K_2 as discriminating measures and to compute these measures, we make use of the automated algorithmic schemes proposed by us recently [9,10]. The scheme involves creation of an embedding space of dimension M with delay vectors x_j constructed from the time series. One then counts the relative number of data points in the embedded attractor within a distance R from a particular i^{th} data point

$$p_i(R) = \lim_{N_v \to \infty} \frac{1}{N_v} \sum_{j=1, j\neq i}^{N_v} H(R - |\boldsymbol{x_i} - \boldsymbol{x_j}|) \tag{1}$$

where N_v is the total number of reconstructed vectors and H is the Heaviside step function. Averaging this quantity over N_c number of randomly selected centres gives the correlation sum

$$C_M(R) = \frac{1}{N_c} \sum_{i}^{N_c} p_i(R) \tag{2}$$

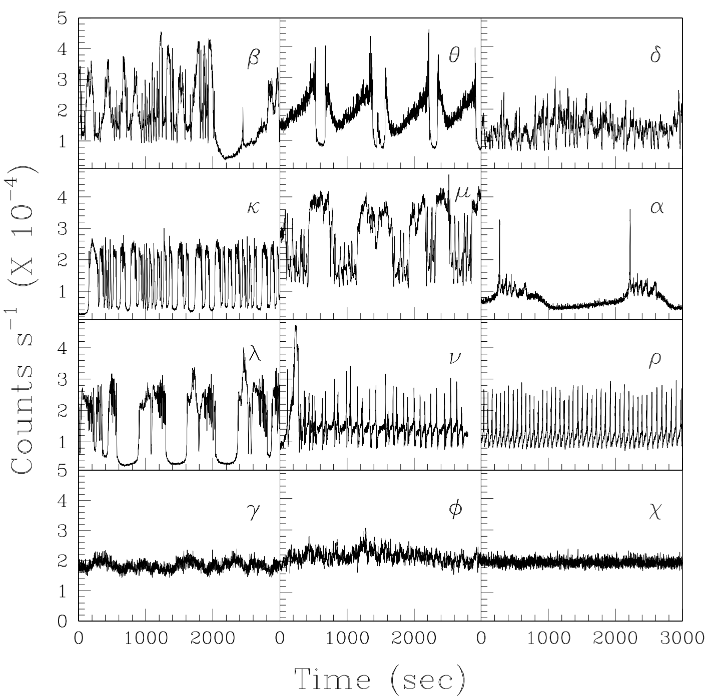

Fig. 1. Light curves from the 12 temporal states of the black hole system GRS 1915+105. Only a part of the generated light curve is shown for clarity.

The correlation dimension $D_2(M)$ is then defined to be,

$$D_2 \equiv \lim_{R\to 0} d(logC_M(R))/d(log(R)) \tag{3}$$

which is the scaling index of the variation of $C_M(R)$ with R as $R \to 0$. In our scheme, D_2 is computed by choosing a scaling region algorithmically.

To compute K_2, one measures the ratio at which the trajectory segments are increased as M increases, using the formal expression

$$K_2 \Delta t \equiv \lim_{R\to 0} \lim_{M\to\infty} \lim_{N\to\infty} log(C_M(R)/C_{M+1}(R)) \tag{4}$$

To generate the surrogate data sets, we apply the IAAFT scheme [12,13] using the TISEAN package [8]. Finally, in order to visualise the qualitative features of the underlying attractor, we use the singular value decomposition (SVD) analysis (for details, see [10]). The SVD analysis computes the dominant eigen vectors whose projection, called the BK projection, shows the reconstructed attractors from the time series. Here we use the TISEAN package to generate the SVD plots.

The black hole source under investigation in this work, GRS 1915+105, is unique among all such sources in that it shows a wide range of variability in the light curves. Belloni et al. [14] have classified the light curves into

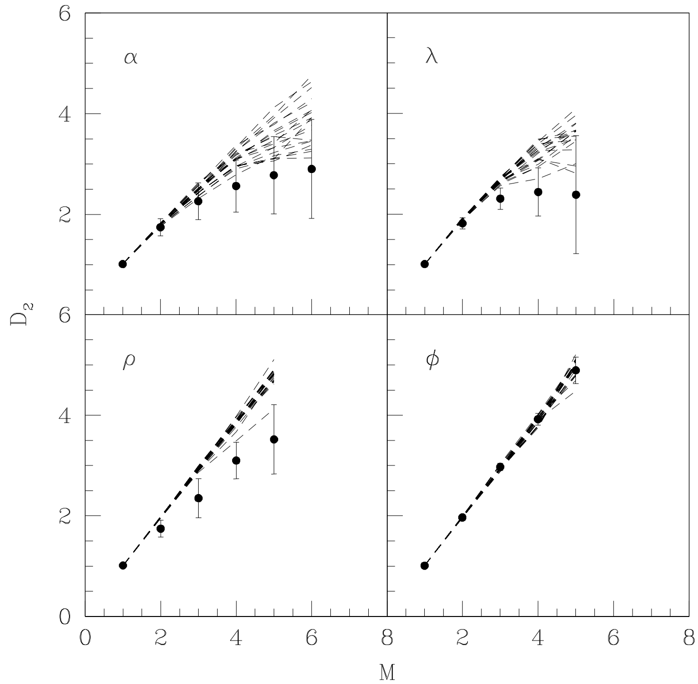

Fig. 2. Surrogate analysis with D_2 as a discriminating measure for the light curves from four states of GRS 1915+105. Surrogates are represented by dashed lines without error bar. Here we have done surrogate analysis with 10 surrogates. For more detailed surrogate analysis with 20 surrogates, see [1]. Note that the null hypothesis can be rejected in all cases except the ϕ state.

12 spectroscopic classes based on the RXTE observations. The nature of the light curves changes completely as the system flips from one temporal state to another. We have chosen a representative data set from each temporal class and extracted continuous data streams 3200 seconds long from it. The light curves were generated with a time resolution of 0.5 seconds resulting in approximately 7000 continuous data points for each class. More details regarding the data are given elsewhere [15].

Fig.1 shows all the 12 light curves used for the analysis, which are labelled by 12 different symbols representing the 12 temporal states of the black hole system. An earlier analysis of these light curves has shown that more than half of these 12 states deviated from a purely stochastic behavior [16]. Here we combine the results of D_2, K_2 and SVD analysis to get a better understanding regarding the nature of these light curves.

Fig.2 shows the results of surrogate analysis on four of the 12 states. It is clear that, of the states shown in the figure, only ϕ shows purely stochastic behavior. Of the remaining 8 states not shown, three more, namely γ, δ and χ, are found to belong to this category. Thus, only four out of the 12 states show behavior consistent with white noise in the D_2 analysis.

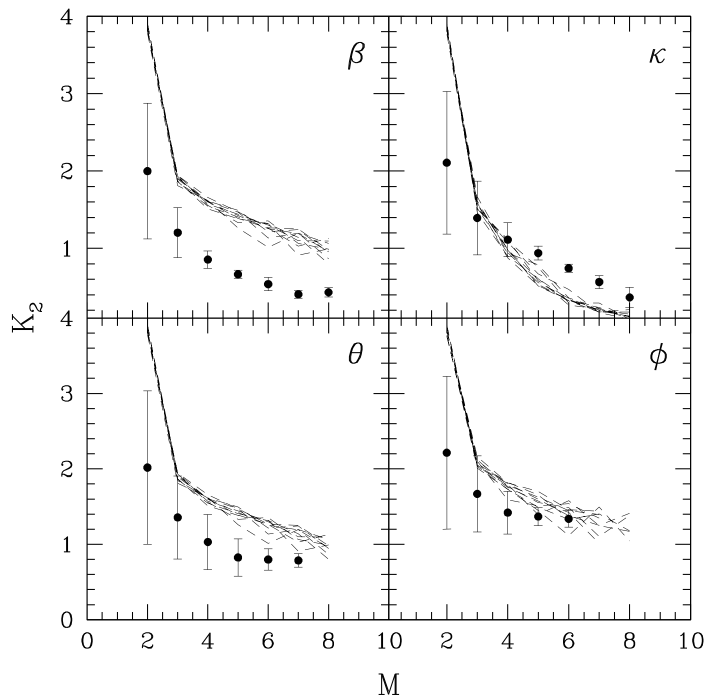

Fig. 3. Surrogate analysis of the light curves from four states with K_2 as the discriminating measure. While the data and the surrogates can be distinguished for β and θ, κ and ϕ behaves like colored noise and white noise respectively.

It is known that the X-ray emissions from the accretion discs may also involve colored noise. The colored noise gives a saturated value of D_2 and hence it is difficult to identify it in D_2 analysis. For this, we undertake surrogate analysis with K_2 as discriminating statistic. While data involving nontrivial structures give a saturated value of K_2, for pure colored noise, $K_2 \to 0$ as the embedding dimension M is increased. Results of K_2 analysis for four representative states are shown in Fig.3. While the behavior of β, θ and ϕ are consistent with earlier D_2 analysis, the bahavior of κ suggests that it is contaminated with colored noise. In fact 3 of the 8 states - κ, λ and μ - which showed deviation from stochastic behavior in the D_2 analysis are found to be contaminated with colored noise in the K_2 analysis. For more details regarding D_2 and K_2 analysis, see [1].

Finally, we perform a SVD analysis on all the states which clearly shows the qualitative nature of the underlying attractors. The plot of attractors for selected states is shown in Fig.4. The most interesting plot is for the ρ state which shows a typical limit cycle type attractor added with noise. Also, note that the SVD plot for the κ state has nontrivial appearance, eventhough the surrogate analysis suggested the presence of colored noise. This may be an indication that the state is not a pure colored noise. The same behavior is

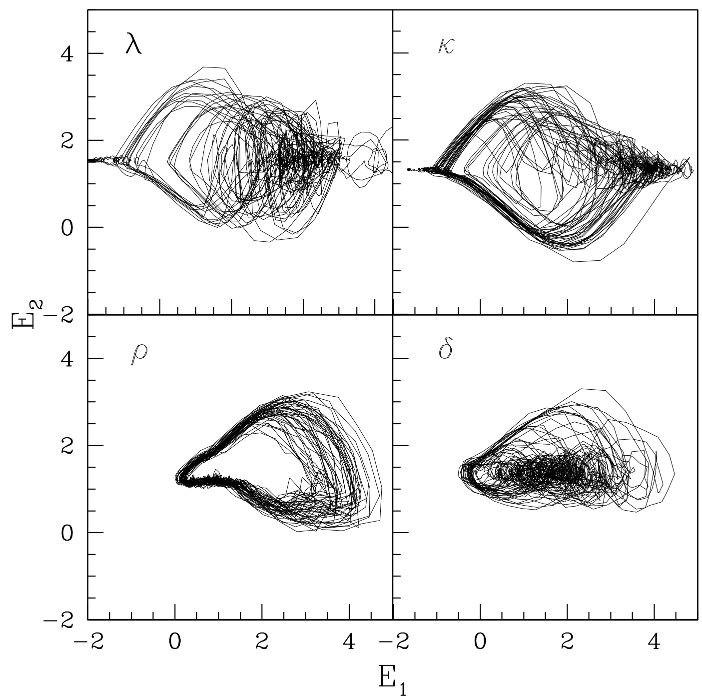

Fig. 4. The plot of attractors underlying four states of the black hole system reconstructed via SVD analysis. Except the ϕ state, which behaves as a white noise, all the others indicate the presence of underlying attractors, the most interesting being the ρ state.

found for two other states, λ and μ. Thus, these 3 states are likely to be a mixture of deterministic nonlinearity and colored noise.

Based on our results, the 12 states can thus be divided into 3 broader classes from the point of view of their temporal properties. It turns out that some of the states which are spectroscopically different, behave identically in their nonlinear dynamics characteristics. This may be an indication of of some common features in the mechanism of production of light curves from these states.

3 Conclusion

Identifying nontrivial structures in real world systems is considered to be a challenging task as it requires a succession of tests using various quantitative measures. Eventhough a large number of potential systems from various fields have been analysed so far, the results remain inconclusive in most cases. Here we present an interesting example of an astrophysical system, which we analyse using several important quantifiers of nonlinear dynamics. We find that out of the 12 spectroscopic states of the black hole system, only 4 are purely stochastic. The remaining states show signatures of deterministic nonlinearity, with 3 of them contaminated by colored noise. All these 8 states are found to have $D_2 < 4$ so that their complex temporal behavior can be approximated by 3 or 4 coupled ordinary differential equations. Based on our results, the 12 states can be broadly classified into 3 from a dynamical perspective: purely stochastic with $D_2 \to \infty$, affected by colored noise and those which show deterministic nonlinear behavior with $D_2 < 4$.

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