Control of irregular cardiac rhythm

Mounira Kesmia[1], Soraya Boughaba[1], and Sabir Jacquir[2]

[1] Département de Mathématiques, Université de Constantine I, Constantine, Algérie (kesmia.mounira@gmail.com, soraya.boughaba@hotmail.com)
[2] Laboratoire Le2i FRE CNRS 2005, Université de Bourgogne Franche-Comté, Dijon, France (sjacquir@u-bourgogne.fr)

Abstract. The aim of this work is to investigate the chaos control of the one dimensional map which modelizes the duration of the current cardiac action potential (APD) as a function of the previous one. Using OGY control method, we obtain very satisfactory numerical results to stabilize the irregular heart rhythm into the normal rhythm.
Keywords: Action Potential Duration (APD), chaos, control, equilibrium point, normal rhythm, irregular heart rhythm.

1 Introduction

The interest in the chaotic control systems has been largely initiated by E.Ott, C. Grebogi and J.York in 1990 [1]. The key idea is that a considerable change in the behavior of a chaotic system can be obtained through a very small change in one or more of its parameters. This known process, namely OGY control method, was the first method introduced to control chaotic systems. When the control is activated, the unstable periodic orbit converges to an approximation of the desired orbit. Since the work of Garfinkel [2], a number of theoretical and experimental studies were performed to control irregular rhythms through various methods of nonlinear dynamics control [3–6]. Control algorithms based on the OGY method were applied to the first to the ventricle of a rabbit [2], but the desired rhythm did not occur. The OGY control method uses external electrical stimulation to irregular heart rhythms in order to recover normal rhythm [4]. The application of this method requires firstly, the analysis of the steady-state system to determine the fixed points and, secondly, the identification of the appearance of chaotic behavior through the bifurcation diagram [1]. The one dimensional map model of APD gives useful information in order to understand the evolution of regular cardiac rhythm into irregular one, mainly the fibrillation ventricular arrhythmia which leads to sudden cardiac arrest [7]. The one-dimensional map model of APD may be helpful and suitable model to control irregular rhythm. The present paper is organized as follows. After this introduction, section 2 reviews the dynamical properties of the one-dimensional

Received: 8 September 2017 / Accepted: 10 January 2018

ISSN 2241-0503

map (APD) of an electric cardiac model describing the propagation of the cardiac action potential. In order to control chaotic behavior, we apply in Section 3 the OGY control method. We show that, the OGY chaotic control method is sufficient to stabilize the iterative model (APD) to the desired fixed point or normal rhythm. Numerical simulation results are very satisfactory, indicating the effectiveness of the proposed method to control irregular cardiac rhythms. The section 4 is devoted to the conclusion.

2 The mathematical model

2.1 The one dimensional map of the APD

The effects of a periodic stimulation on a strand of ventricular muscle have been investigated experimentally by Lewis and Guevara [8]. Electrical stimulations applied at a regular time intervals t_s generated an action potential. At arbitrary t_s, the current duration of any given action potential is given by the previous one:

$$APD_{i+1} = A - B_1 \exp\left(\frac{APD_i - nt_s}{\tau_1}\right) - B_2 \exp\left(\frac{APD_i - nt_s}{\tau_2}\right) \tag{1}$$

Where APD_{i+1} is the APD of $(i+1)st$ action potential and let n a parameter block in the production of an action potential on condition that $(APD_i - nt_s < -DI_{\min})$. The constants A, B_1, B_2, τ_1, τ_2 are related to the heart electrophysiological constraints defined in[1]: $A = 270\ ms$, $B_1 = 2441\ ms$, $B_2 = 90.02\ ms$, $\tau_1 = 19.60\ ms$, $\tau_2 = 200.5\ ms$, and $DI_{\min} = 53.5\ ms$.

2.2 Model Dynamics

The $N : M$ rhythm $(N \geq 1; M \geq 0)$ is periodic with Nt_s periods [8], which contain the repeating $N : M$ cycles, each exhibiting N stimulus pulses and M action potentials or M beats. The dynamics of the eq (1) is graphically studied in [8]. The following sequence rhytms is obtained depending on the stimulation frequency value: [$1 : 1 \rightarrow 2 : 2$(alternans)$\rightarrow 2 : 1 \rightarrow 4 : 2 \rightarrow$chaos$\rightarrow 3 : 1 \rightarrow 6 : 2 \rightarrow$chaos$\rightarrow 4 : 1 \rightarrow 8 : 2 \rightarrow$chaos$\rightarrow 5 : 1 \rightarrow 10 : 2 \rightarrow$chaos$\rightarrow 6 : 1 \rightarrow 12 : 2 \rightarrow$total chaos].

The following bifurcation diagram (see Fig. 1) shows the different dynamics of the system. We propose to control the chaotic state at $t_s = 146ms$, to stabilize the system to the fixed point corresponding to a regular heart rhythm $1 : 1$ and $2 : 1$. At $t_s = 146ms$, the heartbeat is irregular, indicating the presence of a cardiac arrhythmia. The evolution of the system is represented by the figures (2,3):

2.3 Determination of unstable fixed points

To apply the OGY control method, it is necessary to know the unstable fixed point value of the dynamical system in chaotic state (unstable fixed point associated with the unstable regular rhythm $1 : 1$ and $2 : 1$). The determination

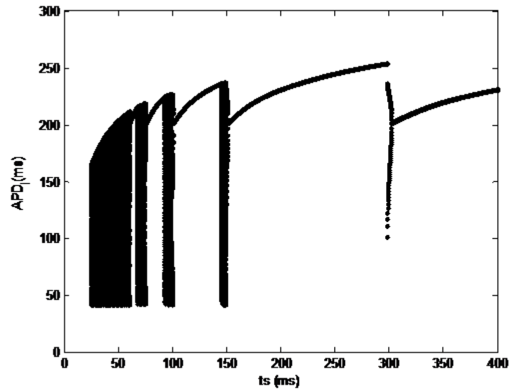

Fig. 1. Bifurcation diagram [8]

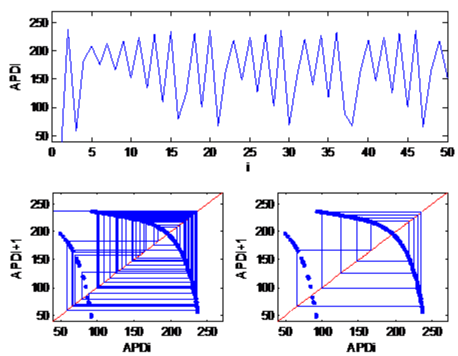

Fig. 2. Aperiodic rhythm at $t_s = 146ms$. Top item is APD_{i+1} function of 50 iteration number i, bottom item $APD_{i+1} = f(APD_i)$, left for 20000 iteration number i, right for 50 iterations.

of the unstable fixed point x^* is based on a numerical method. The fixed point of the map (1) satisfies the equation:

$$F(x^*) = f(x^*) - x^* = 0 \tag{2}$$

We search the root α using Dichotomie method which represents a fixed point for the iteration interval $[0, 270]$. The goal is to try to isolate the root α on the iteration interval $[0, 270](a_0 = 0, b_0 = 270)$. $\alpha \in [a_k, b_k] \subset [a_0, b_0]$, $\alpha \cong x_k$, the middle of the interval $[a_k, b_k]$ such as $|\alpha - x_k| \leq \left|\frac{b_k - a_k}{2^{k+1}}\right| < 10^{-5}$ For $t_s = 146\ ms$, we found two unstable fixed points values corresponding to $1:1$ and $2:1$ rhythms. The $1:1$ rhythm unstable (or period-1 orbit) value is $\alpha \simeq x_{12} = 86.42\ ms$. The $2:1$ rhythm unstable (or period-1 orbit) value is $\alpha \simeq x_{12} = 196.00125\ ms$

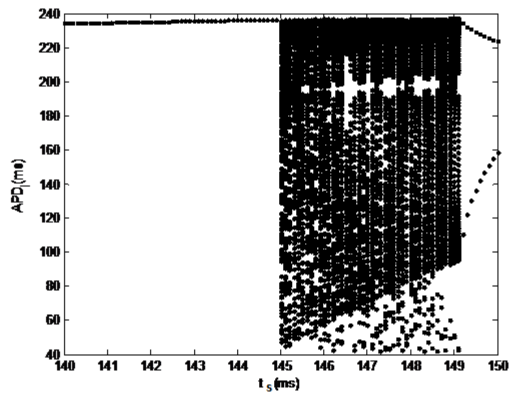

Fig. 3. Enlargement of bifurcation diagram.

3 The OGY control method

Chaos may be desirable since it can be controlled by using small perturbation to some accessible parameter [1,9] or to some dynamical variable of the system [9,10]. The OGY control method is based on the feedback state control which uses the chaos in a dynamical system to stabilize an unstable periodic orbit: the determination of some unstable periodic orbits, reviewing and choosing a representative system performant [1]. Thus, one adjusts the perturbation parameters in a relatively short time, in order to stabilize the unstable periodic orbit.

When the control is activated, the unstable periodic orbit converges to an approximation of the desired orbit. The application of this method requires firstly, analysis of the steady-state system or the periodic orbit and secondly, the identification of the appearance of chaotic behavior through the bifurcation diagram.

3.1 *The mathematical approach*

We consider the following one-dimensional map, where p is the control parameter:

$$x_{i+1} = f(x_i, p) \tag{3}$$

Let x^* a fixed point (or period-1 orbit) of the map:

$$x^* = f(x^*, p^*), \tag{4}$$

with p^* is the nominal parameter value.

Therefore, the control strategy will be to find a control law stabilizing with the feedback state near the chosen orbit. The studied system is in a chaotic state, the passage near the fixed point is guaranteed and once the system is near of

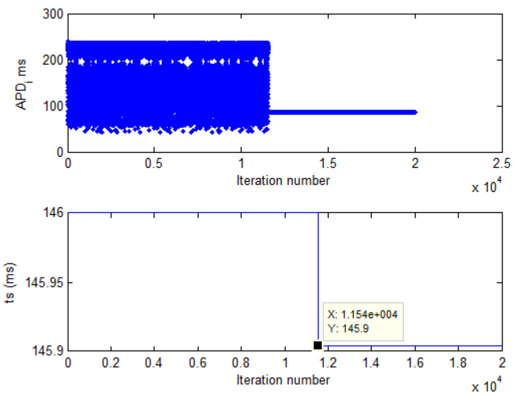

Fig. 4. Stabilization of the chaotic rhythm to an equilibruim point (or period-1 orbit) representing the 1 : 1 normal rhythm with the OGY control method.

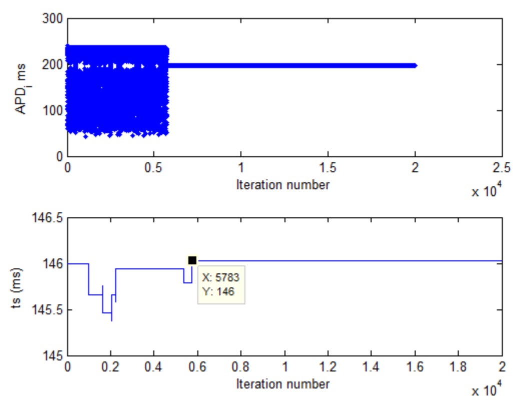

Fig. 5. Stabilization of the chaotic rhythm to an equilibruim point (or period-1 orbit) representing the 2 : 1 normal rhythm with the OGY control method.

the x^*, the control procedure is activated to bring the system into the desired orbit. In this case we have:

$$\delta x_{i+1} = x_{i+1} - x^* \tag{5}$$

And

$$\delta p_{i+1} = p_{i+1} - p^* \tag{6}$$

The linearized dynamics in the neighborhood of x^* is given by:

$$\delta x_{i+1} = \left[\frac{df}{dx_i}\right]_{x^*} \delta x_i + \left[\frac{df}{dp}\right]_{p^*} \delta p_i \tag{7}$$

The strategy of the OGY method is to adjust the control parameter p in order to stabilize the system at the fixed point x^*. This requires that $\delta x_{i+1} = 0$.

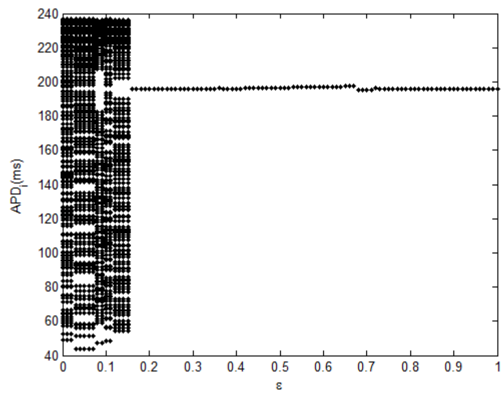

Fig. 6. Bifurcation diagram (APD_i vs ϵ), $APD_1 = 240ms$.

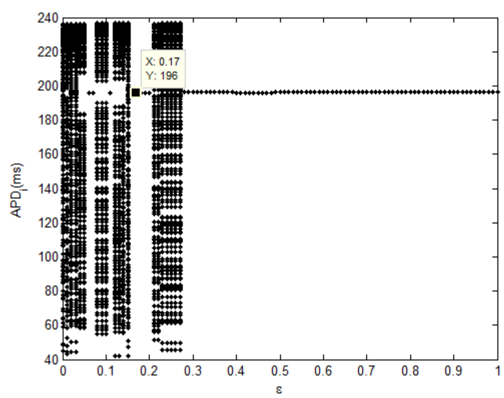

Fig. 7. Bifurcation diagram (APD_i vs ϵ), $APD_1 = 200ms$.

Then :

$$\delta p_i = -\frac{\left[\frac{df}{dx_i}\right]_{x^*}}{\left[\frac{df}{dp}\right]_{p^*}} \delta x_i \tag{8}$$

We set:

$$K = -\frac{\left[\frac{df}{dx_i}\right]_{x^*}}{\left[\frac{df}{dp}\right]_{p^*}} \tag{9}$$

Then:

$$\delta p_i = K \delta x_i \tag{10}$$

The purpose of the control is to satisfy the following condition:

$$|p_i - p^*| < \epsilon \tag{11}$$

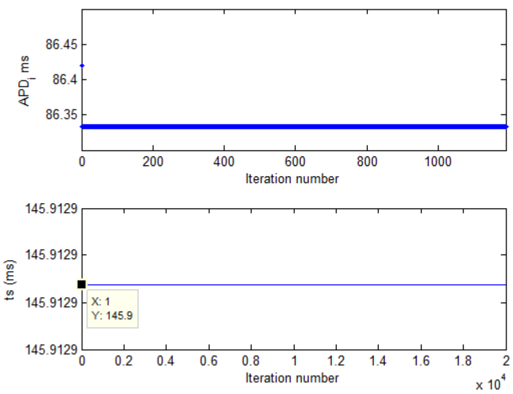

Fig. 8. To prevent chaotic dynamics with the OGY method.

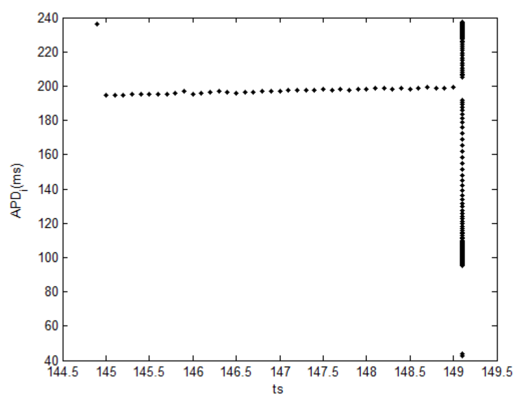

Fig. 9. Bifurcation diagram (APD_i vs t_s), after applying OGY control method with $APD_1 = 240ms$, $\epsilon = 0.9$, $K = 0.5$.

Let ϵ a predefined setting parameter that determines the neighborhood of x^*. We can write:

$$|K\delta x_i| < \epsilon \tag{12}$$

Therefore, the increment of the control is given by:

$$\delta p_i = K(x_i - x^*) \qquad if \qquad |K(x_i - x^*)| < \epsilon \tag{13}$$

$$\delta p_i = 0 \; elsewhere \tag{14}$$

3.2 Numerical results

The control by the OGY method of the map (1) consists in the following operations: The algorithm is applied to control the chaotic state for t_s =

$146ms$. Setting that $x_i = APD_i$, our objective is to stabilize the system to the stable fixed points (or period-1orbit) representing the $1:1$ and $2:1$ rhythms. To stabilize these fixed points, we have carried out a suitable values of K and ϵ, then we have iterated the system 20000 times, starting from any initial condition $x_1 = 240ms$. Obviously, if we change the initial condition value then we must change the ϵ value. The bifurcation diagram (APD_i vs ϵ) under OGY method was presented in figures 6 and 7. At each ϵ eq. (1) was iterated 20000 times. Increment in ϵ was $0.1ms$. The iteration started from initial condition $APD_1 = 240ms$ (see Fig. 6) and $APD_1 = 200ms$ (see Fig.7). To achieve the stability of the $1:1$ rhythm at the value $x_1 = 86.42ms$; the value of the parameter K is 1 and we choose $\epsilon = 0.2$. After 11540 iterations, the system stabilizes at the approximate value $x(11540) = 86.33ms$ (see Fig.4). To achieve the stability of the $2:1$ rhythm at the value $x_2 = 196.00125ms$, the value of the parameter K is 0.5 and we choose $\epsilon = 0.8$. We see that in Fig.5 the triggering of OGY control occurs several times, because x_i is not in the vicinity of the fixed point x^*. After 5783 iterations, the system stabilizes at the value $x(5783) = 196ms$. To prevent the chaotic behavior (or irregular rhythm, see Fig.8), it must choosing the fixed point value as an initial condition $x_1 = x^*$. Hence, the OGY contol triggers in the next iteration. We can stabilize the chaotic states represented in figure 3 with suitable value of K and ϵ (See Fig. 9).

4 Conclusion

Our objective is to investigate the control of the chaotic dynamics of the one-dimensional map (APD). In this paper, by applying the OGY method, the chaotic dynamics stabilize at fixed points (or period-1 orbits). This method needs only small perturbations of an accessible control parameter to stabilize a desired fixed point. On the other hand, it is possible to prevent chaos using OGY method. If the controlled map will start iteration from the unstable fixed point value as an initial condition, the dynamic evolves directly into a desired fixed point from the first iterated. We obtain very satisfactory numerical results to stabilize unstable equilibrium points of the map (APD). The application of this method in the 2D case is the current object of work.

References

1. E. Ott *et al.*, Controlling chaos, Phys. Rev. Lett., **64(11)**, 1196-1196 (1990).
2. A. Garfinkel, ML. Spano, WL Ditto, JN Weiss, controlling cardiac chaos. Science, **257**, 1230-1235 (1992).
3. M. Kesmia, S. Boughaba, S. Jacquir, Predictive Chaos Control for the 1D-map of Action Potential Duration, Chaotic Modeling and Simulation Journal, **3**, 387-398 (2016).
4. DJ. Christini, KM. Stein, SM. Markowitz, S. Mittal, DJ, Slotwiner, BB Lerman, The role of non linear dynamics in cardiac arrhythmia control, Heart Dis., **1**, 190-200 (1999).

5. P.N. Jordan, D.J. Christini, Adaptive diastolic interval control of cardiac action potential Duration alternans, J. Cardiovasc. electrophysiol, **15**, 1177-1185 (2004).
6. B. Xu, S. Jacquir, G. Laurent, J-M. Bilbault, S. Binczak, A hybrid stimulation strategy for suppression of spiral waves in cardiac tissue, Chaos, Solitons and Fractals, **44(8)**, 633-639 (2011).
7. MR. Guevara. F. Alonso, D. Jeandupeux, AG. Antoni, V. Ginneken: Alternans in periodically stimulated isolated ventricular myocytes: Experiment and model, In: Cell to Cell Signalling: From Experiments to Theoretical Models, edited by Goldbeter A. Academic Press, London, (1989).
8. TJ. Lewis, MR Guevara, Chaotic dynamics in an ionic model of the propagated cardiac action potential, J. Theor. Biol. **146(3)**, 407-432 (1990).
9. S. Boccaletti, C. Grebogi, Y.C. Lai, H. Mancini, D. Maza, The control of chaos: theory and applications, Physics Reports. **329**, 103-197 (2000).
10. S. Hayes, C. Grebogi, A. Mark, Experimental Control of Chaos for communication, Phys. Rev. Lett, **73**, 1781-1784 (1994).