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Experimental evidence of wave chaos signature in a microwave cavity cylinder resonator with fractures of side surface

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Abstract. The billiards microwave (microwave cavity resonators) in which the boundary has a surface fractures and contains both dispersing and focusing portions; there may be signs of chaos in its frequency spectrum. The common feature of such billiard's border is its small smooth surface (at least in the absence of the second derivative). Since the billiard system breaks the side surface is non-integrable, its chaotic properties have been studied experimentally. We use the spectral approach, when signs of wave chaos are shown in the properties of the inter-frequency intervals distribution system spectrum. With this approach, in the absence of surface fractures the spectral lines of the resonator are independent, and the distribution of inter-frequency intervals is a Poisson distribution. In the case of existence of surface fractures, the spectral lines are correlated and inter-frequency intervals in spectrum allocation approaches the Wigner distribution.

Keywords: Wave chaos, Cavity microwave resonator, Small smooth surface, Probability distribution of inter-frequency intervals, Wigner distribution.

1 Introduction and problem statement

The work is devoted to the study of quantum (wave) chaos (QC) in linear Hamiltonian systems. This area has recently acquired a great development. This is evidenced by numerous publications - monographs and articles (see [1] and references therein), which contains the results of theoretical and experimental studies of the problem. It should be noted that QC is the sufficiently general phenomenon. It covers a wide range of tasks associated with the quantum mechanical description of the systems, chaotic in the classical limit. For a description of experiments with billiard systems also used the term "wave chaos." However, it is not widely used yet.

Thus, QC research of general interest associated with the implementation of the principle of correspondence between classical and quantum systems. The QC system is non-integrable system, invariant with respect to the inversion operation time. It has a classical analog possessed

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chaoticity. According to the hypothesis of Bohigas, Giannoni, and Schmit [2], the spectral properties of such a system can be described by random matrix theory [3]. The effluent from this theory, the main feature of the QC is as follows. The system has chaotic frequency spectrum with a strong correlation between the spectral lines, which leads to a peculiar effect of "repulsion" and Wigner character of probability distribution of inter-frequency intervals.

In theoretical and experimental studies of quantum chaos scattering Sinai and Bunimovich billiards are commonly used. In Sinai billiards the chaotic state in the classical limit provided strong instability of the trajectories of material particles produced by reflection from its border with negative curvature. In Bunimovich billiards cause of instability is related to the scattering and defocusing of the trajectories of particles with reflections from the lateral limits.

Such scattering billiards relate to systems with mixing [4] (K-systems), when the time correlation of the phase trajectories of the motion of the particle tend to zero. A common property of the system as a scattering billiard dynamic system is its non-integrability related to the fact that it lacks spatial symmetry and besides of the integral of the total energy, there are no other integrals of motion. the experimental study, they use QC modeling by means of microwave resonators, the shape of which is similar to scattering billiards. For this purpose, also they use regular microwave resonators belonging to integrated systems [5-7].

Randomness in the resonator spectra are generated by filling the cavity volume by randomly distributed therein dielectric inhomogeneities [5]. In this cavity through a random arrangement of inhomogeneities the spectrum is random as well. Since, in the distribution of irregularities there is no symmetry the system is non-integrable. Owing to that [2] in the resonator spectrum occurs Wigner (or close to it) shape of the probability distribution of inter-frequency intervals. the inhomogeneity (roughness) of the boundary of the microwave resonator is also influenced on the QC signs. They create the conditions for the emergence of the state, having the QC features [6, 7].

The effect of inhomogeneities on the spectrum of the resonator has recently attracted considerable interest due to the applied aspects relating to quantum electronics. They are directly connected with the creation of a micro-laser with disk semiconductor resonator with super high Q-factor [8]. In this resonator due to the extremely low dielectric losses at optical frequencies and high uniformity of the semiconductor crystal, which it is made of, ultra-high Q-factor (about 10^6 and above) for “whispering gallery” fluctuations has been achieved. Thus, the laser resonator has a very high monochromaticity and low angular divergence of radiation. It is also important that the ultra-high Q -factor of the resonator substantially (of the order of magnitude or more) are reduced laser threshold currents.

Upon reaching ultra-high Q-factor of the laser resonator the great importance has the nature and the value of inhomogeneities in the dielectric disk resonator, as well as the deviation of its shape from a strictly cylindrical. These inhomogeneities cause additional radiation losses, significantly lowering the Q-factor of the resonator. Thus, they adversely affect the basic characteristics of the laser. Therefore, the study of influence of inhomogeneities on the spectral properties of the laser resonator and the manifestation of QC properties is an interesting physical problem, having great practical importance.

Instability of the billiard system (a microwave resonator) and the related with it the frequency chaoticity can be caused not only by the reflection of waves with negative curvature border in Sinai billiards or defocusing trajectory as in billiards Bunimovich. The chaoticity and growing of correlation between resonant lines can arise in billiards in which the boundary has breaks the surface and contains a dispersing and focusing portions. The common feature of this billiard border is a small smooth (at least in the absence of the second derivative). According to [9] a billiard of low smoothness border refers to an unstable scattering billiards (K-systems). It lacks the spatial symmetry. As a result, it is non-integrable system. This billiard through instability and dissipation at inhomogeneities, which serve areas with low surface smoothness, becomes a chaotic system. This suggests that in its spectrum can detect signs of QC.

It should be noted, however, that the statement in [9] to the low smoothness boundary billiard supplies a K-system. It is actually based on the instability of the trajectories of particles traveling in this billiard. This instability leads to chaotic motion of particles in the system. On the other hand, on the basis of the hypothesis in [2] we can assume that the spectra of chaotic systems that are invariant with respect to time reversal (their classical counterparts are K-systems) have statistical properties that are predicted by the random matrix theory to systems belonging to a Gaussian orthogonal ensemble (GOA). Because of this, they must be QC signs. Therefore, the establishment in the system under study a chaotic spectrum with non-zero correlation between the spectral lines and the Wigner probability distribution of inter-frequency intervals give a reason to believe that the signs of QC occur here. Since the billiard system with side surface breaks is non-integrable, its chaotic properties have been studied experimentally.

The aim of this work is an experimental study of billiard systems with small smooth borders and the possibility of finding manifestation QC signs in them. For this was we used a common method of QC-effects modeling with a quasi-two-dimension microwave resonator. Such a resonator is described by the scalar Helmholtz equation for a given type of oscillations, which coincides with the stationary Schrodinger equation. This makes it possible to consider the resonator as a quantum system and to simulate the conditions of existence in it

QC signs. This is the subject of our experiments. In them, we use the spectral approach, when signs of quantum chaos are shown in the properties of the probability of inter-frequency intervals distribution of the resonator spectrum. With this approach, in the QC absence of spectral lines of the cavity resonator are independent and probability of inter-frequency intervals distribution is a Poisson distribution: $P(s) = \exp(-s)$, where *s* is the length of an normalized average inter-frequency interval. A different situation arises when QC sign exists. In this case, the spectral lines are correlated, and probability of inter-frequency intervals distribution of the resonator spectrum approaches the Wigner distribution: $P(s) = \pi/2\, s \exp\{(-\pi/4)s^2\}$.

Thus, the establishment of a Wigner distribution of probability of inter-frequency intervals distribution in a resonator frequency spectrum, which side walls have a low smoothness, is a good argument in favor of the resonator system with low side smoothness has QC signs.

2 Experimental setup and discussion

A quasi-two-dimensional cavity microwave cylindrical resonator was used for modeling QC systems. It has the height of 14 mm and diameter of 130 mm. To concentrate the electromagnetic field in the cavity resonator it was closed with two metal upper and lower disks. To create the various conditions of low surface smoothness border in the resonator, when the second derivative disappears in certain surface shape points, we introduced aluminum inserts in the form of segments, the shape and dimensions of which allow the simulation of a particular type of scattering billiards.

The aluminum inserts made of pure aluminum, a close fit with its outer cylindrical surface of the side wall of the cavity resonator. The size of the inserts along the axis of the resonator was exactly equal to its height. This allowed, on the one hand, to minimize the electromagnetic field losses in the cavity, and on the other hand, to provide a given excitation oscillation mode therein. Spectral studies were performed in eight millimeter cavity at frequencies range 27-38 GHz. To excite microwave oscillations in the resonator a diffraction waveguide antenna has been used. It was a hole of 2 mm in diameter in a thin (0.1 mm thick) diaphragm closing an end of a standard rectangular waveguide soldered to the resonator body. The wide waveguide side is oriented along the axis of the resonator. In these conditions, the vibrations are excited in it predominantly of H-type with magnetic field along the same axis.

Due to this electric the microwave-excited oscillation currents do not cross the boundary between the body and the upper (lower) of the resonator cover. The result was achieved in the resonator relatively high Q-factor, the value of which was 10^3 in the high frequency spectrum part. It should be noted that the shape of the cavity model has been selected with only excited oscillation of H-type. This is necessary to provide the appropriate correlation between resonant frequencies. The frequency spectrum of the cavity was defined by a wideband microwave signal using strength meter R2-65, which allows measurement of the spectrum in a wide microwave range. Spectral measurements were carried out in the "on pass" regime by measuring the intensity of the signal transmitted through the resonator. We recorded numerous (the number of the order of hundreds) spectral lines.

In order to ensure the stability of the signal characteristics and, accordingly, the spectral lines, the measurements were made in a short time. For this purpose, the measurement process was fully automated. The whole range, or some of its sections were recorded for 40 seconds and recorded by a computer. Spectrum processing is carried out using a specially designed program which allows you to determine the quality factor and the natural frequency of each spectral line with high accuracy. The relative errors in the measurement of the resonant frequency and the quality factor of the spectral line using a special calibrator are of the order of 10^{-5} and 10^{-3}, respectively. Fig.1 shows a portion of the spectrum of a smooth cylindrical resonator containing no inserts. This range includes about 50 high quality factor spectral lines.

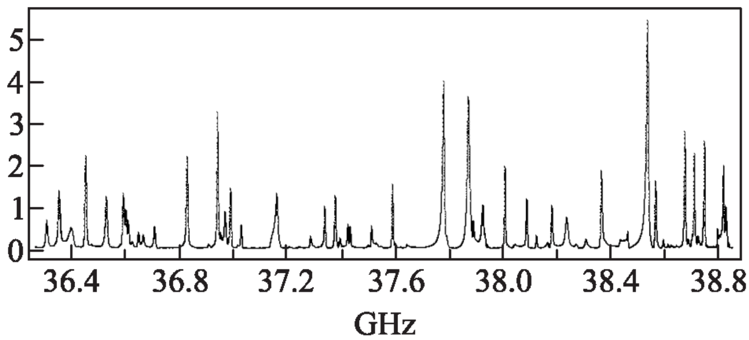

Fig. 1. The portion of a quasi-cylindrical cavity with a smooth boundary frequency spectrum. Vertically (in arbitrary units) is plotted signal intensity on resonant frequency of the spectral lines in GHz.

3 Statistical treatment of frequency resonator spectrum

Let us first consider the results of the spectral measurements of an empty (without inserts) of the cylindrical cavity resonator. Such a resonator having symmetry elements is an integrable system and its electromagnetic field can be represented as a set of independent spectral modes (in this case, the H-mode). Owing to this the resonator spectrum is regular, and spectral lines are independent, and the probability distribution of inter-frequency intervals close to Poisson distribution, Fig. 2.

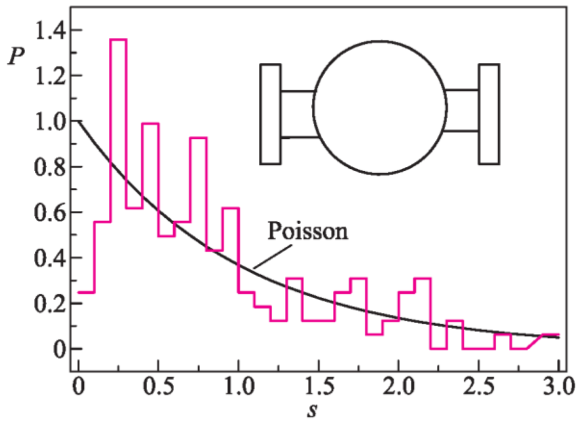

Fig. 2. Histogram of probability distribution of inter-frequency intervals *s* for the empty cylindrical cavity resonator spectrum. The smooth curve is Poisson distribution.

The deviation from the Poisson distribution is observed only at small intervals inter-frequency intervals. It is connected with a finite spectral line width. Fig. 3 shows the histogram for various inserts sector configurations in the resonator cavity. The configuration of the cavity with two sectorial inserts located at an angle of about 20° corresponds to an asymmetric Bunimovich geometry billiard. On the lateral surface of the resonator cavity, wherein the flat inserts are jointed a cylindrical surface, there are fractures. Thus, this resonator has instability and chaotic frequency spectrum, which there is the effect of the repulsion of the spectral lines, significantly affects the distribution of inter-frequency intervals. The number of inter-frequency intervals is significantly reduced, and the probability of large inter-frequency intervals increases, respectively. As a result, the probability of inter-frequency intervals distribution in the resonator spectrum is close to the Wigner distribution. It is seen that the frequency spectrum has still regular component along with dominating the random component. This feature in the probability distribution of inter-frequency intervals occurs when the resonator configuration with one segment insert, which corresponds unbalanced billiards **completely**, Fig.3a, when the second derivative is not in the area of fracture surfaces. Here, just as in the case of asymmetric Bunimovich billiards with two inserts, Fig.3b, the probability of the inter-frequency intervals distribution close to the Wigner distribution. At the same time, the presence of sufficiently large of the probability values at small *s* means that in this case the regular component of the spectrum increases compared to probability values of inter-frequency intervals of the resonator like Bunimovich asymmetrical billiard (two segment inserts). Fig. 3c shows the probability values of inter-frequency intervals distribution for the resonator cavity in which the insert as asymmetrical sharp ledge.

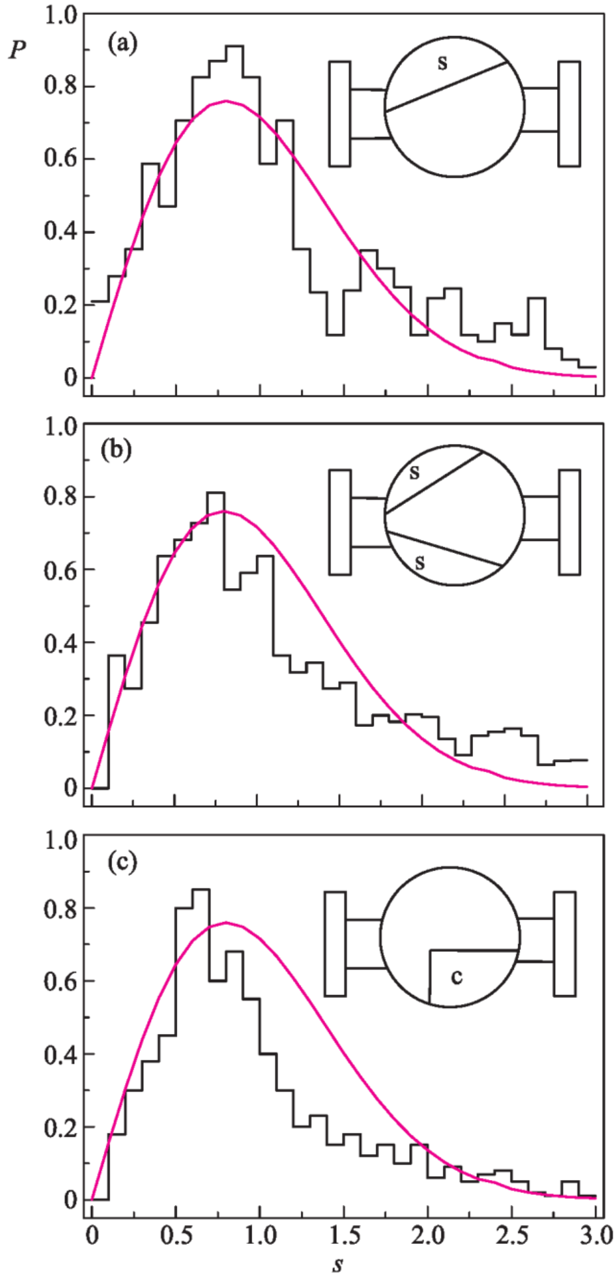

Fig. 3. The histograms of inter-frequency intervals probability distribution resonator spectra with aluminum inserts segments, by which the surface of the By comparing the resulting spectrum with the spectra of the side wall of the resonator becomes small smoothness; (a) is for the resonator with one segment

insert; (b) is for the resonator with two-segment insert, and (c) is for the resonator with an angular ledge. The location of segments in the resonator is shown schematically in the plot inset. The probability distribution of inter-frequency intervals is close to the Wigner distribution (red solid line).

The resonators discussed above with one or two single-ended segment inserts, we can notice that the sharp angle contributes significantly in probability distribution of inter-frequency intervals compared with the resonator without inserts. This distribution is also close to the Wigner one. In this case, probability distribution at small inter-frequency intervals *P(s)* is close to zero. Consequently, the regular component is not detected in the spectrum with a sharp ledge cavity side surface.

Sign of wave chaos presents in in the spectral rigidity dependence $\Delta_3(L)$, where L is the length of the spectrum, normalized by the average inter-frequency interval (Fig. 4). This function characterizes the ordering of the spectral levels in the system at long frequency distance. For a regular system (resonator without inserts) with uncorrelated spectral lines $\Delta_3(L)$ increases proportionally to L such as $\Delta_3(L) = L/15$.

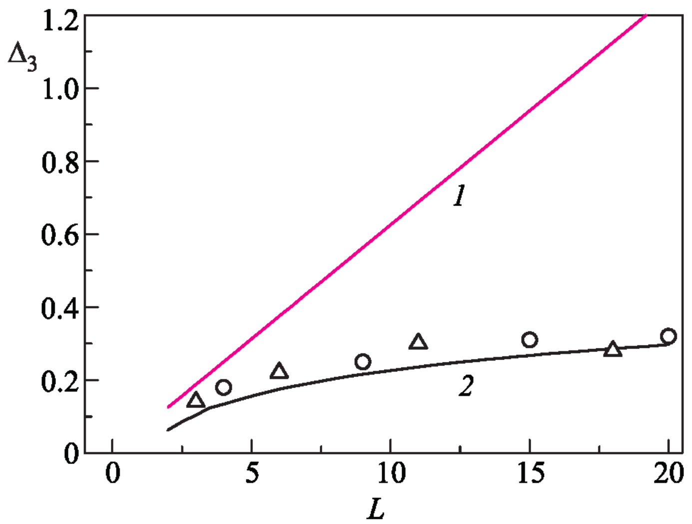

Fig. 4. The spectral rigidity for the spectra of cylindrical resonators: 1 is for the empty resonator, in the spectrum of which there is no correlation between the spectral lines, 2 is for a chaotic system, belonging to the Gaussian orthogonal ensemble with correlated spectral lines. Icons are experimental data for resonators with inserts-segments, providing a small smooth lateral surface: circles are for two inserts segments, triangles are for the corner bench.

The spectra of billiards with small smooth side boundaries the spectral rigidity dependence $\Delta_3(L)$ first increases and then reaches a plateau according the following relation $\Delta_3(L) = 1/\pi^2 \left[Ln(2\pi L) + \gamma - 5/4 - \pi^2/8 \right]$, where γ is Euler's constant. According to [6] this behavior of spectral rigidity $\Delta_3(L)$ is a typical sign of correlation of spectral lines. In addition, for the character of the dependence of $\Delta_3(L)$ for the studied resonators to the plateau is different, which is apparently due to the presence in the system along with the chaotic the regular component.

Conclusions

So, in this paper, using the microwave resonators as a billiard model for the first time detected experimentally that in the billiard system with small smooth side of the border (existence of breaks the surface, where there is no second derivative) is a source of instability and makes a regular, stable billiard system into the scattering K-system. Owing to that in such a system (in the appropriate model microwave cavity resonator) there are signs of quantum chaos: the random nature of the frequency spectrum and the Wigner distribution of the probability of the inter-frequency interval distribution.

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