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Pairs of Vortex-Antivortex and Higgs Boson in a Fractal Quantum System

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Abstract. The stochastic deformation and stress fields inside the fractal multilayer system with active nanoelements are investigated. It is shown that in a coupled system (fractal layer – fractal quantum dot) a decrease of semi-axes of the quantum dot leads to a decrease of the amplitude and the appearance of "influx" from the main peak. With increasing the semi-axes a broadened peak is formed on the background of the stochastic base (signal in the form of halo type). As the active nanoelement a set of ultracold ^{23}Na atoms in an optical trap is selected. It is shown that some of the physical properties (speed, quantization of the flow; hysteresis) of excitations such as a pair of vortex-antivortex associated with the influence on their Bose-Einstein condensate a superfluid (where excitation is the Higgs boson). The analysis of the experiment and the relations with the black hole model is carried out for the ring-shaped trap.
Keywords: Fractal quantum nanosystem, Stochastic deformation and stress fields, Optical trap, Ultracold atoms, Pairs of vortex-antivortex, Higgs boson.

1 Introduction

For creating various technical schemes in atomtronics atoms (instead of electrons in electronics) are applied (Eckel *et al.*[1]). At that the set of ultracold atoms, superfluids, where elementary excitations can be vortices, antivortices, couples vortex-antivortex are used. Fractal dislocations (V. Abramov[2]), oscillators (V. Abramov [3, 4]), traps (O. Abramova, S. Abramov[5]) may also be as active objects. Separate electron, atom, dipole, quadrupole, spin placed into a trap exhibit quantum and statistical properties (Balewski *et al.*[6], Anderson *et al.*[7]). Studying the influence of different stochastic fields on physical properties of individual objects in a trap represents one of the fundamental problems of quantum systems. In (Balewski *et al.*[6]) the relationship of properties of a single electron with the Bose-Einstein condensate (BEC) is established. The experiments at the Large Hadron Collider (LHC) and the 2013 Nobel Prize in Physics by F. Englert and P. Higgs confirmed the mechanism of the origin of the mass of subatomic particles, which is associated with spontaneous breaking of electroweak gauge symmetry, with the mass of the Higgs boson (Higgs [8]). Modern technologies in atomtronics allow to create multilayer nanosystems, synthetic mediums (metamaterials) [9], the Bose-Einstein condensate in traps (Eckel *et al.* [1], Anderson *et al.*[7]). In the work (Eckel *et al.*[1]) such condensate was created from a gas of laser-cooled ^{23}Na atoms by evaporation, first in a magnetic trap and then in a ring-shaped

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optical dipole trap. In the work (V. Abramov[10]) the features of behavior of deformation and stress fields in a fractal multilayer nanosystem with active nanoelement (^{23}Na ultracold atoms in an optical trap) were investigated. It is shown that the excitation type of vortex-antivortex pair and Bose-Einstein condensate of superfluid liquid influence each other. Coupled systems based on fractal quasi-two-dimensional structures were considered in (O. Abramova, S. Abramov[5, 11]). In cosmology a black hole is considered as an active object (Hawking [12]). Coupled system from two black holes can generate gravitational waves. In Sept. 14, 2015 the two detectors of the LIGO simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of $1.0\text{x}10^{-21}$ (Abbott *et al.*[13]). Transient processes in the multilayer nanosystem with nonlinear fractal oscillator were studied in (V. Abramov[4]).
The aim of this article is to investigate the behavior of deformation and stress fields of the coupled system (fractal layer – fractal quantum dot), of the fractal multilayer nanosystem with pairs of vortex-antivortex and the active nanoelements (set of ultracold ^{23}Na atoms in an optical trap).

2 The deformation field of the coupled system: fractal layer – fractal quantum dot

We consider a model multilayer nanosystem: discrete lattice $N_1 \times N_2 \times N_3$, whose nodes are given by integers n, m, j ($n = \overline{1, N_1}$; $m = \overline{1, N_2}$; $j = \overline{1, N_3}$). Inside this nanosystem we investigate the behavior of the deformation field of the coupled system: fractal layer – fractal quantum dot. The nonlinear equation for the dimensionless displacement function u_1 of lattice node of fractal layer j is given in the form (O. Abramova, S. Abramov[5, 11], V. Abramov[14])

$$u_1 = (1-\alpha_1)(1-2sn^2(u_1 - u_{01}, k_1))/Q_1; \quad Q_1 = p_{01}. \tag{1}$$

Here α_1 is the fractal dimension of the deformation field u_1 along the Oz-axis ($\alpha_1 \in [0,1]$); u_{01} is the constant (critical) displacement; the modulus of the elliptic sine k_1 and constant p_{01} characterize the different states of the layer j. In the general case, the parameters $k_1(j)$ and $p_{01}(j)$ depend on the index layer j. When $k_1 = 1$ the nonlinear equation (1) has the form

$$u_1 = (1-\alpha_1)(1-2th^2(u_1 - u_{01}))/p_{01}. \tag{2}$$

The solution of this equation for $p_{01} = 0.1523$ is the separate plane with the negative value $u_1 = -u_{11} = -3.2829$. Changing module k_1 in equation (1) leads to the formation of a stochastic layer instead of a separate plane. For the values of module $k_1 \in [k_{1C}$, 1) (k_{1C} is the critical value of module close to unity) the condition $\mathrm{sn}^2(u_1 - u_{01}, k_{1C}) < 0.5$ is satisfied. Therefore, the whole stochastic layer is close to the lower boundary plane $u_1 = -u_{11}$. For values of module $k_1 \in [0, k_{1c}]$ the displacement of lattice nodes change in interval $(-u_{11}, u_{11})$. The sign change in the parameter $p_{01} = -0.1523$ leads to the appearance at $k_1 = 1$ of the separate plane with the positive value $u_{11} = 3.2829$. Now for $k_1 \in [k_{1C}, 1)$ the whole stochastic layer is close to the upper boundary plane $u_1 = u_{11}$.

Dimensionless displacement function u_2 of lattice node of fractal quantum dot is determined by solving a nonlinear equation (O. Abramova, S. Abramov[5, 11])

$$u_2 = (1-\alpha_2)(1-2sn^2(u_2 - u_{02}, k_2)) / Q_2;$$

$$Q_2(n,m) = p_{02} - (n - n_{02})^2 / n_{c2}{}^2 - (m - m_{02})^2 / m_{c2}{}^2, \qquad (3)$$

where α_2 is the fractal dimension of the deformation field u_2 along the Oz-axis ($\alpha_2 \in [0,1]$); u_{02} is the constant (critical) displacement; the modulus of the elliptic sine k_2 and constant p_{02} characterize different states of the layer j; n_{02}, m_{02} are the centre coordinates and n_{c2}, m_{c2} are half-axis of the quantum dot. Here parameters $n_{c2}(j)$ and $m_{c2}(j)$ depend on the index layer j.

Separate layer and quantum dot, which are in the same layer or in different layers, influence each other and to the state of the whole multilayer nanosystem. Therefore, the dimensionless displacement function u of the such fractal coupled systems, in contrast to (1) and (3) is determined by solving the nonlinear equation (O. Abramova, S. Abramov[5, 11])

$$u = (1-\alpha_1)(1-2sn^2(u - u_{01}, k_1)) / Q_1 + (1-\alpha_2)(1-2sn^2(u - u_{02}, k_2)) / Q_2. \qquad (4)$$

The solution of the nonlinear equation (4) is performed by iteration method over the variable m at fixed values $N_1 = 120$; $N_2 = 162$; $\alpha_1 = \alpha_2 = 0.5$; $k_2 = 0.5$; $u_{01} = u_{02} = 29.537$; $p_{02} = -3.457 \cdot 10^{-5}$; $n_{02} = 59.1471$; $m_{02} = 80.3267$. Influence of a single layer and the quantum dot at each other is shown in fig. 1, 2.

The behavior of the deformation field of the coupled **system I** (separate plane with parameters $k_1 = 1$, $p_{01} = 0.1523$ and quantum dot with semi-axes $n_{c2} = 29.8793$, $m_{c2} = 40.4295$) is given in fig. 1 a, b, c. In this case, the main peak (fig. 1 a) and the stochastic behavior of the core at the quantum dot (fig. 1 b, c), the emergence of "influx" (almost a regular convex region) near the main peak (fig. 1 b) are observed.

The behavior of the deformation field of the coupled **system II** (stochastic layer with parameter $k_1 = 0.5$, different p_{01} and quantum dot with semi-axes $n_{c2} = 29.8793$, $m_{c2} = 40.4295$) is given in fig. 1 d, e, f. Increasing the parameter p_{01} in stochastic layer leads to a decrease in the amplitude of the deformation field out the region of localisation of the quantum dot in a coupled **system II**.

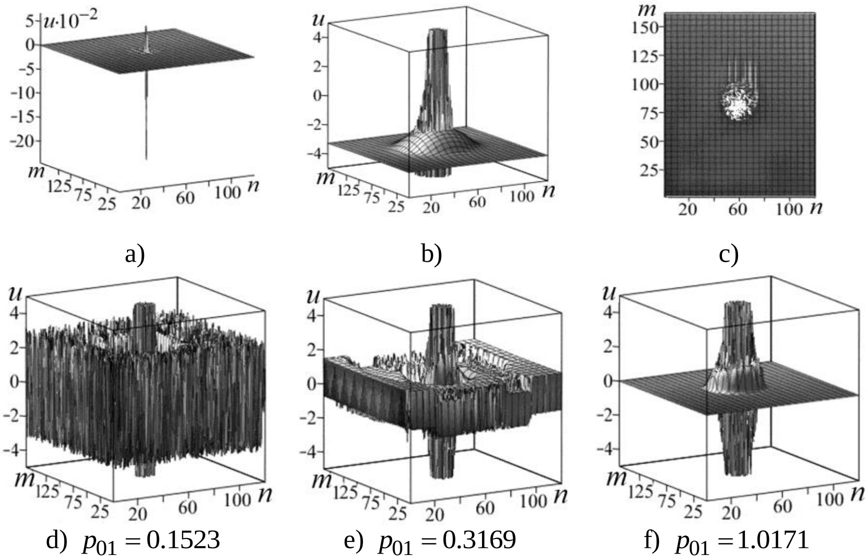


Fig. 1. The behavior of the displacement function u of coupled systems **I**, **II** on n, m: general view (a), cross-section $u \in [-5; 5]$ (b), top view of cross-section (c) for system **I**; cross-section $u \in [-5; 5]$ for different p_{01} (d, e, f) for system **II**.

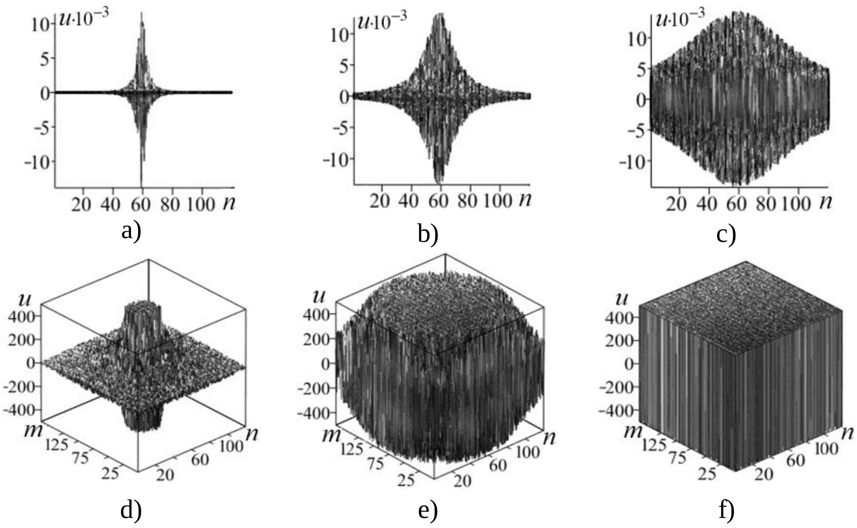


Fig. 2. The behavior of u (a, b, c) and cross-sections $u \in [-500; 500]$ (d, e, f) system **III** on n, m for different semi-axes: n_{c2}, m_{c2} increased by 16 (a, d), 64 (b, e), 256 (c, f) times as compared with fig. 1.

The behavior of the deformation field of the coupled system **III** (stochastic layer with parameters $k_1 = 0.5$, $p_{01} = 0.1523$ and quantum dot with increased semi-axes in comparison with fig. 1) is shown in fig. 2. Increasing the semi-axes by the quantum dot leads to the expansion of influence zone of stochastic behavior of quantum dot on the whole stochastic layer (fig. 2 a, b, c). For the displacement function of the coupled system **III** (fig. 2 c) the presence of broadened peak in the background of a pronounced stochastic base with large amplitudes (form a halo-type signal) is characteristic. These results can be used for modelling the phenomenon of BEC in nanosystems with vortex-antivortex pairs.

3 The deformation field in the nanosystem with pairs of vortex-antivortex

In paper (Eckel *et al.*[1]) an expression for the energy E of a vortex-antivortex pair in a ring trap in the presence of a velocity field is given

$$E = E_{\rm v} + E_m \ln(d\sin(\pi s/d)/\pi\xi)\,; \quad E_{\rm v} = \pi\rho d R {\rm v}^2 + 2\pi\hbar\rho s {\rm v}/m_a\,;$$

$$E_m = 2\pi\rho\hbar^2/m_a^2\,; \quad \xi = (\hbar^2/2m_a g N|\psi|^2)^{1/2}\,. \qquad (5)$$

Expression (5) is written in the limit, when the narrowing width d is considerably smaller than the radius of the trap R ($d \ll R$). Here s is the separation between the vortices and antivortices in a pair; v is the velocity of superfluid; $\hbar$ is Planck's constant; m_a is rest mass of the ^{23}Na atom; ρ is the effective two-dimensional mass density in the area of narrowing; ξ is the healing length of the condensate; g is the interaction strength; N is the number of atoms in the trap; ψ is the BEC wavefunction. From (5) we find

$$\sin(\pi f_s) = \pi\xi\exp((E - E_{\rm v})/E_m)/d\,; \quad f_s = s/d\,; \quad f_s' + f_s = 1\,. \qquad (6)$$

On the other hand, in our model for the nonlinear lattice parameter is defined as

$$f_s = 0.5 - 2\varphi_0/\pi n_{\varphi 0}\,; \quad u_0 = {\rm F}(\varphi_0; k_u)\,; \quad k_u^2 = (1-\alpha)Q^{-1}\,; \quad k_u^2 + (k_u')^2 = 1\,;$$

$$Q = p_0' + p_3' j - b_3 (j - j_0)^2 j_c^{-2}\,. \qquad (7)$$

Here $|\alpha|$ is the fractal dimension of the deformation field u along the Oz-axis ($\alpha \in [-1;1]$); k_u is the variable modulus of the elliptic sine; u_0 is the critical displacement, dependent on the angle φ_0, k_u; F is an incomplete elliptic integral of the first kind; p_0', p_3', b_3, j_0, j_c are some of the governing parameters; $n_{\varphi 0} = 0, \pm 1, \pm 2, \ldots$ are integers, defining a set of different states of the deformation field. When $n_{\varphi 0} > 0$ the state with the left polarisation is realised, when $n_{\varphi 0} < 0$ the state with the right polarisation is realised.

Description of the deformation field in the nanosystem with a vortex-antivortex pairs leads to two branches ($r = 1, 2$) for the dimensionless displacement u_r of lattice node. Model nonlinear equations have the form (V. Abramov [14])

$$u = u_1 = k_u^2 (1 - 2\,\mathrm{sn}^2 (u_1 - u_0, k_u)) ; \quad u = u_2 = k_u^2 (1 - 2\,\mathrm{sn}^2 (u_2 - u_0, k'_u)) . \quad (8)$$

The alteration of the deformation field states can be carried out by changing φ_0 , $n_{\varphi 0}$, u_0 and parameter f_s . The analysis of the behavior of the deformation field is carried out in terms of averaged (along n, m) complex functions $M_r = M'_r + iM''_r$

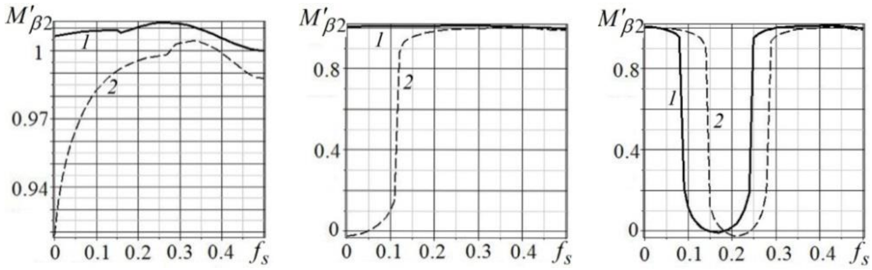
$$M_r(j) = \mathrm{Sp}(\hat{\rho}\hat{u}_r) ; \quad \hat{\rho} = \hat{\xi}^T_{N2} \hat{\xi}_{N1} / N_2 N_1 ; \quad M'_r = \mathrm{Re} M_r ; \quad M''_r = \mathrm{Im} M_r , \quad (9)$$

where Sp is an operation of calculating the trace of a square matrix; i is an imaginary unit; « T » denotes transposition; $\hat{\xi}_{N1}$, $\hat{\xi}_{N2}$ are row-vectors with elements equal to one. On the basis of dependencies M_r on j , f_s from (9) for the inverse states with $\alpha = -0.5$ and the fixed layer $j = 30$ the numerical simulations can be carried out. Initial parameters are $N_1 = 30$, $N_2 = 40$, $N_3 = 67$, $j_0 = 30.5279$; governing parameters are $p'_0 = -1.5123$, $p'_3 = 0$, $b_3 = 1$. The solution of equations for u_r from (8) we find by iteration method over the variable m with the initial conditions $u_1 = u_2 = 0$. Further, the description of hysteresis is conveniently carried out in terms of the functions $M_{\beta 2}$, $m_{\beta 2}$ (normalised half sum, half difference averaged functions M_2 from (9), fig. 3, 4)

$$M_{\beta 2} = M'_{\beta 2} + iM''_{\beta 2} = -\beta_{20} [M_2(f_s; n_\varphi) + M_2(f_s; -n_\varphi)] / 2 ;$$

$$m_{\beta 2} = m'_{\beta 2} + i\,m'_{\beta 2} = -\beta_{20} [M_2(f_s; n_\varphi) - M_2(f_s; -n_\varphi)] / 2 ;$$

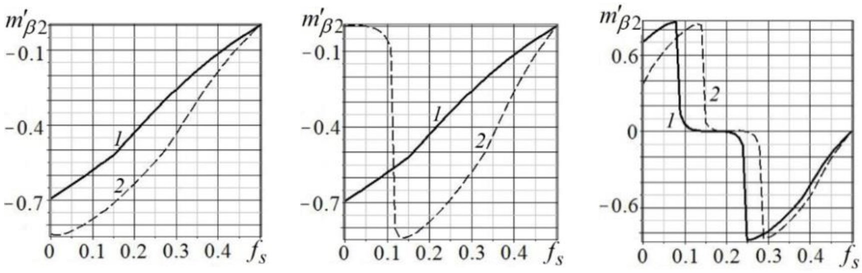
$$\beta_{20} = \left((M'_{20})^2 + (M''_{20})^2 \right)^{-1/2} ; \quad M'_2(f_s; 0) = M'_{20} ; \quad M''_2(f_s; 0) = M''_{20} . \quad (10)$$



a) $n_{\varphi 0} = 1 ;$ $n_\varphi = 1.5$ b) $n_{\varphi 0} = 1 ;$ $n_\varphi = 2$ c) $n_{\varphi 0} = 3 ;$ $n_\varphi = 3.5$

Fig. 3. Behavior $M'_{\beta 2}$ on f_s : *1* is solid line for $j_{c1} = 11.8247$;
2 is dash line for $j_{c2} = 2.9562$.

According to the calculated values $M'_{20} = -0.5404$, $M''_{20} = 1.1062 \cdot 10^{-16}$ the parameter of normalisation $\beta_{20} = 1.8505$ is found. Examples of the behavior of the hysteresis curves for integer $n_{\varphi 0}$ and semi-integer values n_{φ} for two values of semi-axes $j_c = j_{c1} = 11.8247$ and $j_c = j_{c2} = 2.9562$ are given in fig. 3, 4.



a) $n_{\varphi 0} = 1;\ n_{\varphi} = 1.5$ *b)* $n_{\varphi 0} = 1;\ n_{\varphi} = 2$ *c)* $n_{\varphi 0} = 3;\ n_{\varphi} = 3.5$

Fig. 4. Behavior $m'_{\beta 2}$ on f_s : *1* is solid line for $j_{c1} = 11.8247$;
2 is dash line for $j_{c2} = 2.9562$.

Alteration of the structural states due to changes f_s; $n_{\varphi 0}$, φ_0; n_{φ}, φ is accompanied by changes in the position of the local maxima and minima, wells and barriers, polarisation on dependences M_2 from (9). The choice of two different semi-axes j_{c1}, j_{c2} according to (7) leads to two different $Q = Q_{c1}$, $Q = Q_{c2}$ and, respectively, to two different modules $k_u = k_{c1}$, $k_u = k_{c2}$. The alteration of the structural states due to changes j_c is accompanied by hysteresis phenomena (fig. 3 b, c; 4 a, b, c). Double hysteresis loops (fig. 3; 4 b, c) indicate the presence of coupled states (vortex-antivortex pairs).

4 Coupled nanosystem: pairs of vortex-antivortex and Higgs boson

Experimentally vortex-antivortex pairs were created in the ring-shaped BEC which contains $N \approx 4 \cdot 10^5$ ^{23}Na atoms (Eckel *et al.*[1]). This condensate is created from a gas of laser-cooled ^{23}Na atoms by evaporation, first in a magnetic trap and then in a ring-shaped ($R = 19.5\mu\text{m}$) optical dipole trap. The area of the ring narrowing, where there was a vortex-antivortex pair, was created by a blue laser. The estimates of radius R_{0a} of the trap, the wave length λ_4 for the laser transition and their relationship with the parameters of the ^{23}Na atoms we will get by the formulas

$$R_\rho - R_{0a} = \delta_\rho;\quad R_\rho = 2GM_a / c^2;\quad R_{0a} = Nr_{0a} / 2;\quad R_k = Nr_k / 2;$$
$$R_{0a} - R_k = \lambda_4;\quad \lambda_4 = \hbar\pi c / \varepsilon_4;\quad r_{0a} - r_k = \delta_k;\quad \delta_k = 2\lambda_4 / N. \qquad (11)$$

Here $R_\rho = 20.5688\mu\text{m}$ allows the interpretation as the Schwarzschild radius of the black hole with a mass $M_a = N_a M_\rho$; $M_\rho = N_a m_a = 22.99\text{g}$ is the molar mass of the atom ^{23}Na; N_a is Avogadro's number; c is speed of light in vacuum; $G = 6.672 \cdot 10^{-8} \text{cm}^3 \text{g}^{-1} \text{s}^{-2}$ is Newton's gravitational constant. The energy spectrum ε_x (where $x = 1, 2, 3, 4$) of elementary excitations is defined by formulas (V. Abramov and Kopvillem[15])

$$\varepsilon_x = \pm 2\varepsilon_{01} S_x ;\ \ \varepsilon'_x = \pm 2\varepsilon_{02} S_x ;\ \ 2\varepsilon_{01} = (\varepsilon_{0g}^2 + 4\Delta_q^2)^{1/2} ;\ \ 2\varepsilon_{02} = (\varepsilon_{0g}^2 - 4\Delta_q^2)^{1/2} ;$$

$$\varepsilon_{0g} = g_F b_1 \gamma_n n_g (F(F+1))^{1/2} ;\ \ 2\Delta_q = b_0 \varepsilon'_{0g} ;\ \ n_g = 3\cos^2 \theta_g - 1 . \qquad (12)$$

Here $b_1 = 117.68308\,\text{kOe} \cdot \text{meV(MHz)}^{-1}$; $\gamma_n / 2\pi = 1.12677\,\text{MHz(kOe)}^{-1}$ is the nuclear gyromagnetic ratio of the ^{23}Na atom; $g_F = 2/3$ is the spectroscopic splitting factor. The ^{23}Na atom has the nuclear spin $I = 3/2$. We consider the basic electron shell $1s^2 2s^2 2p^5 3s^2$ with angular momentum $J = 3/2$. Then, the total angular momentum F of atom can take integer values $F = 0,1,2,3$.

We consider the state with $F = 3$, $n_g = -1$, $\varepsilon'_{0g} = 2\varepsilon_{0g}$. Then, on the basis of the spectrum (12) the values wavelength $\lambda_4 = 535.5224\text{nm}$, parameter $\delta_k = 0.0268\text{A}$ are obtained. The numerical values parameters b_0, S_x of the theory are given below. If to accept that $R_{0a} = R = 19.5\,\mu\text{m}$, then by (11) we obtain estimates of the radii $r_k = 0.9489\text{A}$ and $r_{0a} = 0.9757\text{A}$, which are close to the ionic (for $^{23}\text{Na}^+$) and atomic (for $^{23}\text{Na}^0$) radii $r_k = 0.95\text{A}$ and $r_{0a} = 0.98\text{A}$, respectively, (Kittel[16]). Then we obtain parameter $\delta_\rho = 1.0688\,\mu\text{m}$ and radius $R_k = 18.978\,\mu\text{m}$. Atoms in the state with $F = 0$ can enter the Bose condensate superfluid with the formation of elementary excitations of the Higgs boson type, and the atoms in the excited states with $F = 1, 2, 3$ can enter elementary excitations of a vortex-antivortex pair type. A superposition of atom states with different F is possible. The physical parameters of these excitations are related as follows (V. Abramov [10])

$$b_0 E_{g1} / N_a = m_a \text{v}_c^2 ;\ E_{g1} = M_{g1} c^2 = M_{H0}\, c_{H0}^2 ;\ c_{H0} = c S'_{\mu 1} ;\ \ E_{H0} = M_{H0}\, c^2 ;$$

$$E_a = m_a c^2 ;\ \ \text{v}_a = 2\text{v}_s M_{H0} / m_{e\mu} ;\ \ m_{e\mu} = 2 m_e m_\mu / (2m_e + m_\mu) ;\ \ b_0 \text{v}_s = \sqrt{2} \cdot \text{v}_c . \qquad (13)$$

Here M_{H0}, m_e, m_μ are rest Higgs boson, electron, muon masses; $S'_{\mu 1} = 1 - S_{\mu 1}$ is parameter of the theory. According to the calculation of the lower critical speed of movement $\text{v}_c = 4.9791 \cdot 10^{-2} \text{cm} \cdot \text{s}^{-1}$ on the basis relations from (13) the estimates of the basic parameters of the theory are obtained: $E_{g1} = 115.1183\,\text{GeV}$ (such value earlier had expected in the theory of the supersymmetry for the Higgs boson); $E_{H0} = 125.0324\text{GeV}$ is consistent with the experimental value at the LHC for the Higgs boson; the upper critical speed is $\text{v}_s = 22.7824 \cdot 10^{-2}\text{cm}\cdot\text{s}^{-1}$; $E_a = 21.4072\text{GeV}$. Further we give the detailed formulas for calculating the parameters of the theory S_x, $S_{\mu x}$, b_0, b_μ, b_q, ξ_q. For the quasi-one-dimensional lattice with two atoms per unit cell (such as the electron and proton rest masses m_e and m_p), when the condition $b_q^2 < 1$ are fulfilled, parameters S_x are given in (V. Abramov and Kopvillem[15])

$$4S_1 = 1 - g_{q1}; \quad 4S_3 = 1 + g_{q1}; \quad 4S_2 = g_{q2} - 1; \quad 4S_4 = g_{q2} + 1;$$

$$g_{q1} = (1 - b_q^2)^{1/2}; \quad g_{q2} = (1 + b_q^2)^{1/2}; \quad b_q^2 = 4\xi_q(\xi_q + 1);$$

$$\xi_q = k\,\text{sn}(u_q,k)/[1+\text{dn}(u_q,k)] = \omega_{2q}/\omega_{1q}, \tag{14}$$

where u_q is effective dimensionless displacement of the lattice node, which depends nonlinearly on the wave vector $\vec{q}$, the lattice parameter $\vec{r}_k$; ω_{1q} and ω_{2q} are frequencies of the optical and acoustic branches of the vibrational spectrum

$$(\hbar\omega_{1q})^2 = (\hbar\omega_{u0})^2(1 + \text{dn}(u_q,k))/2; \quad (\hbar\omega_{2q})^2 = (\hbar\omega_{u0})^2(1 - \text{dn}(u_q,k))/2; \tag{15}$$

$$\omega_{u0}^2 = 4g_{u0}/m_1; \quad m_1 = m_e m_p/(m_e + m_p). \tag{16}$$

Here g_{u0} is the lattice force constant between the particles with the rest mass m_e, m_p. In the centre of the Brillouin zone at $|\vec{q}| = 0$ the condition $\text{dn}(u_0,k)=1$ is fulfilled. Then from (15) follows that the energy of the optical and acoustic modes are equal $\hbar\omega_{1q} = |\hbar\omega_{u0}|$ and $\hbar\omega_{2q} = 0$, respectively, parameter $\xi_q = 0$. At the boundary of the Brillouin zone at $|\vec{q}| = \pi/|\vec{r}_k|$ the condition $\text{sn}^2(u_q,k) = \sin^2(\vec{q}\cdot\vec{r}_k/2)=1$ is fulfilled. Then from (15), (14) we find

$$(\hbar\omega_{1q})^2 = (\hbar\omega_{u0})^2(1 + k')/2 = (\hbar\omega_1)^2; \; (\hbar\omega_{2q})^2 = (\hbar\omega_{u0})^2(1 - k')/2 = (\hbar\omega_2)^2;$$

$$\xi_q^2 = \xi_{q0}^2 = \omega_2^2/\omega_1^2 = m_e/m_p = k^2/(1 + k')^2; \quad (k')^2 + k^2 = 1. \tag{17}$$

The relations (17) allow us to express modules k, k' of elliptic functions from (14), (15) through the known rest mass m_e, m_p of the particles or through the main parameter of the theory ξ_{q0} in the following form

$$k^2 = 4\xi_{q0}^2/(1 + \xi_{q0}^2)^2 = 4m_e m_p/(m_e + m_p)^2; \quad (k')^2 = (m_e - m_p)^2/(m_e + m_p)^2. \tag{18}$$

Other parameters from (14) at the boundary of the Brillouin zone have numerical values $b_q = b_0 = 0.30907377$; $S_1 = -1.2240475 \cdot 10^{-2}$; $S_2 = 1.16685 \cdot 10^{-2}$;

$S_3 = 48.775952 \cdot 10^{-2}$; $S_4 = 51.16685 \cdot 10^{-2}$. Now changing modules of elliptic functions in expressions (1), (3), (7), (8) based on the relations (18) can be interpreted in terms of the rest mass of particles both in separate and in various layers of the multilayer nanosystems. We note that in the general case $\mathrm{sn}^2(u_q, k) \neq 1$ and for finding of dependencies u_q on some parameters ($f_s, j, n_{\varphi 0}$) once again come to the nonlinear equations of the type (8). The parameter $2\Delta_{H0}$ can be interpreted as an order parameter and it is associated with an energy gap $2\Delta_{\rho 0}$ in the spectrum of elementary excitations in a coupled system (vortex-antivortex pair and Higgs boson) relationships

$$2\Delta_{H0} = b_0 E_{H0} = M_\rho \mathrm{v}_{\rho 0}^2 = N_a 2\Delta_{\rho 0}; \quad 2\Delta_{\rho 0} = \hbar\omega_{\rho 0} = m_a \mathrm{v}_{\rho 0}^2 . \qquad (19)$$

Here parameters from (19) are the velocity $\mathrm{v}_{\rho 0} = c(M_{H0} b_0 / M_\rho)^{1/2} = 5.1890 \cdot 10^{-2} \,\mathrm{cm} \cdot \mathrm{s}^{-1}$ and energy gap $2\Delta_{H0} = 38.6442\,\mathrm{GeV}$. By analogy with (14) for a quasi one-dimensional lattice (type of the electron and muon rest mass m_e and m_μ) we obtain the parameters $\xi_q = \xi_\mu = (m_e / m_\mu)^{1/2}$; $b_q = b_\mu = 0.54545421$; $S_{\mu 1} = -4.0464845 \cdot 10^{-2}$; $S_{\mu 2} = 3.4771871 \cdot 10^{-2}$; $S_{\mu 3} = 45.9535155 \cdot 10^{-2}$; $S_{\mu 4} = 53.4771871 \cdot 10^{-2}$. For lattice (type of an atom ^{23}Na and Higgs boson) we find the basic parameter of the theory $\xi_a^2 = m_a / M_{H0} = 0.17121315$ .The parameters $\tau_{2H} = 1 + 2|\xi_a|$ and b_{Ha} connected by the relation $\tau_{2H}^2 - b_{Ha}^2 = 1$ from which we find the value $b_{Ha} = 1.5296957$, i.e. in this case following condition $b_{Ha}^2 = b_q^2 > 1$ is fulfilled, and formulas (14) are replaced by

$$4S'_{Ha} = |\sin\theta_{Ha}| - 1; \quad 4S'_{H3} = 1 + |\sin\theta_{Ha}|; \quad 4S'_{H2} = (1 + \cos^2\theta_{Ha})^{1/2} - 1;$$
$$4S'_{H4} = (1 + \cos^2\theta_{Ha})^{1/2} + 1; \quad \tau_{2H}^{-2} = \cos^2\theta_{Ha}; \quad \sin^2\theta_{Ha} = b_{Ha}^2 / (1 + b_{Ha}^2). \qquad (20)$$

Using (20), the calculated values $|\sin\theta_{Ha}| = 0.8370$, the possible values of angles $\theta_{Ha} \approx \pm 56.83^\circ$ and $\theta_{Ha} \approx \pm 123.42^\circ$, parameters $S'_{H1} = -4.0745963 \cdot 10^{-2}$, $S'_{H3} = 45.925404 \cdot 10^{-2}$, $S'_{H2} = 3.4978505 \cdot 10^{-2}$, $S'_{H4} = 53.497851 \cdot 10^{-2}$ are obtained. In the general case $\sin\theta_{Ha} = \mathrm{sn}(u_{Ha}, k_{Ha})$, $\cos\theta_{Ha} = \mathrm{cn}(u_{Ha}, k_{Ha})$, where $u_{Ha} = \mathrm{F}(\theta_{Ha}; k_{Ha})$ is an effective displacement, which depend on angle θ_{Ha} and module k_{Ha} of elliptic functions. Note that when $k_{Ha} = 1$ these functions are converted into $\sin\theta_{Ha} = \mathrm{th}\, u_{Ha}$, $\cos\theta_{Ha} = \mathrm{sech}\, u_{Ha}$. When the condition $b_{Ha}^2 = 1$ is fulfilled, that from formulas (14) and (20) follows the existence of threshold values of the parameters: $\xi_{ac}^2 = m_{ac} / M_{H0}$, τ_{2H}^2, S_x,

S'_{Hx}, θ_{ac}, rest mass m_{ac}, molar mass $M_{\rho c} = m_{ac} N_a$ and energy $E_{ac} = m_{ac} c^2$ of the particle. The numerical values of the parameters are equal: $\xi_{ac}^2 = 42.893219 \cdot 10^{-2}$; $\tau_{2H}^2 = 2$; $S_1 = S_3 = S'_{H1} = S'_{H3} = 0.25$, $4S_2 = 4S'_{H2} = \sqrt{2} - 1$, $4S_4 = 4S'_{H4} = \sqrt{2} + 1$; $\theta_{ac} = \pm\pi/4$ and $\theta_{ac} = \pm 3\pi/4$; $M_{\rho c} = 5.7596\,\mathrm{g}$; $E_{ac} = 5.3630\,\mathrm{GeV}$. Spectra type of $\omega_{ax} = 2\omega_{01} S_x$ and $\Omega_{ax} = 2\omega_{02} S_x$ allow us to obtain a characteristic frequency of rotation Ω_{0a} of superliquid in the ring with radius R_{0a} of optical trap and critical velocities $\mathrm{v}_{cx}, \mathrm{v}'_{cx}$, where

$$2\omega_{01} = [(\omega_a)^2 + b_0^2(\omega_{3n})^2]^{1/2}; \quad 2\omega_{02} = [(\omega_a)^2 - b_0^2(\omega_{3n})^2]^{1/2};$$

$$\hbar\omega_a = E_a / N_a; \quad \hbar\omega_{3n} = m_a \mathrm{v}_{c1}^2; \quad \Omega_{0a} = \omega_{0a} = (S_1 + S_2)\omega_{01};$$

$$R_{0a}^2 = \hbar N_a / M_\rho \omega_{0a}; \quad \mathrm{v}'_{cx} = S_{\mu x} \mathrm{v}_{\rho 0}; \quad \mathrm{v}_{cx} + \mathrm{v}'_{cx} = \mathrm{v}_{\rho 0}; \quad \mathrm{v}_{c1} = \mathrm{v}_c;$$

$$\mathrm{v}_{c1} + \mathrm{v}_{c3} = 3\mathrm{v}_{\rho 0}/2; \quad \mathrm{v}'_{c4} - \mathrm{v}'_{c2} = \mathrm{v}_{c2} - \mathrm{v}_{c4} = \mathrm{v}'_{c1} + \mathrm{v}'_{c3} = \mathrm{v}_{\rho 0}/2. \quad (21)$$

On the basis of (14) and (21) we find the parameters: $\omega_a = 53.985\,\mathrm{Hz}$; $\omega_{3n} = 89.726\,\mathrm{Hz}$; $b_0\omega_{3n} = 27.732\,\mathrm{Hz}$; $R_{0a} = 19.5146\,\mu\mathrm{m}$; $\Omega_{0a} = 72.553 \cdot 10^{-2}\,\mathrm{Hz}$. In experiment (Eckel *et al.*[1]) for changing governing parameter (in our case f_s) two sets of discrete critical velocities are observed. The calculated values of critical velocities $2\mathrm{v}'_{c4} = 5.5499 \cdot 10^{-2}\,\mathrm{cm \cdot s^{-1}}$ and $\mathrm{v}_{c1} + \mathrm{v}_{c3} = 7.7835 \cdot 10^{-2}\,\mathrm{cm \cdot s^{-1}}$ practically coincide with the experimental values $5.56 \cdot 10^{-2}\,\mathrm{cm \cdot s^{-1}}$ and $7.78 \cdot 10^{-2}\,\mathrm{cm \cdot s^{-1}}$ from the first and second set, respectively. These dependencies of critical velocities from different sets intersect near values $6.67 \cdot 10^{-2}\,\mathrm{cm \cdot s^{-1}}$. The effect of the formation of the double hysteresis loop (analog of this effect for the B-phase in helium-3 ([17, p. 43, 48])) is possible. The temperature $T_{ck} = 68.5305\,\mathrm{nK}$ and $T_{cH} = 100.0660\,\mathrm{nK}$ of supercooled and BEC superfluid liquids, respectively, are determined from the expression

$$\hbar\omega_{3n} = m_a \mathrm{v}_{c1}^2 = k_B T_{ck}; \quad M_{H0} \mathrm{v}_{c1}^2 = 2k_B T_{cH}; \quad T_{c0} = N_{H0} T_{ck}; \quad T_A = N_{H0} 2T_{cH};$$

$$T_{ck} / 2T_{cH} = 2m_a / M_{H0} = 2\xi_a^2 = T_{c0} / T_A; \quad T_A / 2T_{cH} = T_{c0} / T_{ck} = N_{H0}, \quad (22)$$

where k_B is Boltzmann constant. The parameter N_{H0} from (22) is defined by the spectrum $N_{Hx} = N_0 S'_{Hx}$, taking into account (20). If $N_{H4} = N/2$, then $N_0 = 3.7385 \cdot 10^5$, $N_{H0} = N_{H2} = 1.3077 \cdot 10^4$, $N_{H1} = 1.5233 \cdot 10^4$. Further we find temperatures $T_{c0} = 0.8961\,\mathrm{mK}$, $T_A = 2.6170\,\mathrm{mK}$, which are close to the phase transition temperatures $T_{c0} = 0.9\,\mathrm{mK}$, $T_A = 2.6\,\mathrm{mK}$ for helium-3 ([17, p. 43, 48]). If $N_{H0} = 1$, then from (22) follows $T_{c0} = T_{ck}$, $T_A = 2T_{cH}$. Therefore

N_{H0}, N_{H1}, it can be interpreted as the number of Higgs bosons, vortex-antivortex pairs in the condensate superliquid. Next, on the basis of expressions

$$E_{H0} = N_a \hbar\omega_{H0}\,;\ \omega_{H0} = \gamma_{H0} H_{M0}\,;\ \gamma_{H0} = m_p \gamma_n / M_{H0}\,;\ \omega_{H0} = 2\pi\nu_{H0}\,, \quad (23)$$

we find the frequency $\nu_{H0} = 50.183\text{Hz}$; $\gamma_{H0}/2\pi = 8.45557\text{Hz}\cdot\text{Oe}^{-1}$ and the effective magnetic field $H_{M0} = 5.935\text{Oe}$ (analogue of Higgs field). The frequency spectrum $\omega_{Hx} = N_{Hx}\omega_{3n} = N_0 \omega_{3n} S'_{Hx}$, which obtained without the influence of vibrational modes in the parameter N_0, allow us to define the frequency $\nu_{H2} = \omega_{H2}/2\pi = 186.738\text{kHz}$ on the basis of relations $\hbar\omega_{H2} = k_B N_{H2} T_{ck} = N_{H2} m_a v_{c1}^2$. This frequency can be measured by methods of continuous and pulsed nuclear magnetic resonance (NMR) superliquid by analogy with ([17, p. 68]). Then the mentioned frequency ν_{H2} can be determined by the shift from the temperature dependence of the NMR frequency. The influence of the vibrational modes ($\omega_{\delta 0} \neq 0$) leads to a change spectrum ω_{Hx} on

$$\Omega_{\delta x} = 2\Omega_{01} S'_{Hx}\,;\quad \omega_{\delta x} = 2\Omega_{02} S'_{Hx}\,;\quad 2\Omega_{01} = (\omega_{N0}^2 + 4\delta^2)^{1/2}\,;$$

$$2\Omega_{01} = (\omega_{N0}^2 - 4\delta^2)^{1/2}\,;\quad \omega_{N0} = N_0\omega_{3n} = \gamma_n H_{N0}\,;\quad 2\delta = \omega_{\delta 0}\,|\cos\theta_\delta|\,. \quad (24)$$

Here the parameters are frequency $\nu_{N0} = \omega_{N0}/2\pi = 5.339\text{MHz}$, the effective magnetic field $H_{N0} = 4.738\text{kOe}$. The angular position of the weakest link (neck, narrowing) in the ring-shaped condensate of superliquid can be controlled by choosing of the frequencies $\omega_{\delta 0}$ and angles θ_δ, using impulse methods (Eckel *et al.*[1]). If $2\delta = \omega_{\delta 0}/2$, then the possible values of angles are $\theta_\delta = \pm\pi/3; \pm 2\pi/3$. If $|\cos\theta_\delta| = 0$, then parameters are $2\delta = 0$ and $\theta_\delta = \pm\pi/2$. The estimation of the molar masses of the weak link excluding (m_ρ) and taking into account the presence of vortex-antivortex pairs in narrowing (m_s) we will perform by the formulas

$$m_\rho = \rho\pi\xi_{ck}^2 = M_\rho S_{\mu 2}^2 / 4\,;\quad m_s = \rho\pi s^2\,;\quad M_\rho = \rho 2\pi R_{0a} d_k\,, \quad (25)$$

where ξ_{ck} is the limiting correlation length. From (25) we find $\xi_{ck} = R_{0a} S_{\mu 2}^2 / 8 = 29.2600\text{A}$; $d_k = 4\xi_{ck} = 117.0401\text{A}$; $m_s = m_\rho f_s^2$. It follows that mass of the narrowing m_s with the presence of vortex-antivortex pairs can be changed by parameter f_s and m_ρ. This is confirmed by the dependencies of the effective displacements on f_s and parameters j_c (fig. 3, 4), n_{c2}, m_{c2} (fig. 2), p_{01} (fig. 1), where there are changes of the potential relief, the appearance of wells and barriers, hysteresis.

The parameters R_{H0}, M'_{H0} are related to the parameters of the black hole from (11) as

$$R_\rho = \xi_a^2 R_{H0}\,; \quad M_a = \xi_a^2 M'_{H0}\,; \quad M'_{H0} = N_a^2 M_{H0}\,; \quad M_{H0} = N_{HG} M_G\,. \quad (26)$$

Here $M_G = N_a m_G$ and m_G are the molar mass and the rest mass of the graviton; $E_G = M_G c^2 = 12.1175$ мкэВ ; $N_{HG} = E_{H0} / E_G = 1.0318 \cdot 10^{16}$. The mass of a single black hole M_{r0}, which appears after the merger of two black holes, defined as

$$M_{r0} = 2N_{r0} M'_{H0}\,; \quad N_{r0} = N_0 N_{ra}\,; \quad N_{ra} = T_r / 2T_{H0}\,; \quad T_{H0} = T_A / 2\,. \quad (27)$$

Here $T_r = 2.72548\text{K}$ is the cosmic microwave background radiation temperature; parameters are $N_{r0} = 7.7867 \cdot 10^8$; $N_{ra} = 1.0414 \cdot 10^3$. The calculated frequency $\nu'_{H0} = \nu_{H0} R_\rho / 4\delta_\rho = 244.769\text{Hz}$ can be interpreted as the frequency of the radiation of gravitational waves after the merger of two black holes. Experimentally, after the merger of two black holes a transient signal with near modulation frequency (Abbott *et al.*[13]) is observed.

Conclusions

In the multilayer nanosystem the displacements of lattice nodes of the layer at module $k_1 = 1$ are a separate plane, while at $k_1 \neq 1$ they form a stochastic layer. The width and the region of localisation of the stochastic layer can be controlled by the changes of module k_1, the parameter p_{01}. For the quantum dot at a module $k_2 \neq 1$ the presence of sharp peaks with high amplitude and stochastic behavior near the core localisation of the basic peaks are characteristic. In the coupled system the separate layer and the quantum dot influence each other. The decrease in the semi-axes of the quantum dot leads to effects: decreasing of the basic peak amplitude; narrowing of the stochastic behavior region; the appearance of "influx" near the basic peak. The increase in the semi-axes of the quantum dot leads to effects: the expansion zone of influence of the stochastic behavior of the quantum dot on the whole stochastic layer; the appearance of broadened peak in the background of a pronounced stochastic base with large amplitudes (form a halo-type signal). For the coupled system the increase of parameter p_{01} in stochastic layer leads to decrease of the deformation field amplitude outside the region of localisation of the quantum dot.

When the temperature $T < T_{cH}$ the behavior of superliquid (a set of ultracold ^{23}Na atoms in a ring-shaped optical trap) is characterised by: the quantized rotation frequency Ω_{0a} of superliquid (or quantization of the flow $n_{\varphi 0}\Omega_{0a}$); basic radius of trap R_{0a}; two sets of discrete critical velocities (type $2\text{v}'_{c4}$ and $\text{v}_{c1} + \text{v}_{c3}$); hysteresis and the appearance of the double hysteresis loops on the dependencies of the effective displacements $M_{\beta 2}$, $m_{\beta 2}$ on f_s (that is associated with the presence of a set of metastable and coupled states at $n_{\varphi 0} \neq 0$). ^{23}Na atom and the Higgs boson form an elementary cell of quasi-one lattice in superliquid. The elementary excitation of the type of vortex-antivortex pairs (with N_{H1}) influence on BEC of superfluid liquid (with parameters N_{H0}, $2\Delta_{H0}$, H_{M0}), where the elementary excitation is the Higgs boson (with an effective mass M_{H0}). The relationship of temperatures T_{ck}, T_{cH} by parameter N_{H0} with temperatures T_{c0}, T_A (analogous temperatures of phase transition for B and A phases of helium-3) are determined. The relations parameters of ^{23}Na atoms, Higgs boson in a ring-shaped optical trap with the parameters of black holes before merger (with a total mass $2M_a$) and after the merger (with mass M'_{H0}) are determined. The estimate of radiation frequency of gravitational waves ν'_{H0} after the merger of two black holes is performed.

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