Cardiorespiratory System with Strong Interaction

Tatyana S. Krasnopolskaya [1]
Evgeniy D. Pechuk [2]

[1] Institute of Hydromechanics NASU, Kyiv, Ukraine
E-mail: t.krasnopolskaya@tue.nl
[2] Institute of Hydromechanics NASU, Kyiv, Ukraine
E-mail: uzuzun@i.ua

Abstract: A cardiorespiratory system with a strong internal interaction, when a respiratory tract was modeled as a self-oscillating system under an impulsive influence of heartbeats, was studied. The internal interaction gives rise to chaotic steady-state regimes. Analysis of bifurcation curves of the largest Lyapunov exponent, projections of phase portraits, temporal realizations and power spectrums showed the basic laws of dynamics of the cardiorespiratory system. The chaotic dynamics of heartbeat and respiratory systems are in good correspondence with an experimental information of healthy man.
Keywords: cardiorespiratory system, heartbeats, chaotic steady-state regimes.

1. Introduction

The human cardiovascular system directly and indirectly interacts with different systems of entire organism. Realized self-oscillations in a cardiovascular system are under an activity of practically all organs (see [2-5, 9-11]). There are numerous interactions of heart rhythms between itself and with an internal and external environment. Cardiac and respiratory rhythms form up during embryo stage, and even the brief break of these rhythms after a birth results in death.
Existence of breathing and heart rhythm synchronization effect, found experimentally in the cardiovascular system, is well-proven by Toledo [10] in 2002. Dynamic process of mutual synchronization can be realized only in a case of presence of a subsystem mechanical interaction. So, this effect display testifies the presence of both direct and feedback interactions between the cardiovascular and respiratory systems. Namely these direct and feedback interactions are main goal of our modelling and study in present paper.

2. The mathematical model with strong interaction

A heart system and organism of man in general have one of major descriptions of activity, such as a blood pressure dynamics. His time-history, along with electrocardiogram (ECG), is an important information generator for research and diagnostics of laws of the cardiovascular system. The task of mathematical model construction, describing the dynamics of arterial blood pressure, is far from completion. The DeBoer model of a cardiovascular system is under direct action of a respiratory system (what corresponds to experimental data) [3]. This model was substantially developed. The sinus node responsiveness (and other detailed factors) is taking into

Received: 7 September 2016 / Accepted: 17 December 2016

ISSN 2241-0503

account in the work of Seidel and Herzel [9] (the so-called SH-model). In this model chaotic dynamics was found in dynamics of a cardiosystem.

The models of DeBoer and SH only considered direct respiratory influence on heartbeats. The SH-model got further development [5], where an effect of heartbeat and the resultant changes in the baroreceptor afferent activity to the SH-model are added and the cardiorespiratory synchronization found due to this modification. Interaction of blood pressure and amplitudes of breathing oscillations revealed in accordance with principles of optimum control in the DeBoer model is investigated in the Grinchenko-Rudnitsky model [2]. This model allowed, in particular, to explain appearance of a peak on the Meyer frequency in the spectrums of pressure oscillations and synchronization of cardiac and respirator rhythms.

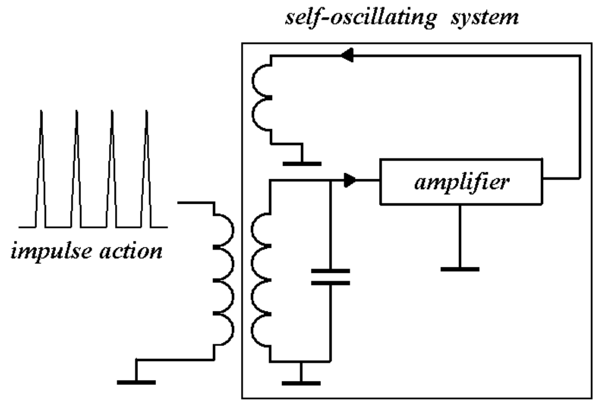


Fig. 1. Schema of a self-oscillating system as a generator of central type.

However, this model does not consider the reverse mechanical influence effect of the heartbeat changes on a breathing phase (frequency). In the present study, we add to the DeBoer model a self-oscillating system (which describes dynamics of the respiratory system as a generator of central type [4], shown in Figure 1) which is under impulsive influence of heartbeat.

The DeBoer model describes the followings main characteristics of the heartbeat system: systolic pressure S , diastolic pressure D, R-R interval I and arterial time constant T (in a state of rest for a healthy man S=120 mmHg, D=80 mmHg, I=800 ms, T=1500 ms). This mathematical model has five discrete nonlinear maps and contains only a direct mechanical influence of the respirator system on the cardiosystem. It can be written in the form:

$$D_i' = c_1 S_{i-1}' \exp\left(-\frac{I_{i-1}'}{T_{i-1}'}\right),$$

$$S_i' = D_i' + \gamma \frac{T_0}{S_0} I_{i-1}' + \frac{A}{S_0} \sin\left(2\pi f T_0 t_i\right) + \frac{c_2}{S_0},$$

$$I_i' = G_v \frac{S_0}{T_0} \hat{S}'_{i-\tau_v} + G_\beta \frac{S_0}{T_0} F(\hat{S}', \tau_\beta) + \frac{c_3}{T_0},$$

$$T_i' = 1 + G_\alpha \frac{S_0}{T_0} - G_\alpha \frac{S_0}{T_0} F(\hat{S}', \tau_\alpha),$$

$$\hat{S}_i' = 1 + \frac{18}{S_0} \arctan \frac{S_0 (S_i' - 1)}{18},$$

where $i \geq 1$, $D' = D / S_0$, $S' = S / S_0$, $\hat{S}' = \hat{S} / S_0$, $I' = I / T_0$, $T' = T / T_0$, $F(\hat{S}, \tau) = 1/9(\hat{S}_{i-\tau-2} + 2\hat{S}_{i-\tau-1} + 3\hat{S}_{i-\tau} + 2\hat{S}_{i-\tau+1} + \hat{S}_{i-\tau+2})$, $t_i = \sum_{k=0}^{i-1} I_k'$ is a real time, A=3 mmHg is a breathing amplitude, f=0.25 Hz is a breathing frequency, $c_1 = D_0 / S_0 \exp(I_0 / T_0)$, $c_2 = S_0 - D_0 - \gamma I_0$, $c_3 = I_0 - S_0 (G_v + G_\beta)$, $\gamma = 0.016$ mmHg, $G_\alpha = 18$ ms/mmHg, $G_\beta = 9$ ms/mmHg, $G_v = 9$ ms/mmHg, $\tau_\alpha = \tau_\beta = 4$, $\tau_v = 0$, is equal to 0 if frequency of heartbeat is less then 75 beat/min, and τ_v is equal to 1, if frequency is more than 75 beat/min. Here we use a parameter $c_1 = D_0 / S_0 \exp(I_0 / T_0)$ as it was in the original DeBoer model [3], it was different in our previous paper [6].

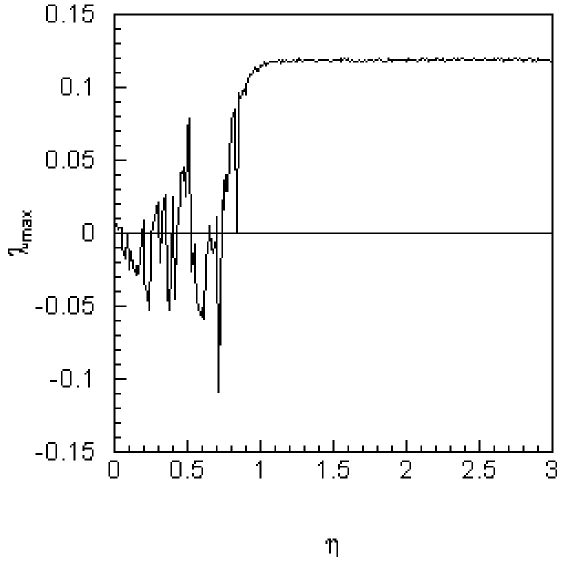


Fig. 2. The largest non-zero Lyapunov exponent of the complex system

We suppose that a healthy man at rest breathes periodically with a permanent frequency and an amplitude of motions of thorax. In that case a breathing process can be described as the self-oscillating system [4], which has a steady limit circle. Thus, equations of the Zaslavskiy map could be used for the mathematical modeling of such system. Famous Zaslavsky map is the system of equations [8, 12] which describes the dynamics of an amplitude r_n and a phase φ_n of the system (in which periodic self-oscillations with a frequency ω are realized) which is under T-periodic impulsive action of constant intensity η. Such impulsive action is very similar to feedback action of the heartbeat system on breathing process. The system has the following form:

$$r_{n+1} = \left(r_n + \eta \sin \varphi_n\right) \exp\{-\kappa T\},$$
$$\varphi_{n+1} = \varphi_n + \omega T + \nu\left(r_n + \eta \sin \varphi_n\right) \frac{1 - \exp\{-\kappa T\}}{\kappa},$$

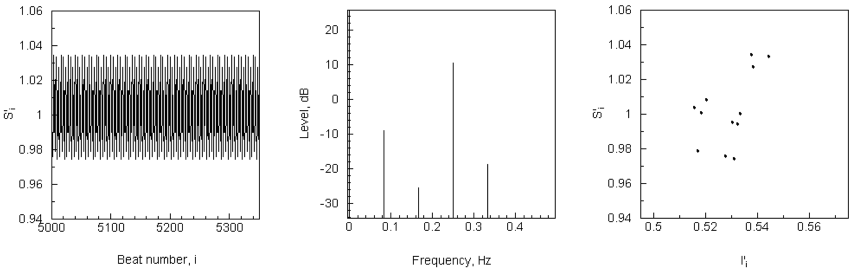
where κ, ν are constant parameters.

In our present model these equations are used to describe changes of an amplitude and phase of a respiratory system effect for every R-R interval with an intensity proportional to systolic pressure: $\tilde{\eta} = -\eta(S_n - S_0)$:

$$r_{n+1} = \left(r_n - \eta(S_n - S_0) \sin \varphi_n\right) \exp\{-\kappa I_n\},$$
$$\varphi_{n+1} = \varphi_n + 2\pi f I_n + \nu\left(r_n - \eta(S_n - S_0) \sin \varphi_n\right) \frac{1 - \exp\{-\kappa I_n\}}{\kappa},$$

where I is R-R interval, $\eta > 0,\ \kappa,\ \nu$ are constant parameters of interaction.

Thus, we study the dynamics of the model of cardiorespiratory system, which consists of the DeBoer model with direct respiratory influence $(A + r_i) \sin \varphi_i$, and with reverse influence modeled by the Zaslavskiy map system.



a b c

Fig. 3. Graphs of simulated systolic pressure data a), power spectra b) and projection of the phase portrait c) for $\eta = 0.2$

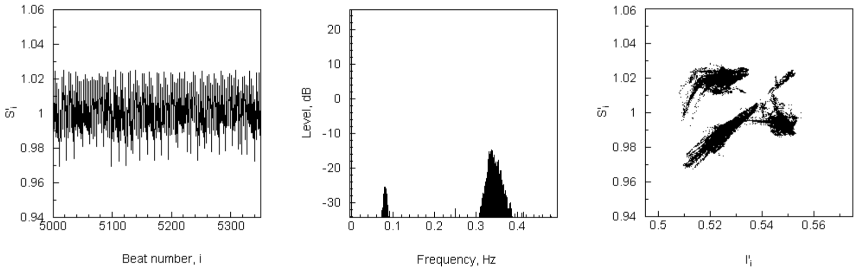


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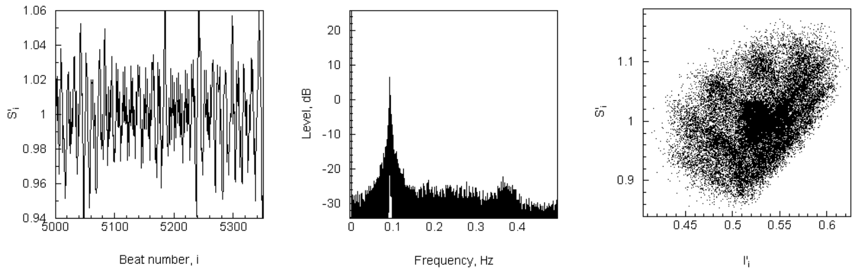
Fig. 4. Graphs of simulated systolic pressure data a), power spectra b) and projection of the phase portrait c) for $\eta = 0.8$



a b c

Fig. 5. Graphs of simulated systolic pressure data a), power spectra b) and projection of the phase portrait c) for $\eta = 1$

3. Numerical simulations results

The following values of variables and constants are used in our numerical simulations: $I'[0]=0.53$, $S'[-j]=1.08$, $j=0,...,6$, $r'[0]=0$, $\varphi'[0]=0$, $\kappa=0.001$ 1/ms, $\nu=0.001$ 1/msmmHg. In order to study steady-state regimes the largest non-zero Lyapunov exponent [1, 7] was calculated. The dependence of the largest non-zero Lyapunov exponent of the system on values of the bifurcation parameter η is shown in Figure 2. The dynamics of the system changes with increasing of this parameter. There is the region where this Lyapunov exponent is equals to zero ($\eta = 0.2$), what means that a limit circle is realized as steady-state regimes [1, 7]. We emphasize that η describes intensity of heart influence on a respiratory system. The next Figure 3 illustrates a behaviour of systolic pressure, power spectra and projection of the phase portrait for this value of intensity. The spectrum in Figure 3 b) has discrete peaks one of which corresponds to a peak on the Meyer frequency in the spectrum. So that, graphs indicate that there are regular regimes in the system. With increasing value of η the transition to chaos occurs. Thus, at η =0.8 chaos is realized in the system, when the spectrum in Figure 4 b) is continuous and the projection of the phase portrait occupies some area in the phase space.

Finally, at $\eta = 1.0$ the largest non-zero Lyapunov exponent is steady positive and the chaotic regime is fully developed, the power spectrum is continuous (Figure 5 b). So we have found such steady-state basic regimes: at $\eta = 0.2$ periodic regime (Figure 3), at $\eta = 0.8$ and for $\eta \geq 1.0$ chaotic regimes (Figure 4 and Figure 5).

4. Conclusions

On the basis of the DeBoer model an interaction of the heartbeat and the respiratory system (as dissipative Zaslavskiy map) is studied and the complex model of cardiosystem is built out. This model takes into account both direct and reverse influence of subsystems – cardiovascular and respiratory. The methods of modern theory of the dynamical systems are used to study laws of the steady-state regimes of the complex model with strong interaction. The chaotic regimes were found out. The dynamics of heartbeat and respiratory systems are in good correspondence with experimental information of healthy man. Found irregularities of phase trajectories of the complex model depend on intensity of heart rhythm influence on breathing, what is well known characteristic for the dynamics of the cardiovascular system of healthy man.

References

1. V. S. Anishshenko. *Acquaintance with nonlinear dynamic,* Institute of Computer Science, Moscow-Izhevsk, 2002.
2. V. T. Grinchenko and A. G. Rudnitskiy. A model of interaction of the cardiovascular and respiratory systems, *Acoustic Journal*, vol. 9, no. 3, 16-26, 2006.
3. R. W. DeBoer, J. M. Karemaker, J. Strakee. Hemodynamic fluctuations and baroreflex sensitivity in humans: A beat-to-beat model, *Amer. J. Physiol*., vol. 253, H680-H689, 1987.
4. L. Glass, M. C. Mackey. *From Clocks to Chaos. The rhythms of life,* Princeton University Press, Princeton, 1988.
5. K. Kotani [et al.]. Model for cardiorespiratory synchronization in humans, *Physical Review E.*, vol. 65, 051923-051932, 2002.
6. T. S. Krasnopolskaya, E.D. Pechuk Chaos In a Modified Cardiorespiratory Model, *CMSIM*, no. 4, 563-570, 2013.
7. T. S. Krasnopolskaya, A. Yu. Shvets. *Regular and chaotic dynamics of the systems with limited excitation,* Institute of Computer Science, Moscow-Izhevsk, 2008.
8. R. Z. Sagdeev, D. A. Usikov and G. M. Zaslavsky. *Nonlinear Physics: From the Pendulum to Turbulence and Chaos*, Harwood Academic Publishers, New-York, 1988.
9. H. Seidel and H. Herzel. Bifurcations in a nonlinear model of the baroreceptor-cardiac reflex, *Acoustic Journal*, vol. 115D, 145-160, 1998.
10. E. Toledo [et al.]. Does synchronization reflect a true interaction in the cardiorespiratory system, *Med. Engng Phys.*, vol. 24, 45-52, 2002.
11. T. Yang, M. D. Jacobstein, M. N. Levy. Synchronization of automatic cells in S-A node during vagal stimulation in dogs, *Am. J. Physiol.*, n. 246, H585-H591, 1984.
12. G. M. Zaslavsky. The simplest case of a strange attractor, *Phys. Lett. A*, vol. 69, 145-147, 1978.