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Bottom Particles Segregation: Experiments and Numerical Simulations Using Non-linear Diffusion Equation.

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Abstract: We report experimental and theoretical results on segregation of particles in sandy bottom under the action of oscillating velocity field due to surface waves. Experiments were performed in-situ and in wave flume. Segregation in the mixtures of light and heavy particles of the same diameter and small and large particles of the same density was observed. It was found that segregation appears on the background of sand ripples generation in the upper «active» zone of the bottom. The thickness of this zone is proportional to the amplitude of velocity oscillation. The topology of segregation patterns depends on particle size differences, on particle density differences as well as on particle percentage composition in the mixture. It was revealed that small particles with small percentage are concentrated below sandy ripples crest. If percentage composition of small and large particles are approximately equal, the large particles are concentrated below the ripples crests. Light particles quite the «active» zone where concentration of heavy particles increases. To investigate the segregation of particles we use non-linear diffusion equations in the presence of gravity field like it was done recently in paper of Fernandez *et al* (Physica A, 2003, 327, 94–98). The main idea of this model is to take into account the different sizes and densities of particles using mobility coefficients in diffusion equations for concentration of particles. We extended one dimensional model of Fernandez *et al* for two dimensional case. Numerical simulations demonstrate good qualitative coincidence with experimental data. In particular, zones with high concentration of small and large particles below ripples crest were obtained in numerical simulations depending on percentage composition of particles. Formation of the layer where the concentration of light particles sufficiently decreases was also found in numerical simulations. Importance of segregation processes for biological and environmental problems is discussed.

Keywords: Segregation, sand ripples, surface wave, sea bottom, non-linear diffusion equation.

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1. Introduction

Interest in segregation of particles is associated with a large number of different practical applications. For example segregation is widely used to sort materials in the mineral processing and building industries. To date, the most thoroughly studied patterns are the ones that result from the segregation in a dry mixture of solid particles with different properties: size, density, and shape. The process of segregation occurs in the mixture of particles subjected by an external field. It can be vibration, gravity forces, rotation, or another external influence. It should be noted that under the influence of external fields, one could observe the appearance of segregation more often than mixing. Mixture of particles in external field can be considered as a non-equilibrium dissipative media that is far from equilibrium. Spatially homogeneous state in this media may be unstable. As a result of this instability of mixture of particles spatial patterns appear. The dynamics of these patterns has been investigated for granular materials when interaction of particles includes a dry friction. This paper is devoted to pattern formation in a mixture of solid particles immersed in a viscous fluid. It should be noted that the number of studies on segregation pattern formation in such systems is sufficiently small. However, these problems appear very important for different processes that take place on the ocean bottom. It is obvious that such segregation is significant for understanding the problems of sediment transport and erosion of the seabed. The segregation may also lead to the redistribution of micro-and meso benthos on the seabed. Segregation of particles is important for environmental issues, because concentration pattern formation can lead to an abnormally high localization of a particular type of particles in limited areas where conditions can change significantly for the flora and fauna on the seabed. This paper aims to examine the conditions under which it is possible to observe the segregation of particles on the seabed and to investigate the characteristics of emerging patterns on the bottom. The paper is organized as follows. At the beginning some experimental results on particle-size and particle-density segregation are presented. Then we discuss a theoretical model describing segregation and present some results of numerical simulations. Numerically obtained results are compared with experimentally observed patterns.

2. Experimental results

Experiments were conducted in a 10.7-m-long, 0.5-m-wide wave flume [1]. Surface waves are generated by an oscillating paddle at one end of the flume; an absorbing beach is located at the other end (Fig.1). The water depth at rest h is 0.24 m. The height and the period of the wave are measured with two resistive probes. Ripples were generated on the bottom from an initially flat bed, which consisted in a 4-cm layer of mixtures. Two kinds of mixtures were used: for the first mixture (mixture 1): sand of density $\rho_1 = 2.65\ 10^3\ Kg/m^3$, diameter $d_1 = 0.17mm$ and polyvinyl chloride (PVC) grains of density $\rho_{22} = 1.35\ 10^3$ kg/.m^3, median diameter $d_1 = 0.17mm$. The volume concentration for this mixture are C_1*=96%*, C_2*=4%.* For the second mixture (mixture 2), we used yellow sand with the following properties: d_1*= 0.34 mm*, ρ_1 *=2.65* 10^3 *Kg/m*^{3} , and red sand: *d2=0.15mm*, ρ_2 *=2.5* 10^3 *Kg/m*^{3} ; the volume concentrations were C_1*=90%*,C_2*=10%* in this case.

Patterns appearing on the bottom are shown in Fig.2. Observations of segregation patterns were possible since the side walls were made of glass and the particles with different properties had different colors. Fig. 2 shows the patterns arising from homogeneous mixtures (Fig. 2ac) of particles under the influence of surface waves. In both mixtures , ripples with spatial period of 10 cm and height 2 cm formed rapidly along the flume once the wave maker was

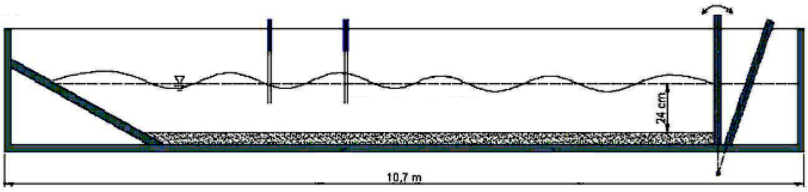


Fig.1 Experimental set-up

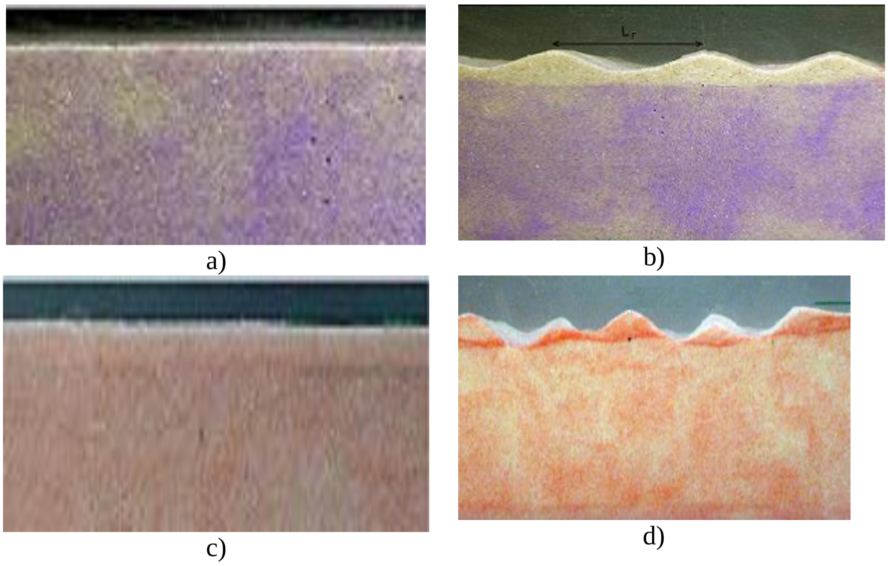


Fig. 2 Segregation patterns appearing from homogeneous mixtures: a) mixture of sand and PVC, b) pattern in mixture (a) appearing after sand ripples generation, c) mixture of yellow sand and red sand, d) pattern in mixture (c) appearing after sand ripples generation.

switched on. The size and shape of ripples were approximately the same along the flume. For the first type of mixture, a layer where concentration of PVC is very low appeared near the boundary between sand and water (Fig. 2ab). For the second type of mixture, the segregation resulted in an increase of concentration of red sand (the finest sand in mixture 2) under the ripples crest (Fig.2cd). It should be emphasized that the generation of sand ripples at the bottom is a quite fast process (characteristic time: tens of minutes), whereas the segregation patterns are forming during several hours.

3. Theoretical model.

To describe the particle segregation by currents induced by surface waves, we use the approach developed in [2,3]. A mixture of particles is considered as a gas with a temperature *T* that is in the field of potential forces. To a mixture of particles can be calculated the entropy *S* and free energy *F*: $F = U - TS$, where *U* is potential energy. The volume concentrations $C_{1,2}$ of particles may be found using variation derivatives of functional $\Im$ [2,3]:

$$\Im = \int F dQ$$

$$\frac{\partial C_{1,2}}{\partial t} + \nabla \cdot \vec{J}_{1,2} = 0, \; \vec{J}_{1,2} = -\Gamma_{1,2} \nabla \mu_{1,2}, \; \mu_{1,2} = \frac{\delta \Im}{\partial C_{1,2}}$$

where $\mu_{1,2}$ are chemical potentials and $\Gamma_{1,2}$ are mobilities of particles.

In these equations particle mobilities $\Gamma_{1,2}$ are not defined. In [2,3] a phenomenological mobility depending on the concentration is used to calculate the concentration of particles. The basic assumption is that there is some limit particle concentration above which the mobility of the particles is zero. Physically, this means that there is a dense packing of particles which totally eliminates their movement. The mobility may be presented as:

$$\Gamma_{1,2} = \Gamma_0 C_{1,2} \Theta(C_c^{1,2} - (C_1 + C_2)) \cdot \left(1 - (C_1 + C_2)/C_c^{1,2}\right)^{\Phi_{1,2}}$$

where $\Phi_{1,2}$ are constants discussed in [2,3].

After these assumptions, the non-linear diffusion equation (NLDE) was found for particle concentrations. In [2] segregation of particles was investigated in a one-dimensional case, when the concentrations depend on vertical coordinate z and on time t: $C_{1,2} = C_{1,2}(z,t)$. It was found that NLDE could describe the effect of Brazil nuts: vibrations of particle mixture leads to the fact that larger particles tend to settle over small ones. A qualitative explanation of this effect is quite simple: the small particles fall between large particles.

In the case of sand ripples the problem is much more complicated, because the system is fundamentally two-dimensional. Experiments have shown that the appearance of sand ripples develops much faster than the process of particle segregation. Therefore, for the correct application of the NLDE it is necessary to take into account the sand ripples. How to take into account the existence of sand ripples in NLDE? Imagine that there is a border between oscillating water and sand ripples. The sandy bottom is a porous medium, where the correlation between the velocity field $\vec{V}$ and pressure field p is determined by Darcy law [4]: $\vec{V} = -\frac{K}{\rho\nu}\nabla p$, where K is the permeability of the porous medium, ρ the water density and ν the kinematics viscosity.

In the sand layer, under the action of oscillating velocity a mean pressure field P is generated. For small amplitude a of sand ripples this mean pressure field may be calculated as:

$$P = \frac{1}{2}\rho U_a^2 ak\cos(kx)\cdot\exp(k(z-h))$$

where a is for amplitude of sand ripples, U_a is the amplitude of oscillating velocity field near the bottom, h the average sand height (see Fig.3), and k the wave number of sand ripples.

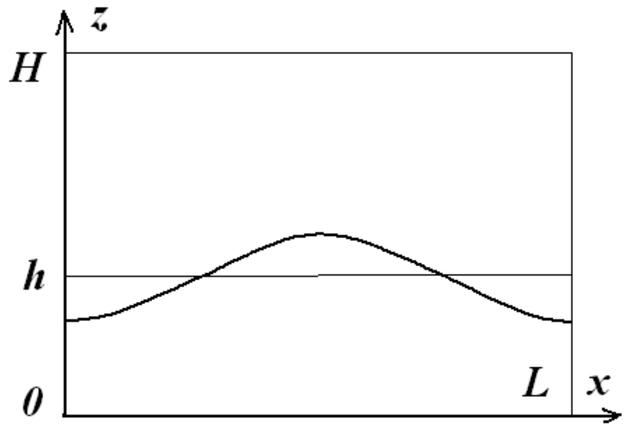


Fig.3 Schematic representation of integration domain (*0<x<L, 0<z<H*) and the boundary between water and sand, *z=h* corresponds to the averaged level of the bottom.

The mean force caused by the pressure field acting on a particle with diameter d is expressed as $\vec{F} = -\frac{1}{6}\pi d^3\nabla P$, whereas the component of the force due to the velocity field is zero. The mean pressure may be considered as potential depending on horizontal and vertical coordinates. For our two dimensional case, NLDE concentrations of particles can be described by the following equation:

$$\frac{\partial C_i}{\partial t} = \frac{\partial}{\partial z}\left[\Gamma_i\left(\frac{1}{C_i}\frac{\partial C_i}{\partial z} + \frac{1}{1-C_1-C_2}\left(\frac{\partial (C_1+C_2)}{\partial z}\right)+\gamma_{i,z}\right)\right]+$$

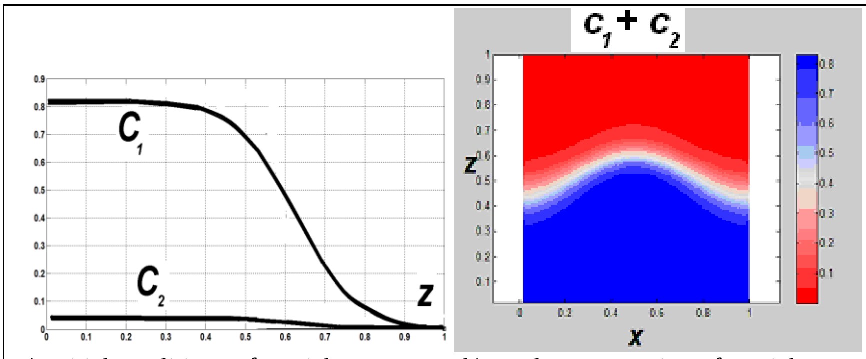
$$\frac{\partial}{\partial x}\left[\Gamma_i\left(\frac{1}{C_i}\frac{\partial C_i}{\partial x} + \frac{1}{1-C_1-C_2}\left(\frac{\partial (C_1+C_2)}{\partial x}\right)+\gamma_{i,x}\right)\right],\ i=1,2$$

$$\gamma_{i,z}=-\frac{4\pi d_i^3}{3k_BT}\left(\frac{\partial P}{\partial x}\right),\ \gamma_{i,z}=-\frac{4\pi d_i^3}{3k_BT}\left(g(\rho_i-\rho_w)+\frac{\partial P}{\partial z}\right),$$ where ρ_w is for water density, k_B is Boltzmann constant, T is effective temperature of particle mixture

Using this system of equations with boundary conditions at $x=0,L$ and $z=H$, different regimes of particle segregation have been found.

4. Numerical results.

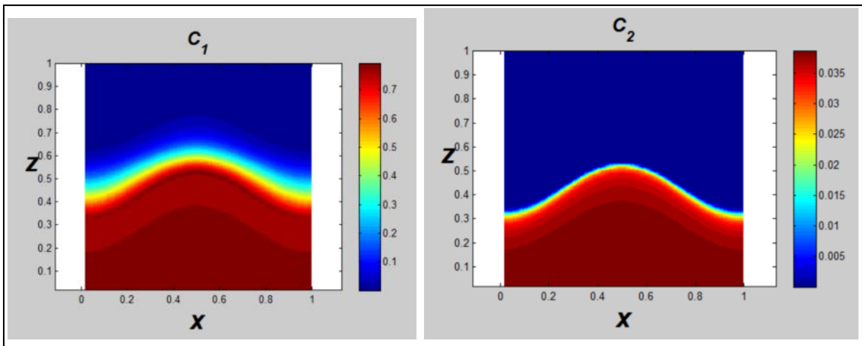
Since the onset of sand ripples in the experiments occurs much faster than the segregation of particles, the initial conditions for the numerical simulations correspond to periodic sand ripples of final amplitude in homogeneous mixture (Fig. 5). We suppose that structure is periodic along x coordinate and $\frac{\partial C_{1,2}(x=0,1)}{\partial x}=0$. For vertical coordinate z conditions of zero particle flux $\frac{\partial C_{1,2}(z=0,1)}{\partial z}=0$, or constant concentration $C_{1,2}(z=0,1)=0$ are used. . The evolution equations for the volume concentrations are discretized with the finite difference method and an explicit time scheme is used.



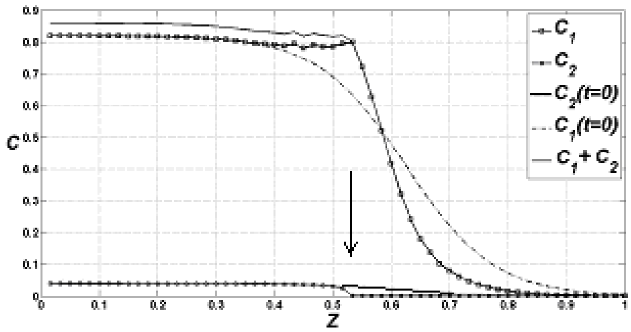
a) Initial conditions of particle concentrations under the crest of ripples b)Total concentration of particles

Fig. 5

The first series of experiments was performed to model the segregation in the mixture of sand PVC particles. In these experiments parameters $\Phi_{1,2}$ determined by dimensions of particle are equal $\Phi_{1,2}=3.1$, as it was done in [2,3]. Critical concentration of particles above which mobility is zero was $C_{cr}=0.84$. Results are presented in Fig. 6. The main feature of final steady concentration is the following. In the upper part of sand layer *x>0.52* (this point is indicated by an arrow in Fig.6c) the concentration of light (PVC) particles decreases sufficiently in comparison with initial concentration Fig.6c. This pattern may be compared with pattern presented in Fig. 2ab. The concentration of PVC in the layer corresponding non-zero mobility deceases with time.



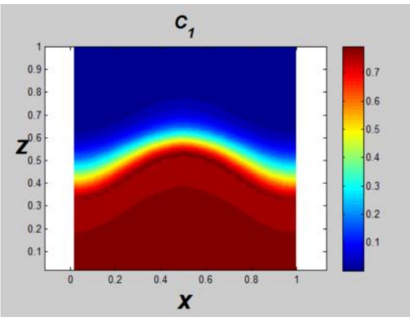
a) Final distribution of heavy particles b)Final distribution of light particles



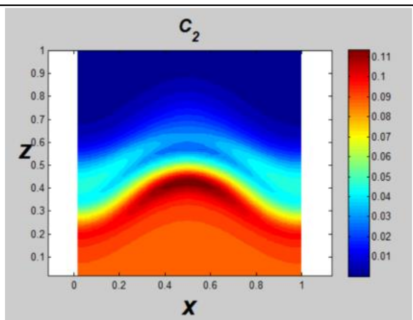
c) Dependence of particle concentrations on vertical coordinate z under the crest of sand ripples x=0.5

Fig. 6

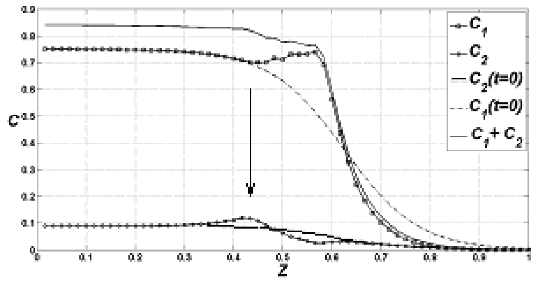
The second series of experiments was performed to model segregation in the mixture of sand particles with different diameters. In this case, we have the following values of the parameters $\Phi_1=4$, $\Phi_2=2$, $d_1=1{,}33d_2$. Results are shown in Fig.7. Segregation leads to the formation of pattern with large concentration of small particles under the crest of ripples (Fig.7b). In this region (indicated by an arrow in Fig.7c), the concentration of large particles decreases. This pattern may be compared with pattern shown in Fig. 2cd. Red sand particles during the process of segregation tend to be concentrated under the crests.



a) Final distribution of large particles



b) Final distribution of small particles



c) Dependence of particle concentrations on vertical coordinate z under the crest of sand ripples x=0.5

Fig.7

Conclusions

Using nonlinear diffusion equation, we have found two dimensional segregation patterns for mixtures of particles with different diameters and different densities. Patterns observed in numerical simulations are similar to patterns investigated in physical experiments. At least it is possible to find the same topological features in experimentally and numerically obtained patterns. The main problem is connected with the choosing of coefficients in differential equations. By now there is no any regular method to determine these coefficients.

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