The Effects of Diffusion of Individuals between Two Single-Species Populations

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Abstract: In the paper we focused on a general model for the growth of a single-species population with non-overlapping generations. The data we have used correspond to Nicholson's blow-flies population and lie in the chaotic regime. The population was divided in two groups. If these groups evolve in distinct locations, their behavior is chaotic and, after a few generations, the initial small difference in number of individuals becomes big enough and behaves randomly. The question I want to answer in the paper is: What happens with the two populations if the individuals can migrate in both directions within the time intervals between their reproduction and death? The effect of coupling the two groups consisted in a rich dynamic behavior depending on the coupling strength. It was found that there is a consistent region where the coupling brings out the full synchronization of the two chaotic systems, two transition regions where an intermittent behavior was observed and two peripheral regions where control of chaos is shown to coexist with quasi-periodic and chaotic regimes.
Keywords: Single-species populations, Synchronization, Intermittent chaos, Control of chaos.

1. Introduction

According to May [1], models for population growth in a limited environment are based on two fundamental premises: a) the populations have the potential to increase exponentially; b) there is a density-dependent feedback that progressively reduces the actual rate of increase. By using a variety of data from field and laboratory populations, some researchers have proposed continuous or discrete models of population growth. The most known of these models is the logistic equation (Verhulst, 1838). Other simple models were introduced by May (1974), Li & Yorke (1975), May & Oster (1976), and Hassel et al (1976). Their models, which refer to single-species population with discrete, non-overlapping generation, predict that most of the populations show monotonic damping back to an equilibrium following a disturbance, with some exceptions of oscillatory damping or some sort of low-order limit cycles. They concluded that high-order limit cycles and chaos appear to be relatively rare phenomena in naturally occurring single-species populations. Guckenheimer et al (1987) have found that more realistic models of population growth, such as these that include overlapping generations, are more likely to exhibit complex behaviors. If data from laboratory population are used, even for these simple models, it was found that some populations will not exhibit stable equilibrium points but stable cycles or chaotic behavior [2]. That is because the laboratory situation (homogeneous environment, constant food supply, no competitors, no predators) make possible an exaggerated non-linear behavior. In this paper we focused on a general model

for growth of a single-species population with non-overlapping generations, namely

$$N_{t+1} = f(N_t) = \lambda\, N_t\, (1 + a\, N_t)^{-b} \qquad (1)$$

where N_t and N_{t+1} are the populations in successive generations, λ is the finite rate of increase and a, b are constant defining the density-dependent feedback term. The values for parameters correspond to Nicholson's blowflies and lie in the chaotic regime [3].

The population of blowflies was divided in two groups. If these groups evolve in distinct locations their behavior is chaotic and, after a few generations, the initial small difference in number of individuals becomes big enough and behaves randomly. The question I want to answer in the paper is: *What happens with the two populations if the individuals can migrate in both directions within the time intervals between their reproduction and death?*
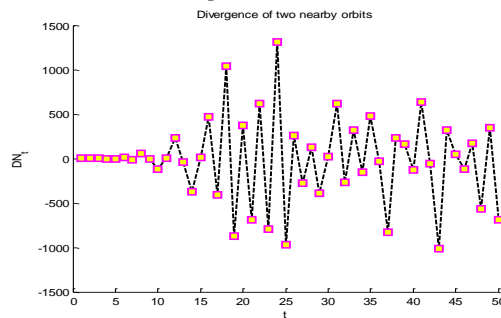

Fig. 1. Divergence of the two isolated populations versus time

2. The Model of Two-coupled Single-species Populations

To answer the above question let us hereafter turn our attention towards the following system of two-coupled single-species populations:

$$N_{t+1} = f(N_t) + c\left[f(M_t) - f(N_t)\right], \quad M_{t+1} = f(M_t) + c\left[f(N_t) - f(M_t)\right] \qquad (2)$$

where the coupling parameter c can be thought as the fraction of the two populations which migrate to the neighboring location. Throughout the paper I used the fixed parameter values $\lambda = 60, a = 0.003$ and $b = 6.$ The total population $\overline{N}_t = 3950$ was divided in two unequal groups, $N_t = 1950$ and $M_t = 2000$. If no change between the groups was permitted, the initial small difference in number of individuals, $\Delta N_t = 50$, increased quickly and behaved chaotically (see Figure 1). The effect of coupling consisted in a rich dynamic behavior having the main features as follows.

2.1. Complete synchronization

If two or more chaotic systems are couple, it is possible that the attractive effect of a suitable coupling to counterbalance the trend of the trajectories to separate due to chaotic dynamics. Synchronization of chaotic systems can be explained by the suppression of expanding dynamics in the state space transversal to the synchronization manifold (here $M_t = N_t$). It is natural then to ask for which

values of coupling strength c the two systems will oscillate in a coherent and synchronized way.

Laureano et al [4] have demonstrated that, for this kind of coupling, the range of synchronization (in the linear approximation) is given by

$$0.5\left(1-\exp(-\lambda_u)\right)<c<0.5\left(1\_\exp(-\lambda_u)\right) \tag{3}$$

where λ_u is the Lyapunov exponent for the uncoupled map f. For our data it was found that $\lambda_u \cong 0.35$, so $c \in (0.15 ; 0.85) = S$. As an example, let consider $c = 0.16$. The synchronization takes place after 200 generations (see figure 2).

Fig. 2. Evolution to synchronous state for $c = 0.16$

Each of the systems shows chaos and their states are identical at each moment in time (full synchronization). To verify that the synchronous state is chaotic, a Lyapunov exponent versus coupling strength diagram was considered (see Figure 3.
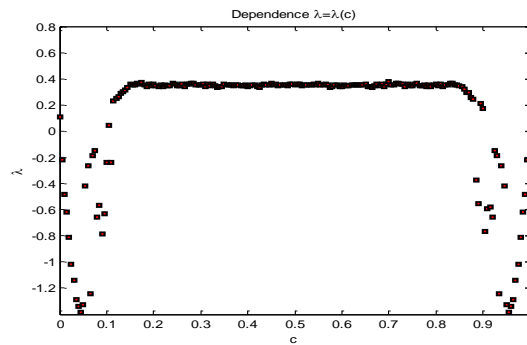

Fig. 3. Lyapunov exponent versus coupling strength

If c is chosen deep inside the interval S, the synchronous state is reached after only few steps (see Figure 4). Otherwise, if c is chosen near the borders of S the synchronization is hard to obtain, a lot of steps being necessary (e.g. 2000 steps for $c = 0.15$.
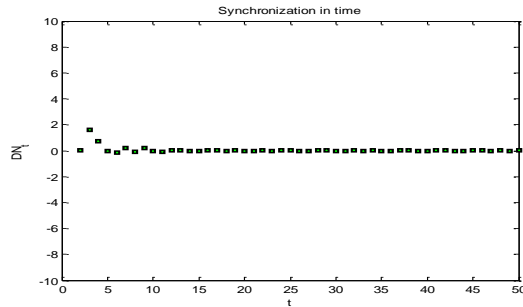
Fig. 4. Evolution to synchronous state for $c = 0.25$

2.2. Intermittent chaos

If the coupling strength c falls short of the critical value $c_{crit} = 0.15$ the synchronized state $M_t = N_t$ becomes unstable and an intermittent dynamics is observed. Figure 5 shows the time evolution of the transverse coordinate $DN_t = N_t - M_t$ for $c = 0.1467$. The time periods of synchronicity are interrupted by aperiodic chaotic bursts.
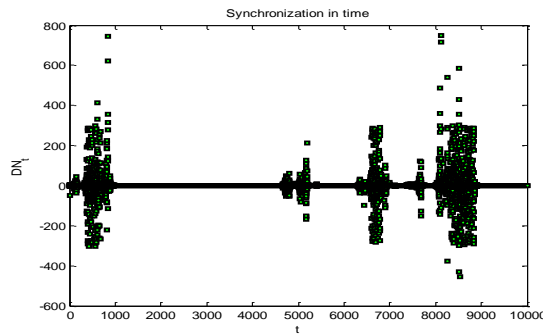

Fig. 5. Time periods of synchronicity interrupted by aperiodic chaotic bursts
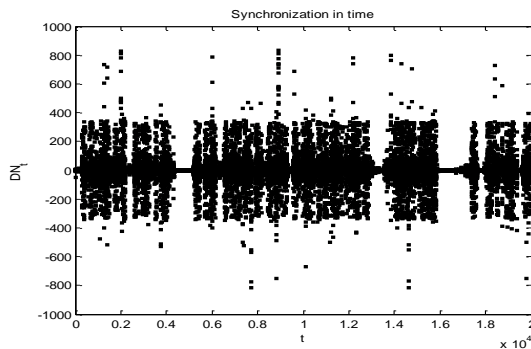($c = 0.1467$)

Fig. 6. A completely erratic state for $c = 0.14$

The basic intermittency mechanism comes from the competition between the trajectory instability of chaotic elements and the synchronization tendency due to the diffusion-type coupling [8]. For $c = 0.14$ the chaotic bursts were already merged so the synchronization started to dissolve into a completely erratic state (see Figure 6).

2.3. Stabilization to an ordered state

Outside the interval of synchronization the dynamics is quite complicated. For very small values of c (weak coupling) the system behaves chaotically, the N_t values being distributed over an entire interval. By increasing c the chaotic distribution of N_t comes undone in strips, thinner and thinner (see Figure 7).
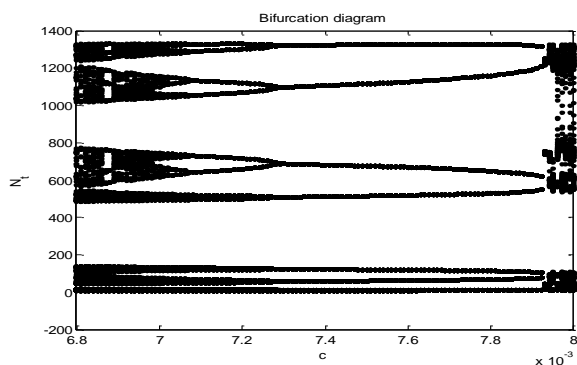

Fig. 7. A part of the bifurcation diagram $N_t = N_t(c)$

At $c \cong 0.007$ the system entered a periodic regime, and was subjected to a sequence of changes from a 2^n - period cycle to a 2^{n-1} - period cycle. A 8-period cycle was obtained for $c \in (0.0072; 0.0080)$ (see Figure 8).

Then, a quasi-periodic regime with two strips appeared (Figure 9) which, in its turn, was changed by a 2-period cycle for $c \in (0.013; 0.11)$. This periodic regime is interrupted by windows corresponding to a 4-period cycle or even to thin windows of chaotic regime.

Beginning with $c \cong 0.1$ the number of steps required for stabilization to the 2-period cycle became bigger and bigger so, finally, the chaotic regime was reached. An analogous discussion can be done for $c \in (0.85; 1)$.
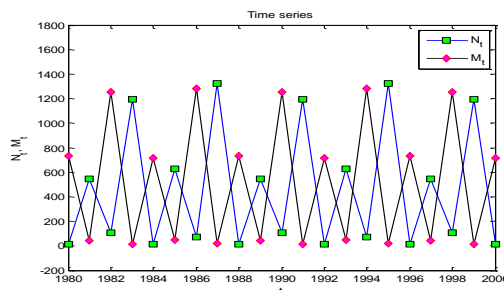

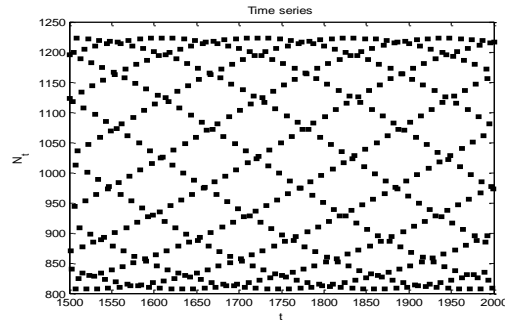Fig. 8. Time evolution of N_t , M_t for $c = 0.0079$ (8-period cycle)

Fig. 9. Time evolution of N_t for $c = 0.99$ (quasi-period regime)

3. Conclusions

The dynamics for many biological populations, which breed seasonally and have non-overlapping generations, are described by a density-dependent relation of the form $N_{t+1} = f(N_t) = \lambda N_t (1 + a N_t)^{-b}$. If data from laboratory tests are used it was found that populations can exhibit even a chaotic behavior. Two almost identical populations, living in distinct locations, evolved so that the initial small difference in number of individuals became big enough and behaved randomly. If the individuals representing the two populations could migrate in both directions within the time intervals between their reproduction and death then a rich dynamic behavior depending on the coupling strength was observed. It was found that there is a consistent region where the coupling brings out the full synchronization of the two chaotic systems, two transition regions where an intermittent behavior was observed and two peripheral regions where control of chaos is shown to coexist with quasi-periodic and chaotic regimes.

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