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Can the multifractal spectrum be used as a diagnostic tool?

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Abstract. We seek the possibility of using multifractal spectrum as a diagnostic tool to differentiate between healthy and pathological time series. The data sets used for the analysis consist of EEG and Heart Rate Variability (HRV) time seres downloaded from Physio Bank archives. We use the automated algorithmic scheme recently proposed by us to compute the multifractal spectrum, which provides a set of parameters to compare different data sets. We show that the set of parameters characterising the multifractal spectrum can distinguish between healthy and pathological states in both EEG and HRV.
Keywords: Time Series Analysis, Physiological Chaos, Multifractal Spectrum.

1 Introduction

Recently, many authors [1,2] have stressed the importance of multifractality in the study of heart rate variability and suggested that it could provide a new observational window into the complexity mechanism of heart rate control. The study also highlights the need for evaluating new nonlinear parameters for a better physiologcal investigation and for finding new clinical applications. The main issues regarding the characterisation of complex physiological signals are discussed in a recent review [3].

Out of the large number of studies done on physiological data, the focus has mainly been on the analysis of EEG and ECG time series data, with the purpose of characterisation and prediction from a dynamical systems point of view. The analysis of EEG data from healthy persons and epiletic patients has lead to a better understanding of various aspects of epileptic seizure activities and the corresponding brain states [4,5], but the question of whether the seizure can be predicted in advance is still an open one [6].

There have been a multitude of studies on ECG data sets recorded from healthy persons as well as during some pathological cases, such as, congestive

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heart disorders and ventricular fibrillation [7–9]. Most of these studies have searched for deterministic nonlinearity in the time series from cardiac system [10,11], and the reliability of these results have also been questioned [12–14] due to various reasons, such as, insufficient data, presence of noise, the subjective nature of the computational techniques and so on.

In this paper, we present some preliminary results for the analysis of physiological data, by computing the $f(\alpha)$ spectrum from the time series using an automated algorithmic scheme. The details of the scheme are presented and tested in the next section and it is applied to physiological data in §3. The conclusions are drawn in §4.

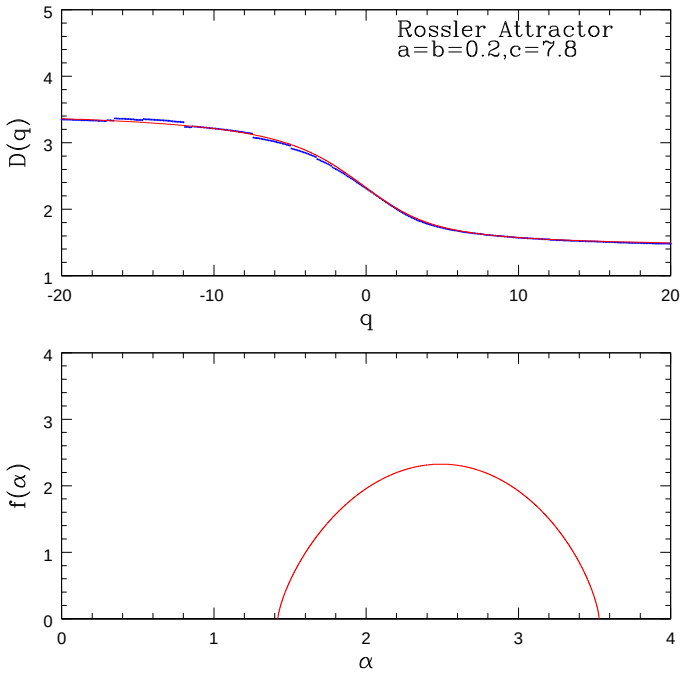

Fig. 1. The D_q spectrum (points) and its best fit curve (continuous line) for the Rossler attractor computed from 10000 data points are shown in the upper panel. The lower panel shows the $f(\alpha)$ spectrum computed from the best fit curve using our scheme.

2 Computing the Multifractal Spectrum

Here we discuss only the salient features of the algorithmic scheme and more mathematical details are presented elsewhere [15], [16]. The scheme provides us with a set of parameters characterising the spectrum which are good quantifiers to compare the changes in the multifractal character as reflected in the time series.

As the first step, the spectrum of generalised dimensions D_q is computed from the time series using the equation

$$D_q \equiv \frac{1}{q-1} \lim_{R\to 0} \frac{\log\ C_q(R)}{\log\ R} \tag{1}$$

where $C_q(R)$ are the generalised correlation sum. This is done by choosing the scaling region algorithmically as discussed earlier [16]. We make the conditions for R_{max} and R_{min} fixed by the algorithm itself so that the comparison between data sets becomes nonsubjective.

We then use an entirely different algorithmic approach for the computation of the smooth profile of the $f(\alpha)$ spectrum. The $f(\alpha)$ function is a single valued function between α_{max} and α_{min} and also has to satisfy several other conditions, such as, it has a single maximum and $f(\alpha_{max}) = f(\alpha_{min}) = 0$. A simple function that can satisfy all the necessary conditions is

$$f(\alpha) = A(\alpha - \alpha_{min})^{\gamma_1}(\alpha_{max} - \alpha)^{\gamma_2} \tag{2}$$

where A, γ_1, γ_2, α_{min} and α_{max} are a set of parameters characterising a particular $f(\alpha)$ curve. It can be shown [16] that only four of these parameters are independent and any general $f(\alpha)$ curve can be fixed by four independent parameters. Moreover, by imposing the conditions on the $f(\alpha)$ curve, it can also be shown that

$$0 < \gamma_1, \gamma_2 < 1 \tag{3}$$

The scheme first takes $\alpha_1(\equiv D_1), \alpha_{min}(\equiv D_\infty)$ and $\alpha_{max}(\equiv D_{-\infty})$ as input parameters from the computed D_q values and choosing an initial value for γ_1 in the range $[0, 1]$, the parameters γ_2 and A are calculated. The $f(\alpha)$ curve is then computed in the range $[\alpha_{min}, \alpha_{max}]$. From this, a smooth D_q versus q curve can be obtained by inverting using the Legendre transformation equations, which is then fitted to the D_q spectrum derived from the time series. The parameter values are changed continuously until the D_q curve matches with the D_q spectrum from the time series and the statistically best fit D_q curve is chosen. From this, the final $f(\alpha)$ curve can be evaluated. An important aspect of the scheme is that it also provides a set of parameters that can completely characterise a given $f(\alpha)$ curve. The parameters can play an important role in the nonsubjective comparison of the multifractal properties of the same system under different conditions, such as, the changes in the chaotic attractor due to parameter variation, changes in the physiological conditions etc.

To illustrate our scheme, we choose the time series from a standard chaotic attractor, namely the Rossler attractor with parameter values $a = 0.2, b = 0.2$ and $c = 7.8$. We use 10000 data points generated with a time step $\Delta t = 0.1$. The D_q spectrum is first computed with embedding dimension $M = 3$, for q values in the range $[-20, +20]$, taking a step width of $\Delta q = 0.1$. Choosing D_{-20}, D_1 and D_{20} as the input values for the $f(\alpha)$ function Eq. (2), the parameters γ_1 and γ_2 are scanned in the range $[0, 1]$ and the statistically best fit D_q curve is chosen. The complete $f(\alpha)$ spectrum is then computed from the best fit D_q curve. The D_q spectrum and the best fit D_q curve are shown in Fig. 1 (top panel). The complete $f(\alpha)$ profile computed from the best fit D_q curve is also shown in Fig. 1 (bottom panel).

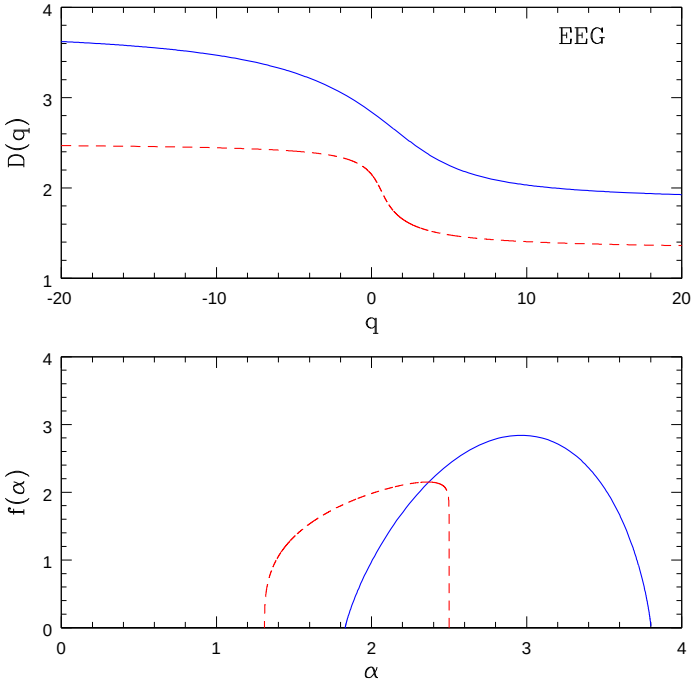

Fig. 2. The top panel shows the D_q spectrum computed using our scheme from representative EEG time series for healthy persons (continuous line) and during epileptic seizure (dashed line). The bottom panel shows the corresponding $f(\alpha)$ spectrum.

3 Application to Physiological Data

Physiological systems are, in general, complex where several nonlinearities are involved. We use physiological data commonly used for this kind of analysis, namely, EEG and HRV. In the case of EEG, we analyse signals from normal state and during epileptic seizure. Four data sets each from both cases are used for the analysis. In the case of HRV, we use three catagories of time series. The first one is from normal healthy persons, while the second and third corresponding to different pathological conditions of the heart, namely, congestive heart failure (CHF) and atrial fibrillation (AF). Four data sets for each of the above mentioned classes of HRV are analysed.

The EEG data were downloaded from the website of the Department of Epileptology, University of Bonn while the ECG data were obtained from http://www.physionet.org/physiobank/archives. The EEG data sets consist of continuous data streams of about 24 secs long and with approximately 5000 data points. The HRV data sets for different catagories consist of continuous data streams of approximately 5400 data points with a time step of 0.04 secs. All computations are done for an embedding dimension $M = 3$ and we show results for representative time series from each class.

The D_q and $f(\alpha)$ spectra for the two classes of EEG signals computed by our scheme are shown in Fig. 2. Similarly, the D_q and $f(\alpha)$ spectra for the three different classes of HRV time series are shown in Fig.3 and Fig. 4 respectively. One result which is clear from the figures is that all these signals show multifractal character. Some earlier studies had suggested that there could be a loss of multfractality for HRV in some pathological states. But we find that there is only a change in the multifractal character from healthy to pathological states.

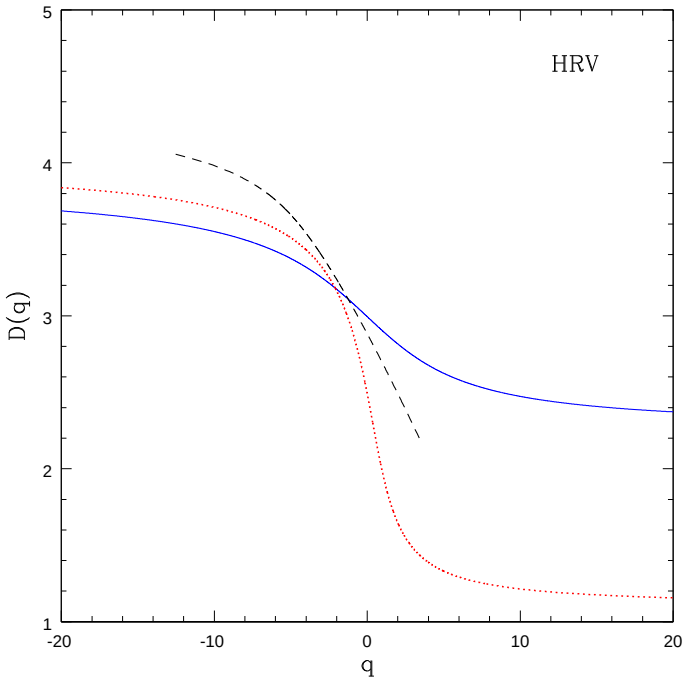

Fig. 3. Typical D_q spectra for HRV signals computed from healthy persons (continuous line), persons with CHF (dotted line) and those with AF (dashed line).

Of course, the difference between healthy and pathological time series is evident even visually, with the healthy signals appearing much like random fluctuations and the pathological ones do have some spiky nature. So we expect that these differences are also reflected in their D_q and $f(\alpha)$ spectra. The question is whether these qualitative changes can be quantified using our algorithmic scheme. It is quite evident from the figures that the nature of the $f(\alpha)$ profile is different for healthy and pathological states, in the case of both EEG and HRV. There is significant change in the profile of the spectrum and the parameter values between healthy and pathological states, for both EEG and HRV.

The range of α values, $|\alpha_{max} - \alpha_{min}|$, generally tend to change from healthy to pathological states in all cases. But the changes in the other three parameter

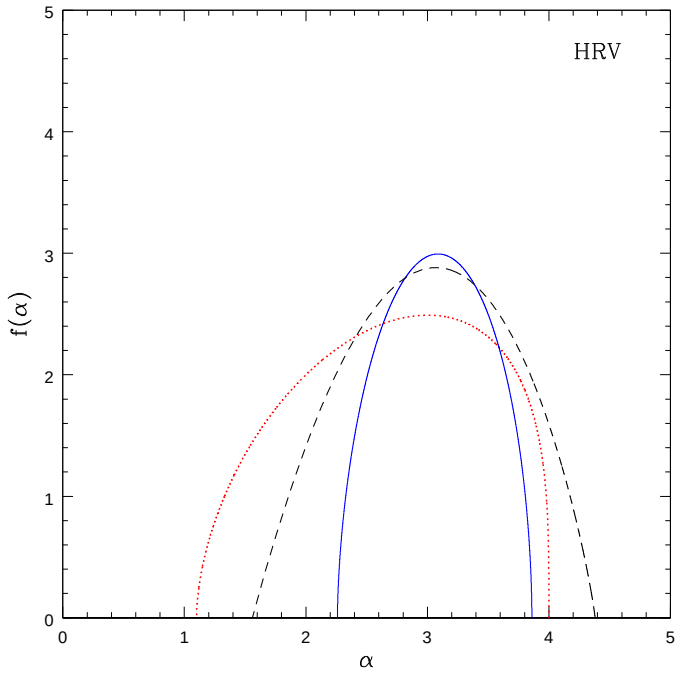

Fig. 4. The $f(\alpha)$ spectrum corresponding to the three cases of HRV signals shown in the previous figure.

values seems to be more significant. The values of γ_1 and γ_2 appear to be more sensitive to the changes in the multifractal character of the time series, especially since the range of γ_1 and γ_2 is limited ($0 < \gamma_1, \gamma_2 < 1$). For example, for the healthy data sets, the values of γ_1 and γ_2 are very close and always $\gamma_1, \gamma_2 > 0.8$. But in the case of pathological states, their values are generally found to be much less, with the difference $|\gamma_1 - \gamma_2|$ increasing. This, in turn, increases the asymmetry between the two branches of the $f(\alpha)$ profile.

Thus our results clearly indicates the importance of computing the multifractal spectrum using an algorithmic scheme and the utility of the associated parameters in differentiating signals from different physiological conditions. But we have used only limited number of data sets for the analysis. Whether all the trends shown by the parameters as discussed above are genuine and whether they can be used as diagnostic tools from a practical point of view will have to be confirmed by a much more comprehensive data analysis.

4 Conclusion

In this paper, we analyse an ensemble of physiological signals generated from different physiological conditions and try to distinguish them based on their multifractal properties. We use the automated algorithmic scheme recently proposed by us to compute the $f(\alpha)$ spectrum from the time series. The scheme

provides a set of parameters to characterise a given $f(\alpha)$ spectrum. The scheme is first tested and illustrated using synthetic time series from standard chaotic systems. It is then applied to two catagories of physiological data, namely, EEG and HRV. The signals from healthy and pathological states in both catagories are analysed. Our analysis indicates that the set of parameters characterising the $f(\alpha)$ spectrum show systematic difference between healthy and pathological states in both catagories. Thus, we find that measures based on multifractal structure can be effectively employed for differentiating signals from healthy and pathological states.

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References

1.R. Sassi, M. G. Signorini and S. Cerutti, *CHAOS*, 19: 028507, 2009.
2.U. Freitas, E. Roulin, Jean-Francois Muir and C. Letellier, *CHAOS*, 19:028505, 2009.
3.L. Glass, *CHAOS*, 19:028501, 2009.
4.N. Pradhan and D. Narayana Dutt, *Comput. Biol. Med.*, 23:425, 1993.
5.W. Latzenberger, H. Preissl and F. Pulvermiller, *Biol. Cybernetics*, 73:477, 1995.
6.M. A. F. Harrison, I. Osorio, M. G. Frei, S. Asuri and Ying-Chang Lai, *CHAOS*, 15:033106, 2005.
7.L. Glass, A. L. Goldberger, M. Courtemanche and A. Shrier, *Proc. R. Soc. Lond. Ser. A*, 413:9, 1987.
8.F. X. Witkowski, K. M. Karnagh, P. A. Penkoske, R. Plonsey, M. L. Spano, W. L. Ditto and D. T. Kaplan, *Phys. Rev. Lett.*, 75:1230, 1995.
9.C. S. Poon and C. K. Merril, *Nature*, 389:492, 1997.
10.F. Ravelli and Anbolin, *Biol. Cybernetics*, 67:57, 1992.
11.R. B. Govindan, K. Narayanan and M. S. Gopinathan, *CHAOS*, 8:495, 1998.
12.L. Glass and P. Hunter, *Physica D*, 43:1, 1990.
13.A. L. Goldberger, *News. Physiol. Sci.*, 6:87, 1991.
14.F. Mitschke and M. Dimming, *Int. J. Bif. Chaos*, 3:693, 1993.
15.K. P. Harikrishnan, R. Misra, G. Ambika and A. K. Kembhavi, *Physica D*, 215:137, 2006.
16.K. P. Harikrishnan, R. Misra, G. Ambika and R. E. Amritkar, *CHAOS*, 19:143129, 2010.