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Stability and Chaos in a Classical Yang – Mills - Higgs System

O. Ozgur Aybar[†•], Avadis S. Hacinliyan[†‡], Ilknur Kusbeyzi Aybar[*], Kamer Koseyan[‡], Berc Deruni[‡]

[†]Yeditepe University, Department of Information Systems and Technologies, Istanbul, Turkey
[‡]Yeditepe University, Department of Physics, Istanbul, Turkey
[*]Yeditepe University, Department of Computer Education and Instructional Technology, Istanbul, Turkey
[•]Gebze Institute of Technology, Department of Mathematics, Kocaeli, Turkey
E-mail: oaybar@yeditepe.edu.tr
E-mail: ahacinliyan@yeditepe.edu.tr
E-mail: ikusbeyzi@yeditepe.edu.tr
E-mail: kaderci.johannes@gmail.com
E-mail: berc_890@hotmail.com

Abstract: A motivation for looking at chaos in the classical realizations of the Yang-Mills or Yang Mills augmented by Higgs equations is the importance of this system in the initial (in)stability at big bang, since in the initial stages all interactions were of the same strength and were based on non abelian gauge theories, of which the SU(2) Yang Mills is a first example.
In this study we consider the following two particle effective Hamiltonian suggested by Biro, Matinyan and Müller:

$$H = \frac{{p_x}^2 + {p_y}^2}{2} + \frac{1}{2}x^2y^2 + \frac{1}{2}(a^2x^2 + b^2y^2) + \frac{1}{2}x^4 + \frac{1}{4}py^4$$

Keywords: Dynamical systems, Yang-Mills, Lyapunov exponents, Chaos.

1. Introduction

Global properties for mappings such as Poincare sections, Lyapunov exponents and other topological properties as introduced by Poincare and Birkhoff are important objects of study in nonlinear dynamical systems in addition to their local properties such as various bifurcations and invariant manifolds[1].
As Matinyan suggested, one of the ways to search for chaos is to investigate Poincare sections[4,5,6]. Since the system is described by a time independent Hamiltonian, the energy integral reduces the four dimensional system into a three dimensional system and a two dimensional Poincare map[1,2,3]. Unfortunately, the Hamiltonian involves the squares of the momentum. Taking the square root leads to missing information since the trajectory should cross into regions where the momentum can have either sign. There are two ways

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known to solve this problem. One of the solutions to this problem is the symplectic numerical integration technique and the other one is to check the energy conservation numerically at every point. Results of these investigations lead to the same results obtained by KAM (Kolmogorov–Arnold–Moser) theory and hence this numerical study is proven to be an indicator for chaos.
If the system is integrable, the trajectory is closed. Hence a torus is obtained. If the system is not integrable, elliptic orbits are observed with a chaotic regime. According to KAM theory the invariant tori of an integrable system retain their topology under a perturbation that destroys the integrability of the Hamiltonian, however chaos is observed in some regions of the phase space of the system with random points on the surface of section.
In addition to the Poincare section study done by Matinyan et. al, a Lyapunov exponent study can reveal the parts of the parameter space in which chaos is observed. Preliminary results indicate that for the case in which the Higgs terms (x^4 and y^4) are absent, all regions for the parameters $a>0$ and $b>0$ give positive maximal Lyapunov exponents that indicate chaos For $a = b = 0$, the chaoticity is maximum. As a or b increase, the system is still chaotic, but the system loses its chaoticity gradually and tends to converge to a limit cycle. On the other hand, for the case the Higgs terms are present with $a=b=0$, the system still has a positive maximal Lyapunov exponent whose value is smaller than that in the Yang Mills case.

2. Chaos in Yang Mills Higgs system

Although there is no universally accepted definition of chaos, most experts think that chaos is the aperiodic, long – term behavior of a bounded, deterministic system that exhibits sensitive dependence on the on initial conditions. Lyapunov Exponents is the mathematical method for the determination of chaos in dynamical systems. It is the measure of the exponential separation of two trajectories with a very small initial separation. A system with positive values of Lyapunov Exponents is chaotic, and the value of these exponents the average rate at which predictability is lost.
In this section, we compute Lyapunov exponents with the aid of Fortran code that implement Wolf algorithm as we discussed before. In addition we also use Reduce code which calculates variational equations needed for the Wolf algorithm. Both programs are included in appendixes. We mostly emphasize on Yang Mills Higgs coupled system in order to demonstrate the corresponding chaotic behavior.
First of all we investigated how exponents are changing with respect to the scale parameter p, we found that system possesses chaotic motion in wide range of value of p. Especially we scan for the interval from $p=0.05$ to $p=4$. Here are some of graphs for the specific values of p.

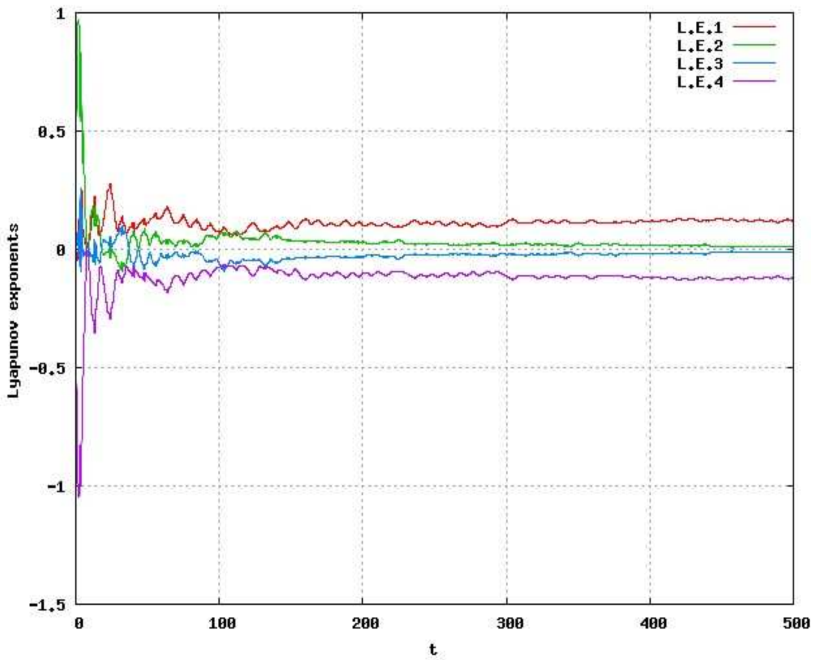

Fig. 1. Lyapunov exponents vs time for *p=0.2*

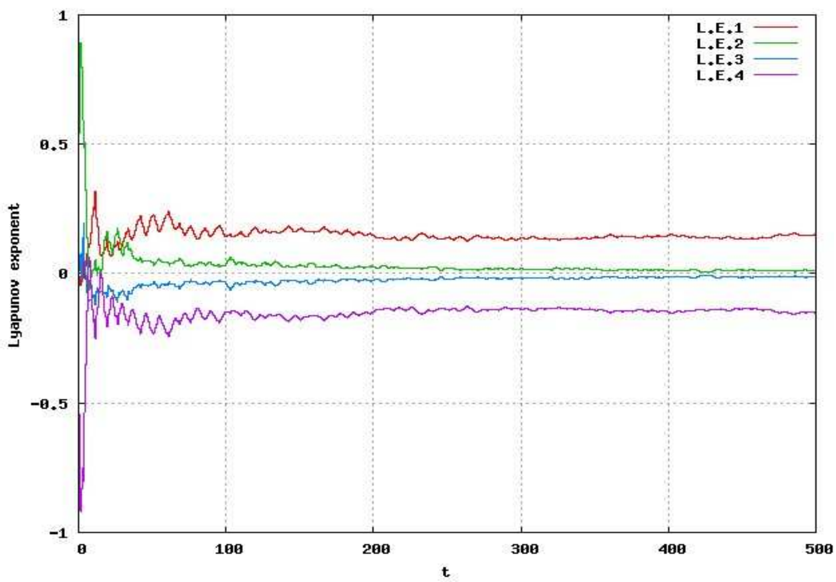

Fig. 2. Lyapunov exponents vs time for *p=0.5*

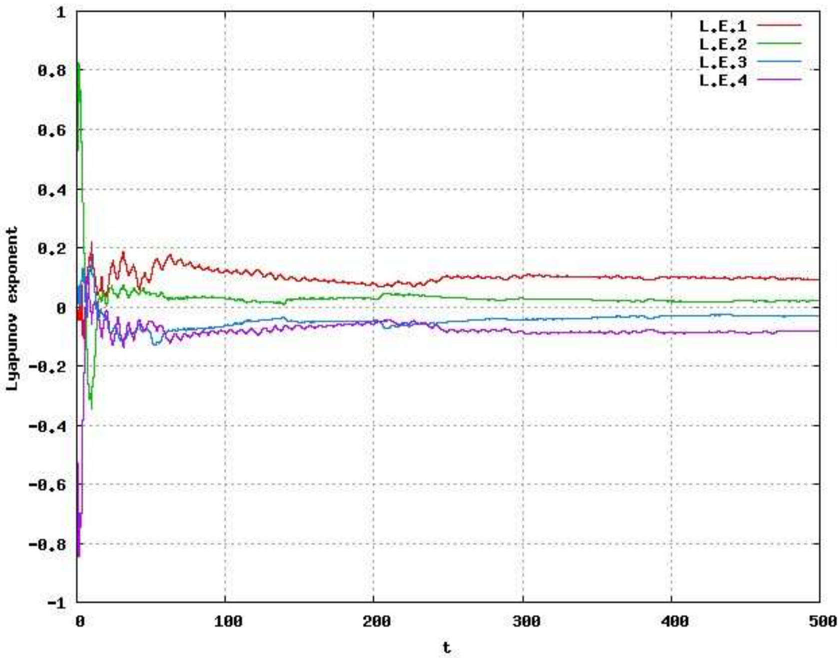

Fig. 3. Lyapunov exponents vs time for *p=0.8*

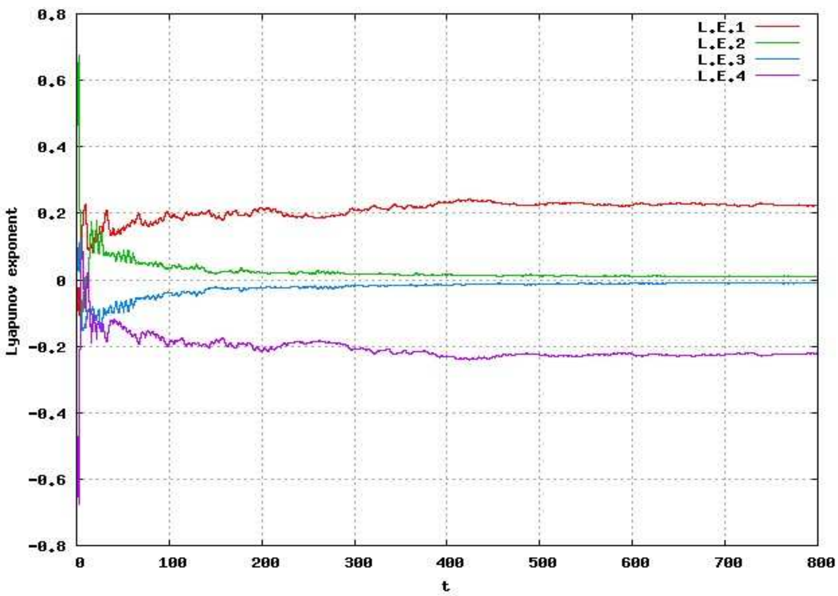

Fig. 4. Lyapunov exponent vs time for *p=2.2*

On the other hand we also analyze the results of adding oscillator term to dynamical system by giving a coefficient "a". We saw that all Lyapunov exponents tend to decrease for bigger value of "a" and there occurs a transition from chaotic motion to periodic or quasi periodic motion. We investigate this transition for the value of parameters where the Lyapunov exponents seem to be maximum. Some of the results are shown below

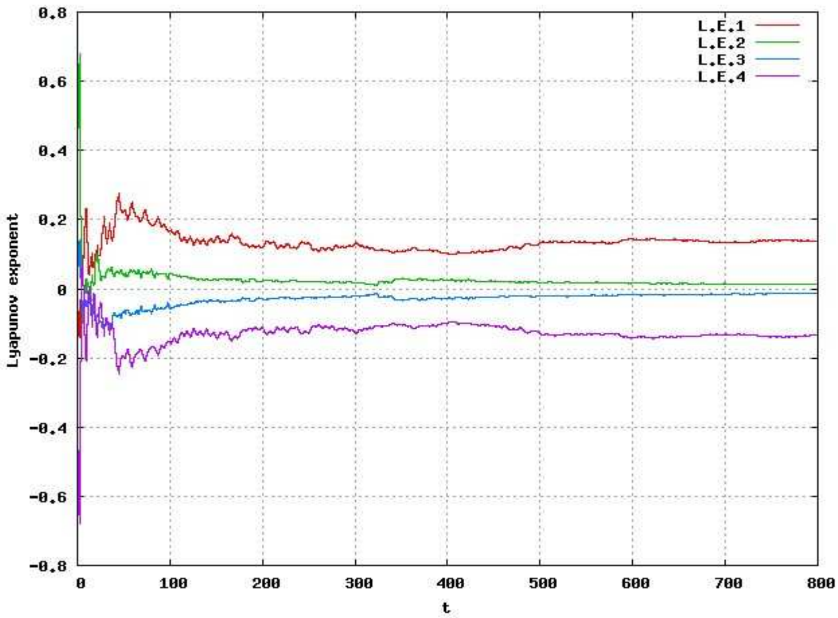

Fig. 6. Lyapunov exponents vs time for *a=0.1* and *p=2.2*

We can deduce from these graphs that, for small values of "*a*" the system persist for a chaotic behavior. But when "*a*" grows system start to possess periodic motion.On the other hand we can see that almost all Lyapunov spectrums are symmetrical which is the expected result since in Hamiltonian systems the sum of Lyapunov exponents must be zero as we stated before so when there is an expanding trajectory in phase space there must be also equally contracting trajectory to compensate for this.

We also investigate phase space trajectories for the corresponding system. Here are some of the trajectories for this system

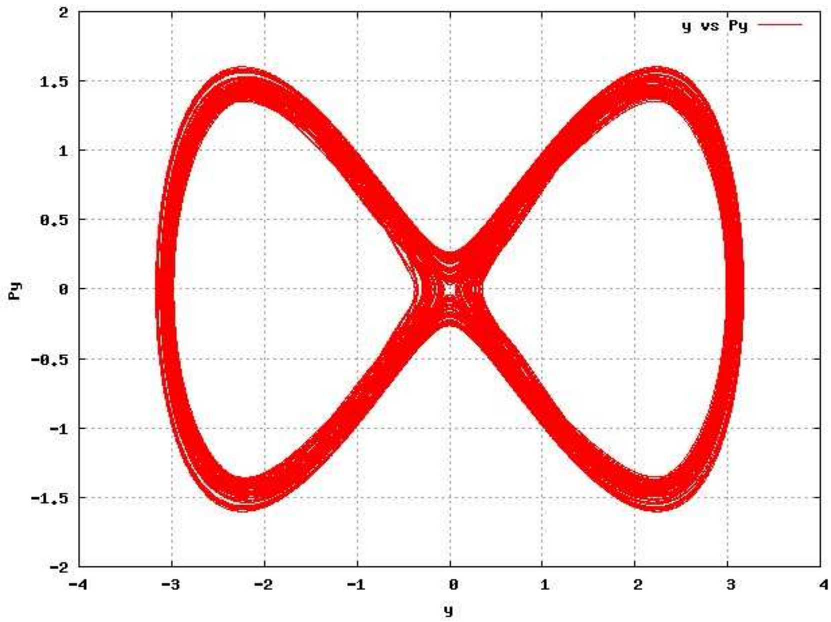

Fig. 7. Trajectory of y vs P_y

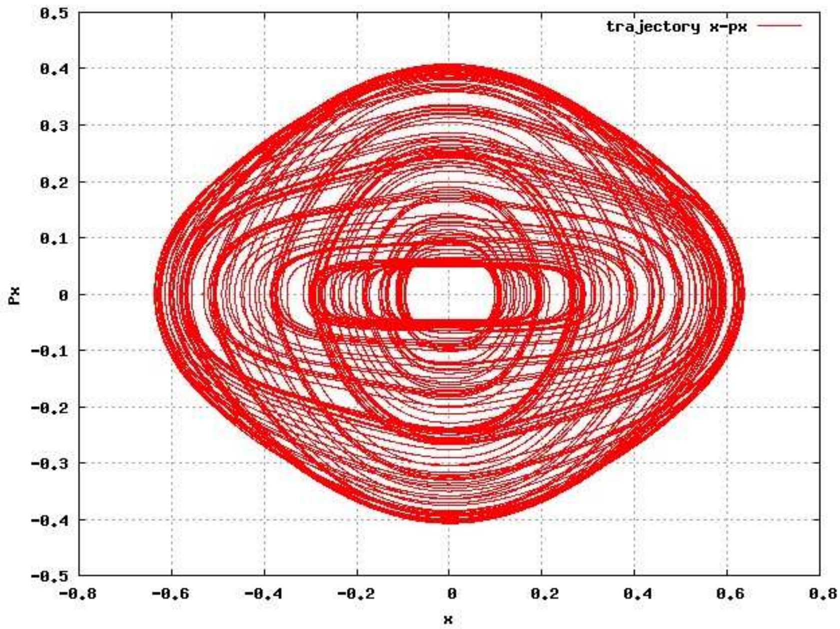

Fig. 8. Trajectory of x vs P_x

3. Conclusions

In this article we try to demonstrate chaotic behavior in the dynamically coupled Yang Mills Higgs system classically. We know that pure Yang Mills fields possess highly chaotic behavior. Although Yang Mills Higgs system also possess chaotic behavior for variety of range of scale parameter, in general the Higgs field is responsible for considerably regularizing motion in the dynamical system[4,6]. So we can say that Higgs mechanism has a stabilizing effect. On the other hand we also consider an additional oscillator term in the Yang Mills Higgs system and it is observed that for small coefficients of the oscillator term chaotic motion still persists. But when oscillator term gets larger chaos disappears and regular motion involving multi periodic motion takes place instead, since the oscillatory motion begins to dominate[5,6].

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