

## **Chaos in Modified CFOA-Based Inductorless Sinusoidal Oscillators Using a Diode**

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**Abstract:** Two modified inductorless sinusoidal oscillators are presented as two chaotic oscillators. The active component employs a current-feedback operational amplifier (CFOA) whereas the nonlinear component employs a simple diode. Numerical and PSpice simulations are demonstrated in terms of chaotic attractors. A bifurcation diagram is also included.

**Keywords:** Chaos, Nonlinear circuit and system, RC oscillator.

### **1. Introduction**

The design and development of autonomous chaotic oscillators over the past three decades have been increasing due to a variety of applications in, for example, spacecraft trajectory control, stabilization of the intensity of a laser beam, noise radars and sonar [1], synchronization [2, 3] and secure communications [4, 5, 6]. One of the best known chaotic circuits is Chua's circuit [7] as well as its variants [8, 9], using a Chua's diode. However, an active nonlinear resistor such as the Chua's diode is not recommended by [10] because it does not follow the design rules of [10]. Instead, a passive nonlinear component for chaos has been suggested using either a diode or a junction field effect transistor (JFET) [10].

A current-feedback operational amplifier (CFOA) is currently recognized as a versatile alternative to the traditional op amp for its excellent performance in high-speed and high slew-rates analog signal processing, and therefore does not suffer from the finite gain bandwidth product typically encountered in the conventional voltage op amps [11]. A chaotic oscillator has been designed using a modified CFOA-based sinusoidal oscillator with two capacitors and an inductor for a third-order chaotic system [11]. Such a chaotic oscillator has subsequently been further investigated by [12] using three capacitors. The nonlinear device of both chaotic oscillators has exploited a two-terminal nonlinear resistor formed by a JFET (J2N4338). However, chaos has not successfully found in [12] using a single diode as a nonlinear component.

In this paper, chaos in two modified CFOA-based inductorless sinusoidal oscillators is presented. The active element employs the CFOA whereas the



nonlinear component employs a single diode. Chaos can be found by replacing a JFET resistor of [12] with a sub-circuit consisting of a diode and a resistor.

### 2. Circuit Implementation

Figures 1(a) and 1(b) show two proposed chaotic oscillators using a single diode as a nonlinear device. Both circuits are modified CFOA-based inductorless sinusoidal oscillators which almost resemble the existing circuits reported in [12], except that the JFET nonlinearity of [12] is replaced with a new sub-circuit consisting of a diode  $D_1$  and a resistor  $R_3$ . The latter is connected to a negative DC supply.

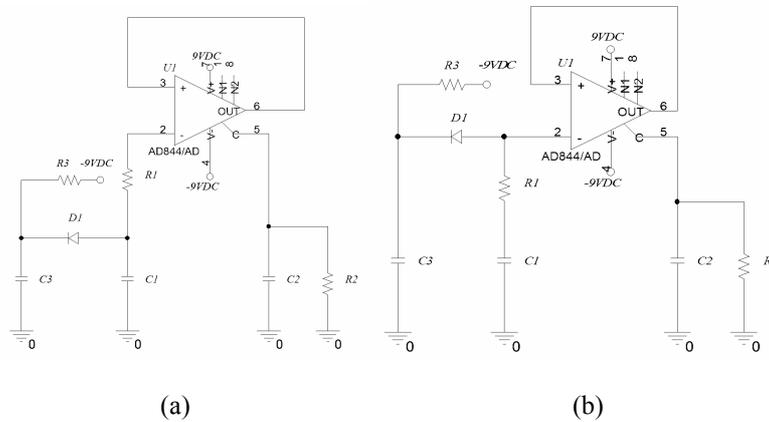


Fig. 1 Modified CFOA-based inductorless sinusoidal oscillators using a diode for : (a) the first chaotic oscillator, (b) the second chaotic oscillator.

The proposed chaotic oscillator shown in Figure 1(a) is described by a set of differential equations as follows :

$$\begin{aligned}
 C_1 \dot{V}_{C1} &= \left( \frac{V_{C2} - V_{C1}}{R_1} \right) - I_D \\
 C_2 \dot{V}_{C2} &= \left( \frac{V_{C2} - V_{C1}}{R_1} \right) - \frac{V_{C2}}{R_2} \\
 C_3 \dot{V}_{C3} &= I_D - \left( \frac{V_{C3} + 9}{R_3} \right)
 \end{aligned}
 \tag{1}$$

where the overdot denotes a time ( $t$ ) derivative. The voltages across capacitors  $C_1$ ,  $C_2$ , and  $C_3$  are  $V_{C1}$ ,  $V_{C2}$ , and  $V_{C3}$ , respectively. A diode current  $I_D = I_S \{ \exp[(V_{C1} - V_{C3})/nV_T] - 1 \}$  where  $I_S$  is the reverse saturation current,  $n$  is the nonideality factor, and  $V_T$  is the thermal voltage of 25.85 mV at room temperature (300K). The proposed chaotic oscillator shown in Figure 1(b) is described by another set of differential equations as follows:

$$\begin{aligned}
 C_1 \dot{V}_{C1} &= \frac{V_{C2} - V_{C1}}{R_1} & (2) \\
 C_2 \dot{V}_{C2} &= \left( \frac{V_{C2} - V_{C1}}{R_1} \right) - \left( \frac{V_{C2}}{R_2} \right) + I_D \\
 C_3 \dot{V}_{C3} &= I_D - \left( \frac{V_{C3} + 9}{R_3} \right)
 \end{aligned}$$

where the diode current  $I_D = I_S \{ \exp[(V_{C2} - V_{C3})/nV_T] - 1 \}$

### 3. Simulation Results

The CFOA can be implemented using the commercially available AD844. The diode  $D_1$  is 1N4001 using PSpice parameters  $I_S = 14.11 \times 10^{-9}$  A and  $n = 1.984$ . The junction capacitance of 1N4001 is typically 15 pF and, for simplicity, may be neglected compared to the much larger values of  $C_1$ ,  $C_2$  and  $C_3$ . For a PSpice simulation, Figure 2(a) shows a circuit diagram of (i) a diode circuit ( $D_4$ ,  $R_2$ ), (ii) a nonlinear JFET resistor ( $J_1$ ,  $R_4$ ), and (iii) a sub-circuit consisting of a diode and resistors ( $D_3$ ,  $R_1$ ,  $R_3$ ). Figure 2(b) shows a comparison of the three simulation results of current-voltage characteristics in (i), (ii) and (iii) where the currents on the vertical axis are through  $R_2$ ,  $R_4$  and  $R_1$ , respectively, and the voltage on the horizontal axis is  $V_S$ , which is swept linearly from -2V to +1V with an increment of 0.01 V. It should be noted that the current in (i) is always positive whereas the current in (ii) can be either positive or negative. This may probably be the reason why the authors in [12] could not find chaos in their proposed oscillators using only a diode in (i). With a new sub-circuit in (iii), the current in (iii) can be either positive or negative, as shown in Figure 2, and chaos can be quickly found without changing the connections of other components.

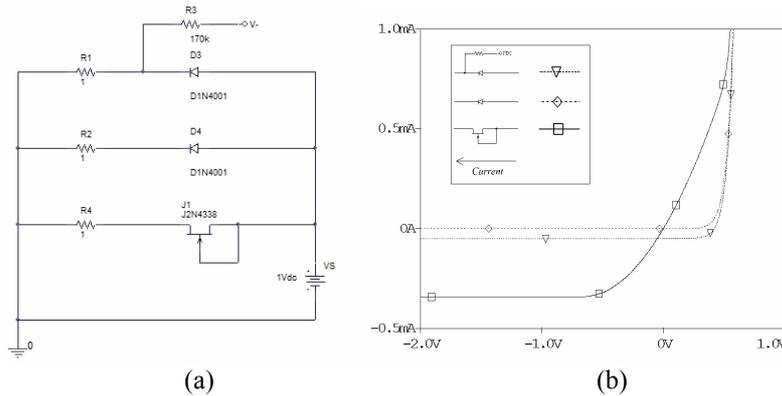


Fig. 2 (a) A circuit diagram of three circuits using (i) a diode circuit ( $D_4$ ,  $R_2$ ), (ii) a nonlinear JFET resistor ( $J_1$ ,  $R_4$ ), and (iii) a sub-circuit consisting of a diode and resistors ( $D_3$ ,  $R_1$ ,  $R_3$ ), (b) A comparison of three simulation results.

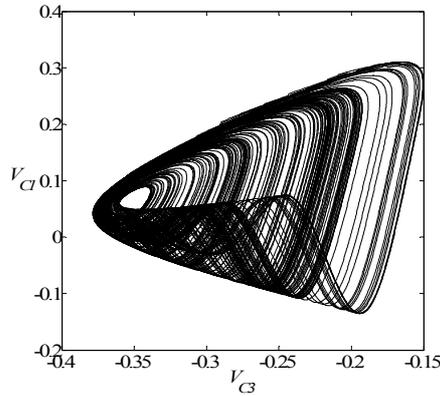


Fig. 3. A numerical result of a chaotic attractor projected onto  $V_{C3}-V_{C1}$  plane of equation (1).

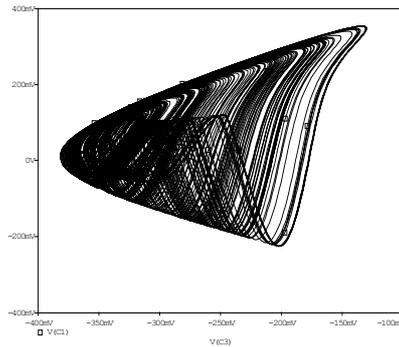


Fig. 4. A PSpice simulation of a chaotic attractor projected onto  $V_{C3}-V_{C1}$  plane of the oscillator shown in Figure 1(a).

Figure 3 shows a numerical result of a chaotic attractor projected onto a  $V_{C3}-V_{C1}$  plane of equation (1) using a fourth-order Runge-Kutta integrator with a fixed step size of  $0.1\mu\text{s}$ . The same values of components reported in [12] are used except  $R_3$ , i.e.  $C_1 = C_2 = 10\text{ nF}$ ,  $C_3 = 18\text{ nF}$ ,  $R_1 = 220\ \Omega$ ,  $R_2 = 1.5\text{ k}\Omega$ , and  $R_3 = 170\text{ k}\Omega$ . Figure 4 shows a PSpice simulation of a chaotic attractor projected onto  $V_{C3}-V_{C1}$  plane of the oscillator shown in Figure 1(a) with the same values of components reported in [12] except  $R_3 = 180\text{ k}\Omega$ . As shown in Figure 4, the PSpice simulation runs up to 30 ms with a fixed step size of  $0.5\ \mu\text{s}$ . The results in the first 20 % are discarded to ensure that the solution is on the attractor. Initial conditions are  $(V_{C1}, V_{C2}, V_{C3})_{t=0} = (0, 0, 0)$ . The numerical and PSpice results are in a similar manner.

It can be seen from Figures 1(a) and 1(b) that  $R_3$  is connected in series with the diode  $D_1$ . This enables  $R_3$  to control the current of  $D_1$  in DC operation (by opening  $C_1$ ,  $C_2$ , and  $C_3$ ). Therefore  $R_3$  can be exploited as a tunable bifurcation parameter. As an example, Figure 5 depicts a bifurcation diagram of the peak of  $V_{C3}$  ( $V_{C3-max}$ ) of Figure 1(a) versus  $R_3$  varied from 140 to 220 k $\Omega$ . A period-doubling route to chaos is evident. There are various periodic windows immersed in chaos.

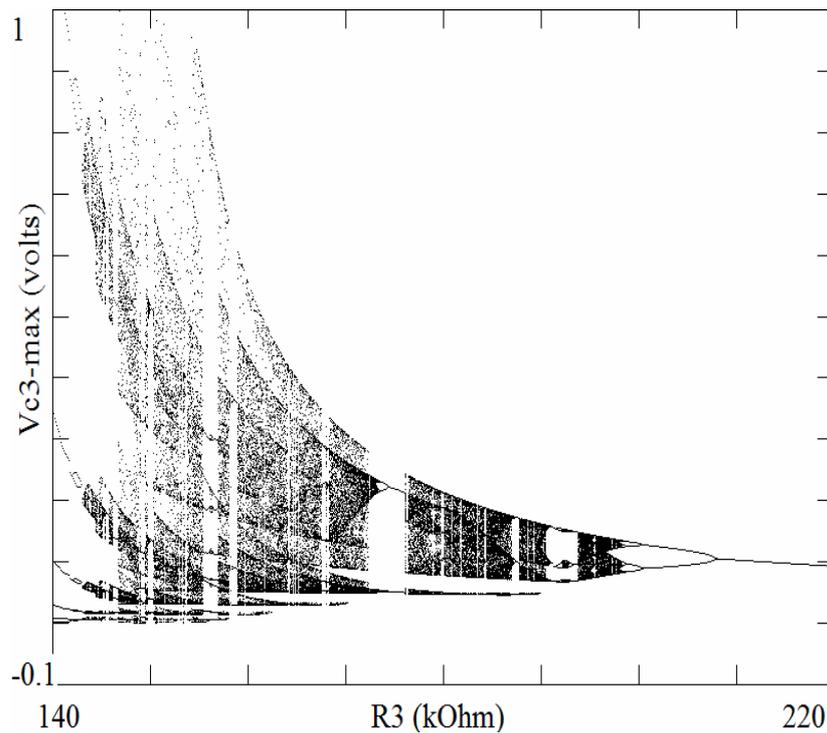


Fig. 5. A bifurcation diagram of the peak of  $V_{C3}$  of Figure 1(a).

Figure 6 shows a numerical result of a chaotic attractor projected onto a  $V_{C3}$ - $V_{C2}$  plane of equation (2) using a fourth-order Runge-Kutta integrator with a fixed step size of 0.1  $\mu$ s,  $C_1 = 10$  nF,  $C_2 = 11$  nF,  $C_3 = 5$  nF,  $R_1 = 220$   $\Omega$ ,  $R_2 = 2.7$  k $\Omega$ , and  $R_3 = 220$  k $\Omega$ . Figure 7 illustrates a PSpice simulation of a chaotic attractor projected onto  $V_{C3}$ - $V_{C2}$  plane of the oscillator shown in Figure 1(b) with the same values of components used in Figure 6. The PSpice simulation runs up to 20 ms with a fixed step size of 0.1  $\mu$ s. The results in the first 20 % are discarded to ensure that the solution is on the attractor. Initial conditions are  $(V_{C1}, V_{C2}, V_{C3})_{t=0} = (0, 0, 0)$ .

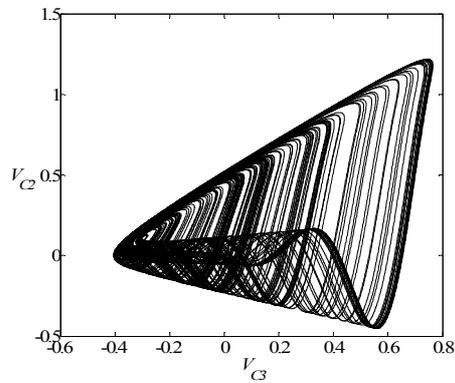


Fig. 6. A numerical result of a chaotic attractor projected onto  $V_{C3}-V_{C2}$  plane of equation (2).

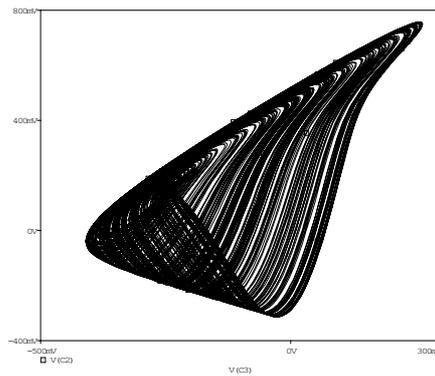


Fig. 7. A PSpice simulation of a chaotic attractor projected onto  $V_{C3}-V_{C2}$  plane of the oscillator shown in Figure 1(b).

#### 4. Conclusions

Two chaotic oscillators have been presented through the use of two modified CFOA-based inductorless sinusoidal oscillators. A CFOA has been exploited as the active component whereas a single diode has been exploited as the nonlinear component. Numerical and PSpice simulations have been demonstrated with chaotic attractors. A bifurcation diagram has been studied.

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