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Collision Frequency leads the Plasma into a Chaotic State. Influence on the Conductivity

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Abstract: In some cases, plasma turbulence exhibits the signature of vortex structures. In this paper, the very important role of stereo-electrostatic waves in those nonlinear behaviors is attempted to be studied. Using the two-fluid model, a set of nonlinear equations is obtained, which are capable of describing the dynamics of long wavelength drift instabilities, as well as of giving a reason for the plasma turbulence rising. It is shown that collision frequency seriously affects the plasma stability, and interferes with all wavy phenomena. The presupposed conditions and the expressions for the growth rates are given as well. Furthermore, it is found that possible stationary solutions of the nonlinear equations can represent the vortex structures.

Keywords: plasma stability, plasma instabilities, turbulence, chaotic state, vortex structure, feed-back state, chaotic simulation.

1 Introduction

The relativity theory, the quantomechanics and the chaos theory were the three greater scientific achievements during the last centuty. The first one has solved the macroworld problems between space and time, the second the causality principle on the microworld and the third one is studying the foresight meaning of how similar initial conditions may give different results. The chaotic theory appears to find a great resonance into researchers and has given an important push to the sciences, as well as other fields. On the sciences, the chaos is defined as the very sensitive influence of the initial conditions on the dynamic system movement. The smallest change on the initial conditions is the cause of the chaos, where a regular and steady physical process is lead to chaotic disorder.
The plasma is another important physical state, which occupies the scientific

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community at the last half of the 20th century. The plasma significance is centralized on obtaining the thermonuclear fusion, as the plasma includes the required high temperatures. However, the plasma instabilities, which cause waves capable to reduce the plasma temperature [1-3], soon appear. So, the plasma waves are considered as one serious obstacle to the thermonuclear fusion process [4-6]. A big variety of these plasma instabilities are the drift waves [1-6], which are characterized by low frequencies. The main cause of the drift waves' rising is the plasma parameters' gradients, such as the electron density gradient [2,3], the rf electric field gradient [5], the dc electric field gradient [7,8] e.t.c. The fluid state of the ionized gas makes it unstable and many nonlinear phenomena may appear into the plasma [9,10]. So, a high frequency instability may affect the drift waves dispersion relation and a decay of these waves occurs [11].

During the past few years, a lot of work has been done on the plasma nonlinear dynamics, which end in chaotic behavior. Recently, in the Plasma Laboratory of N.C.S.R. ''Demokritos'', a sunchronization on the plasma wave has been done and the plasma turbulence has been calmed [12]; earlier the nonlinear dynamics behavior had been constantly studied [13,14].

In the present research, a combination between theory and experiment is carried out; the experimental results appear to agree with the mathematical model, as well as with the theory to justify the experimental data. A persistant effort has been done to show the chaos which is included into the plasma, combining the nonlinear terms with plasma turbulences.A fractal behavior appears to exist in the particles velocities. Furthermore, the collision frequency influence on the drift velocities is researched [5,6], experimentally and theoretically, as these velocities constitute the initial conditions for the dynamic system; the initial conditions significance on the final result is well known. Another study on the wave amplitude and the drift current takes place [7] and the trapped electrons are measured.

The paper is written as following: the experiment description is presented in Sec.2. This contains the description of the apparatus and the experimental data. A detailed mathematic theory on the two fluids' model is presented in Sec.3. Finally, in Sec.4 explanations and conclusions are given.

2. Experiment Description

2.1 Description of the Apparatus

A low temperature rf argon plasma is produced within a quasi Q- machine. A cylindrical cavity made of steady steel is located along the machine axis, which coincides with the restraining magnetic field $\vec{B}$. The cavity dimensions are $6cm$ diameter and $70cm$ length, and it is closed by two metallic disk-like bases; on every base center two metallic antennas with $3mm$ diameter and $25cm$ length are put along the magnetic field direction. On antenna 1 the rf power is enforced, whereas on antenna 2 a driving signal is imposed. The arrangement has a complete cylindrical symmetry, and the mathematical elaboration is carried out in the cylindrical system, consequently. Figure 1.a shows the outline of the device, as Figure 1.b represents the cylinder cut.

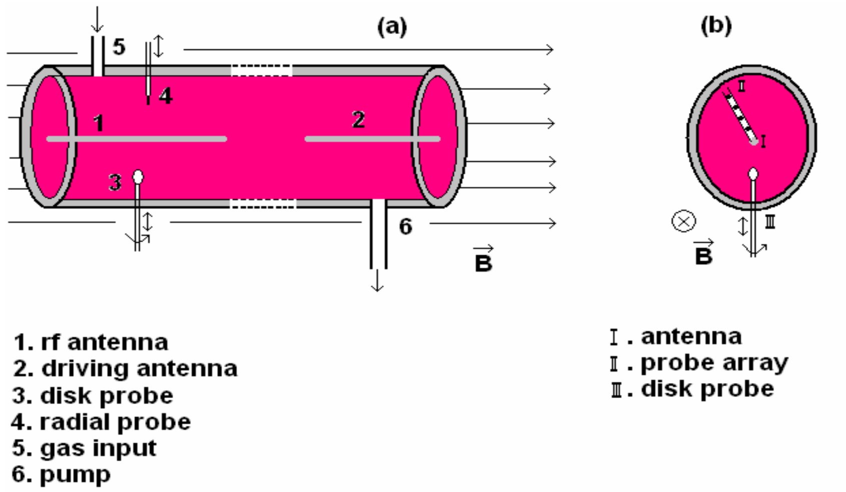

Figure 1a. The cavity outline, Figure 1b. The cylinder cut drawing

A disk-probe , a probe array, as well as other probes, are placed properly so that to measure the plasma parameters at any interesting point.

2 . 2 Experimental data

Plasma is produced easily into the above described cylindrical cavity and plasma waves appear when the parameters satisfy the suitable dispersion relation. These electrical waves are rising into the plasma and are always accompanied by their upper harmonics. Three are the frequency regions for these waves: the first one is the region of $\cong 18kHz$; this has been classified as drift wave and is caused on rf electric field gradient [5]. The second region is of $\cong 70kHz$, and it has been classified as electrical wave caused on dc electric field gradient [8]. Finally, the third one is the region of $\cong 200kHz$ but it has not been studied yet.
In the present experiment, a focus-concentration on the middle wave frequency has been given by researching the plasma chaotic configuration, mainly. The basic plasma parameters are the gas pressure , the absorbed rf power and the plasma restraining magnetic field value. Secondary parameters are the plasma density, the plasma temperature, the electron- neutral collision frequency e.t.c. Typical- average values of these parameters are given in Table 1.

Table 1. Typical-average values of plasma parameters.

Parameters	Average-typical values
Argon pressure p	$0.35Pa$
Argon number density n_g	$6.0x10^{19}m^{-3}$
Magnetic field intensity B	$80mT$
Microwave power P	$80watt$
Electron density n_0	$5x10^{16}m^{-3}$
Electron temperature T_e	$2.2eV$
Ion temperature T_i	$0.07eV$
Ionization rate	0.1%
Electron drift velocity u_e	$1.3x10^4 \, m/s$
Electron-neutral collision frequency v_e	$2.2x10^7 s^{-1}$

Firstly, an electrostatic wave is given in Figure 2.a (video) and in Figure 2.b its' spectrum of frequencies is presented. The wave figure shows the plasma found in dynamic equilibrium and the parameters have the following values: $p = 0.32Pa, B = 75mT, P = 75watt$. The wave inclination from the sine form gives the reason for the upper harmonics appearance.

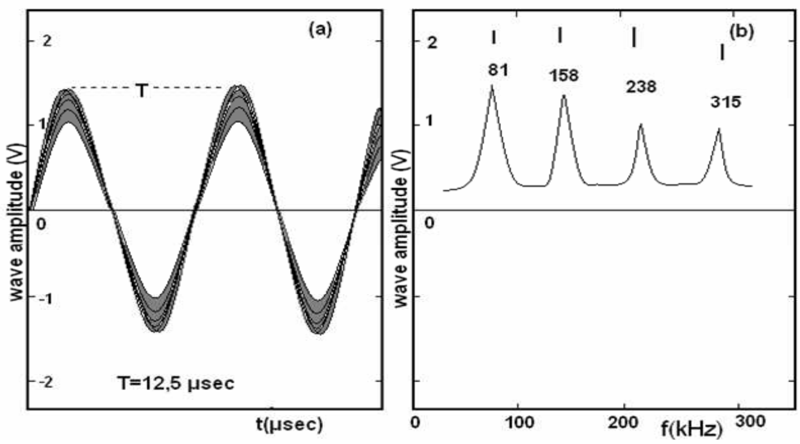

Figure 2a. The wave form (video) and Figure 2b. The frequencies spectrum are shown

Afterwards, by increasing the gas pressure at the value $p = 0.38Pa$ and the rf power at the value $P = 85watt$, since the magnetic field intensity remains constant-unchanged, the plasma equilibrium disappears and a wave turbulence appears clearly. Figure3.a and Figure 3.b show the wave turbulence (video) and the frequencies spectrum, respectively.

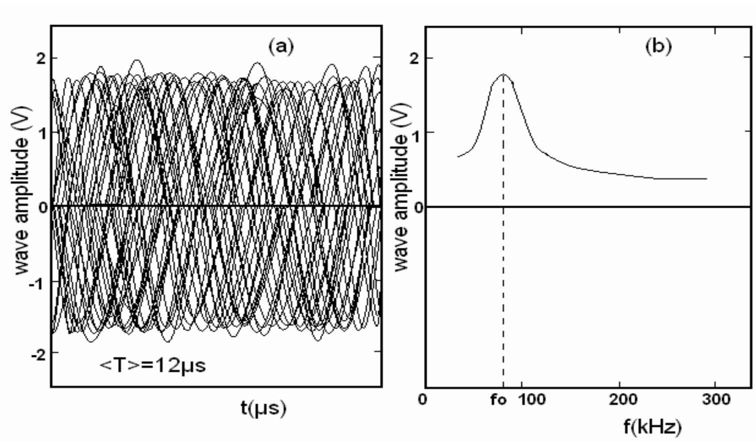

Figure 3a. The wave turbulence (video) and Figure 3b. The frequencies spectrum are shown

It must be noted that, in the previous parameters' values, the turbulence frequencies are gathered around a central frequency f_0 (Figure 3.b), since by a small change on the rf power the turbulence spectrum was extended into a big frequency region. This can be seen in Figures 4.a, 4.b

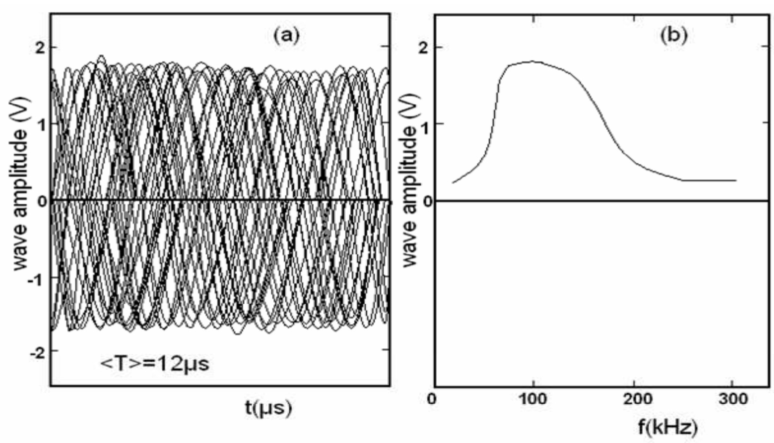

Figure 4a. The wave turbulence (video) and Figure 4b. The frequencies spectrum is shown

Notice: the turbulences in Figures 3.a, 4.a appear not to be different, since they have distinguishing spectra. Another interesting question was how the electron-neutral collision frequency affects the drift velocities which constitute the initial conditions of the plasma wavy state. Trustworthy measurements can be carried out on the azimouthal drift velocity only when $\nu \prec \omega_c$, as the radial drift is not measurable. So, the experiment is limited on the measuring of the azimouthal drift and wave frequency, as the gas pressure increases the collision frequency, as shown in Figure 5.

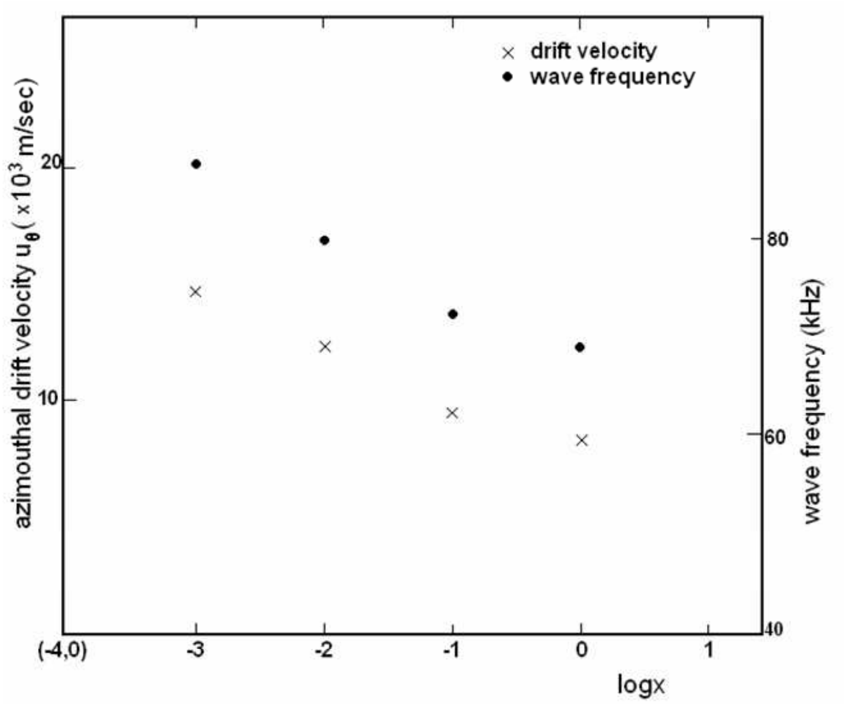

Figure 5. Azimouthal drift and wave frequency vs collision frequency

However, a computer simulation gives a numerical-theoretical study on this issue, by using realistic values on the physical quantities; involving the relations (see mathematical model),

$$u_\theta = \frac{q\varepsilon}{m} \cdot \frac{\omega_c}{\omega_c^2 + \nu^2} = \frac{\varepsilon}{B} \cdot \frac{1}{1 + (\frac{\nu}{\omega_c})^2} = \frac{\varepsilon}{B} \cdot \frac{1}{1 + \chi^2}$$

and $$u_r = \frac{q\varepsilon}{m} \cdot \frac{\nu}{\omega_c^2 + \nu^2} = \frac{\varepsilon}{B} \cdot \frac{\frac{\nu}{\omega_c}}{1 + (\frac{\nu}{\omega_c})^2} = \frac{\varepsilon}{B} \cdot \frac{\chi}{1 + \chi^2}$$

$$(\chi = \frac{\nu}{\omega_c})$$

and by taking the velocity $\frac{\varepsilon}{B} = 1{,}25.10^4 \, {m}/{s}$ and the gyrofrequency $\omega_c = 1{,}4.10^{10} s^{-1}$

Then, Table 2 is formed

Table 2. Numerical values for drift velocities study

$\chi = \frac{\nu}{\omega_c}$	$\log \chi$	$1 + \chi^2$	$1/(1+\chi^2)$	$\chi/(1+\chi^2)$	$u_\theta\ (10^4\ m/s)$	$u_r\ (10^4\ m/s)$
0,001	-3	1,000001	0,9999990	0,0099990	1,24999	0,00124
0,01	-2	1,0001	0,99990	0,0099990	1,24987	0,01249
0,1	-1	1,01	0,9900	0,09900	1,2375	0,12375
0,3	-0,52	1,09	0,9174	0,2752	1,14675	0, 344
0,5	-0,301	1,25	0,8	0,4	1,000	0,5
0,7	-0,155	1,49	0,671	0,4697	0,83875	0,587125
0,9	-0,046	1,81	0,552	0,4968	0,69	0,621
1	0	2	0,5	0,5	0,625	0,625
1,5	0,176	3,25	0,307	0,4605	0,38375	0,575625
2	0,301	5	0,2	0,4	0,25	0,5
3	0,47712	10	0,1	0,3	0,125	0,375

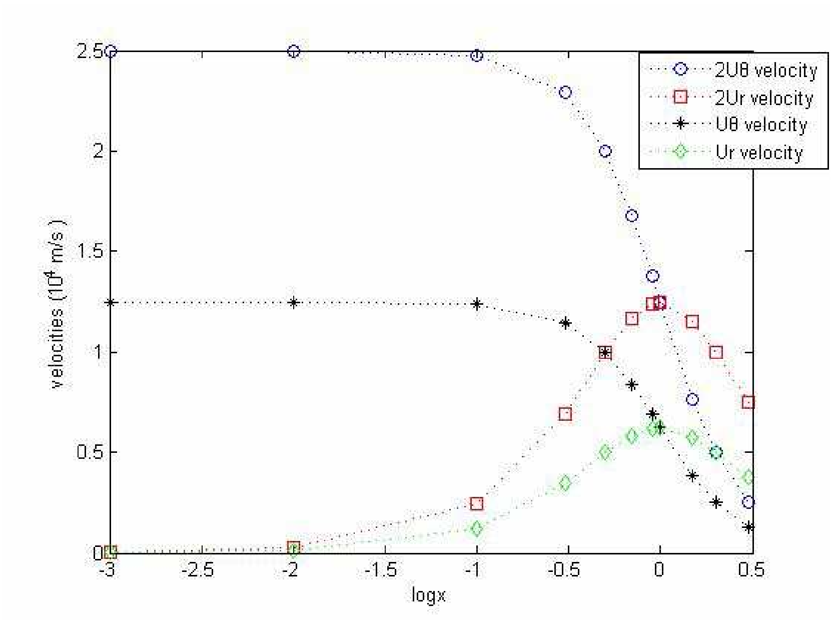

Figure 6. The drift velocities' computer simulation is presented

Finally, the wave affection on the azimouthal drift current has been examined, just as the trapped electrons move along with the wave phase velocity. When there is no wave into the plasma, the radial electric field is unimportant and the measured azimouthal drift current is neglectable as well. The plasma behavior is similar when a turbulence appears in the place of the clear wave. So, the steady state is resulted from some-concrete initial conditions by the existence of the wave, on which the measurements have been taken. The transition in the final state is chaotic and very hard to be solved; so, measurements must be taken when the wave is clear and the physical quantities are stable. An increase of the gas pressure gives a decrease of the wave amplitude and frequency. The drift current can be measured by using the disk probe, while the trapped electrons can be calculated from the relation, $I = qAn_0u_0$, (A is the disk probe plain surface). Figure 7 the wave frequency, the drift current and the trapped electrons density, versus the wave amplitude are shown.

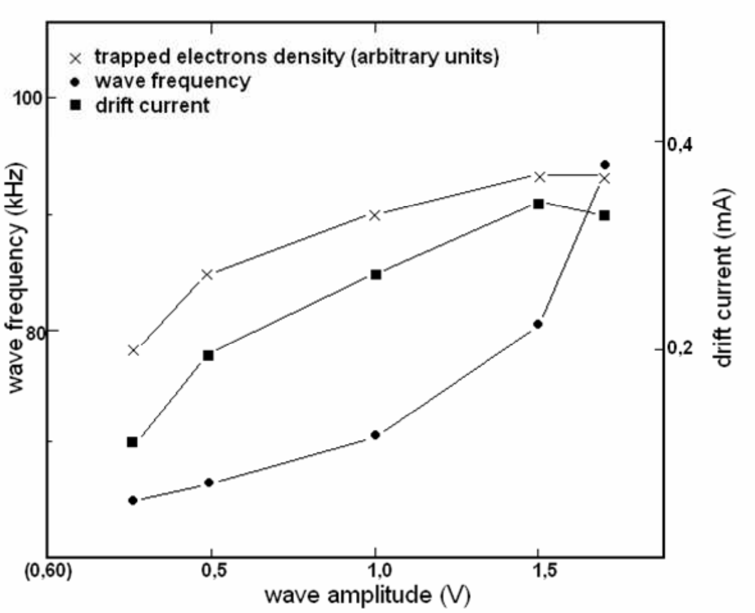

Figure 7. Drift current, wave frequency, and trapped electrons density vs wave amplitude are given

3. Mathematical model

3.1 Stability-Instability-Turbulence-Chaos

Using the two fluids theory in a non-local slab, the momentum equation for both kinds of charged particles is written as,

$$Nm\Big[\frac{\partial}{\partial t}+\vec{V}.\vec{\nabla})\Big]\vec{V} = Nq(\vec{\varepsilon}+\vec{E})+Nq\frac{\vec{V}x\vec{B}}{c}-Nm\nu\vec{V}-\vec{\nabla}p \quad (1)$$

where $N = n_0 + n(\vec{r},t), \vec{E}_{tot} = \vec{\varepsilon} + \vec{E}(\vec{r},t)$, and $\vec{V} = \vec{u}_0 + \vec{\upsilon}(\vec{r},t)$,

where $n(\vec{r},t), \vec{E}(\vec{r},t)$, and $\vec{\upsilon}(\vec{r},t)$, the perturbed qualities with harmonic influence $\propto e^{j(\vec{k}\vec{r}-\omega t)}$

when no perturbation exists, the drift velocity u_0, is obtained;

$$0 = n_0 q\vec{\varepsilon} + n_0 q\frac{\vec{u}_0 x\vec{B}}{c} - n_0 m\nu\vec{u}_0 \quad (2)$$

With the separation on the $\vec{r}$ and θ axis the drift components are given,

$$u_\theta = \frac{q\varepsilon}{m}.\frac{\omega_c}{\omega_c^2+\nu^2} \quad \text{and} \quad u_r = \frac{q\varepsilon}{m}.\frac{\nu}{\omega_c^2+\nu^2} \quad (3)$$

(drift velocities are represented by the 0-order equation).

i) If the perturbation is taken into account, eq.(1) gives,

α) $$n_0 \frac{\partial}{\partial t} + n_0 \vec{u}_0 \vec{\nabla} \vec{\upsilon} = n_0 \frac{q\vec{E}}{m} + n\frac{q\vec{\varepsilon}}{m} + nq\frac{\vec{u}_0 x\vec{B}}{mc} + n_0 q\frac{\vec{\upsilon} x\vec{B}}{mc} - n\nu\vec{u}_0 - n_0\nu\vec{\upsilon} \quad (4)$$

(the 1st order equation)

β) $$n_0 \vec{\upsilon}.\vec{\nabla}\vec{\upsilon} + n\frac{\partial\vec{\upsilon}}{\partial t} + n\vec{u}_0.\vec{\nabla}\vec{\upsilon} = \frac{nq\vec{E}}{m} + nq\frac{\vec{\upsilon} x\vec{B}}{mc} - n\nu\vec{\upsilon} - \frac{\vec{\nabla}p}{m} \quad (5)$$

(the 2vd order equation)

γ) And finally,

$$n\vec{\upsilon}.\vec{\nabla}\vec{\upsilon} \cong 0 \quad (6)$$

(the 3rd order equation).

From the equilibrium state (zero order equation), the drift velocity components are easily obtained,

$$u_\theta = \frac{q\varepsilon}{m}.\frac{\omega_c}{\omega_c^2+\nu^2} \quad \text{and} \quad u_r = \frac{q\varepsilon}{m}.\frac{\nu}{\omega_c^2+\nu^2}$$

From the first order equation, the perturbent velocity components may be given as,

$$\upsilon_\theta = \frac{qE}{m}.\frac{\omega_c}{\omega_c^2+\Pi^2} \quad \text{and} \quad \upsilon_r = \frac{qE}{m}.\frac{\Pi}{\omega_c^2+\Pi^2} \quad (7)$$

with , $\Pi = j(ku-\omega)+\nu$

If it is considered that $ku-\omega \prec\prec \nu$, then it is taken $\Pi \cong \nu$ and the perturbed velocity components become,

$$\upsilon_\theta = \frac{qE}{m}.\frac{\omega_c}{\omega_c^2+\nu^2} \quad \text{and} \quad \upsilon_r = \frac{qE}{m}.\frac{\nu}{\omega_c^2+\nu^2}$$

as the drift velocity components with the replacement of the dc electric field ε with the perturbed one E .

The second order equation gives the dispersion relation eq.(5) of Ref. [8] under some conditions; one of these conditions is the omission of the second order term $n_0\vec{\upsilon}.\vec{\nabla}\vec{\upsilon}$ as approaching zero.

The third order equation (as the second order one) is responsible for the plasma chaotic behavior, which is presented as a wavy turbulence. If these terms are approached by a very small quantity $c(\vec{r},t)$, then the differential equation, $n_0\vec{\upsilon}.\vec{\nabla}\vec{\upsilon} = c(\vec{r},t)$ is formed , and the relation ,

$$n_0\int \vec{\upsilon}.d\vec{\upsilon} = \int c(\vec{r},t)d\vec{r} + c_1$$

is seeking its' solution.

3. 2 Collision frequency affects the drift velocities

As eq.(3) shows, the drift velocities include the collision frequency ν, the dc electrical field ε, and the gyrofrequency ω_c. In the present experiment, the ω_c remains constant, and the ν varies where ε is the parameter. Then the study indicates that the azimouthal velocity $u_\theta = \frac{q\varepsilon}{m} . \frac{\omega_c}{\omega_c^2 + \nu^2}$ is decreaced as ν is increased continuously, where the radial velocity $u_r = \frac{q\varepsilon}{m} . \frac{\nu}{\omega_c^2 + \nu^2}$ is increasced firstly for $\nu \prec \omega_c$, while a maximum appears when it is $\nu = \omega_c$, where $u_\theta = u_r$.

3. 3 Waves affection on plasma conductivity

Without minutenes, as the transit examination is very hard to follow, we consider the steady state by the existance of the stable wave, and then the trapped electrons constitute the drift current. If the drift electron current is measured (by using the disk probe), then the electron density, which composes the drift current, may be calculated from the relation $I = qAn_0u_0$, (A is the disk probe plane surface). As the wave amplitude is decreased, even less electrons are trapped and the measured current decreases.

4. Explanations-Conclusions

1. When no wave exists, the drift velocity is not measurable (as the radial dc electric field is unimportant) and the plasma state is characterized as stability.
2. When plasma waves appear, the dispersion relation [8] is valid and the state is characterised as equilibrium (dynamic equilibrium).
3. When the 2nd order term ($n_0\vec{\upsilon}.\vec{\nabla}\vec{\upsilon}$) is valid and not neglected/ omitted from Eq.(5), the plasma state leaves the dynamic equilibrium and a turbulence appears.
4. When the 3rd order term ($n\ \vec{\upsilon}.\vec{\nabla}\vec{\upsilon}$) is taken into consideration, the turbulence becomes chaotic, as this can appear in the frequency face.

Due to low plasma temperature ($\cong 2{,}2eV$), the collision time τ_e is small [6] and then the collision frequency becomes comparable to gyrofrequency ω_c. If the collision cross section is σ, an incoming particle travels a distance

$l = 1/n\sigma$, the average free path. If the particle moves with velocity υ and the collision time is τ_e, then the collision frequency is $\nu_e = n.\sigma.\upsilon$. As the cross section is a function of the velocity υ, the effective collision frequency is the average value of the product of cross section and velocity [4], $\nu_e = n\langle.\sigma(\upsilon).\upsilon\rangle$.

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