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Complexity in Theory and Practice: Toward the Unification of Non-equilibrium Physical Processes

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Abstract. During the last two decades our scientific group has developed new non-linear methods of analysis applied to various physical systems. In this review study we present our scientific contribution to nonlinear science, including also some novel concepts as for the constructive role of complexity in modern physical theory. The experimental verification of chaos existence in physical systems remains one of the most significant problems of non linear science and complexity. The extended chaotic algorithm presented in the following as well as the results concerning its application at different experimental time series reveal the universal character of the complexity theory for the far from equilibrium dynamics of spatially extended physical systems. The developed methodology that was used compromises different types of computational tools as well as theoretical concepts for the physical interpretation of the experimental information. As we present here the strong dispute and criticism of chaos hypothesis in physical systems during the last two decades was fruitful and challenged us to develop a novel composition of experimental and theoretical knowledge of universal character for the far from equilibrium dynamics. The solar and magnetospheric dynamics included in space plasma processes, the environmental and seismic dynamics, the human brain or the on–chip workload are distinct systems which were studied by our group revealing common chaotic characteristics and chaotic phase transition processes. Moreover, the intellectual struggle for the comprehension of the theoretical presuppositions of the experimentally observed universal chaotic character of spatially distributed systems lead us to the fundamentals of complexity theory as manifested at the macroscopic and microscopic level of physical reality. From this point of view, some common characteristics of macroscopic and microscopic complexity included in the scientific knowledge of the recent two or three decades can be used as a road for the physical theory unification. That is complexity, scaling, chaos, quanticity and fractality could be supported as different manifestations of a unified physical law from the microscopic to the macroscopic and cosmological level. As we can argue, determinism and probabilism can also be unified through chaoticity. Moreover, the rising of new physical knowledge reveals that under the macroscopic or the microscopic physical phenomena there exist a fundamental and multilevel acting unit physical process that produces physical reality rather than a fundamental essence or simple

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substance from which cosmos can be build.
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Contents

1 Introduction

After the historical work by Prigogine, Nicolis, Glansdorff and others [43], [78]-[83], [110], [111] the science of Complexity is rapidly growing providing the opportunity, combined with the computational power, for the development of new methods of analysis, modeling and prediction of various processes with intense stochastic or random–chaotic character. This is related to areas of great interest, such as space plasmas, environment, material mechanics, bio–medicine, economy, human society, psychology, urban development, e.t.c. On the other hand, the nonlinear time series analysis as it was developed by Takens, Grassberger & Proccacia, Theiler, Tsonis and others [44], [120], [121], [124], [125] and was systematically used and extended by Pavlos and Athanasiou [6], [7], [90]-[104] is the road for the experimental verification of complex dynamics. The Complexity theory and the experimental time series analysis concerning spatiotemporal and far from equilibrium nonlinear dynamics include significant collective phenomena such as: fractal and multifractal structures, power law distribution and critical scale invariance, nonequilibrium fluctuations causing spontaneous nucleation and evolution of turbulent motion from metastable states, defect mediated turbulence and localized defects changing chaotically in time and moving randomly in space, spatiotemporal intermittency, chaotic synchronization, anomalous diffusion and directed percolation causing levy-flight spreading processes, turbulent patches and percolation structures, threshold dynamics and avalanches, chaotic itinerancy, or stochastic motion of vortex like objects [10], [11], [29]-[31], [48], [67], [73], [118], [133].

In particular, in the phase space of a complex system various finite dimensional attractors can exist such as: fixed points, limit cycles - torus or more complicated structures as strange attractors. Strange attractors can correspond to chaotic dynamics as self-organized critical dynamics (SOC) or strong chaos. Generally, spatiotemporal chaos includes early turbulence with low effective dimensionality and few coherent spatial patterns or states of fully developed turbulence. These states are out of equilibrium steady states related to bifurcation points of the nonlinear distributed dynamics, as well as to first and second order non-equilibrium phase transition processes. Also, other spatial and temporal patterns as well as spatially localized structures are possible solutions of the nonlinear spatially extended dynamics. With regard the exploration of space plasma complexity by experimental time series analysis, the primary studies by Vassiliadis, Pavlos and other scientists including nonlinear analysis of magnetospheric data [12], [90]-[92], [129], [130], as well as the hypothesis of magnetospheric chaos went through a strong and fruitful criticism [107]-[109]. This criticism was a general dispute about the experimental verification of chaos. In the following we present the main points of the criticism against low dimensional chaos: a) The correlation dimension of experimental time series cannot be distinguished from a stochastic signal with the same power spectrum and amplitude distribution as the original data. b) There is no evidence for the existence of low-dimensionality according to their estimate of correlation dimension obtained using Takens method. c) When there is some evidence of nonlinearity in the experimental time series it is not clear whether it is the result of intrinsic dynamics or nonlinearities in the external pertubations of the system. d) When the system is open and externally largely controlled this alone should provide evidence against the existence of a strange attractor in the observed signals as the system is a randomly driven non-autonomous system.

The refutation of the concept of low dimensional chaos in spatially extended systems as the space plasmas it was further strengthened after the introduction of the concept of self organized criticality (SOC) [10], [25], [64]. Under this circumstantial evidence, Pavlos et al. [95] introduced the term "pseudo–chaos" to discriminate the real low-dimensional chaotic dynamics from the stochastic and nonchaotic colored noise processes. Furthermore, the Thrace group created an extended algorithm for the detection of low dimensional chaos by time series analysis and the discrimination of chaos from stochastic processes which can mimic low dimensional chaos. This algorithm was based on the Wold's decomposition theorem, the theory of input–output dynamics and the embedding theory of Takens, as well as the fruitful contribution of Theiler concerning the method of surrogate data [2]-[7], [44], [95]-[98], [120], [121].

During the last two decades the chaotic algorithm was used to analyse experimental time series corresponding to various physical or technical systems such as: The solar corona and the Earth–Jovian magnetospheres, the solid outer crust of the earth, the atmospheric system, the human brain and the Network–on–chip Architecture of the mobile technology [53]-[55], [60]-[62], [98]-[101], [126]. The far from equilibrium distributed input–output dynamics is the common character in all these distinct cases of physical systems in which the chaotic analysis of experimental time series revealed significant characteristics such as: multiscale and critical dynamics phase transition processes as well as the noticeable coexistence of intermittent turbulence, high dimensional SOC and low dimensional chaotic processes.

After all, a necessity was raised for a theoretical interpretation of the universal and experimentally observed characteristics that lead us to the fundamentals of complexity theory, according to which phase transition processes or intermittent turbulence phenomena and low dimensional self organized chaos are at the edge of macroscopic and microscopic complexity. Moreover, the universality of Tsallis non-extensive statistics, verified at the macroscopic and the microscopic level, indicates also the universality of complexity theory at both macroscopic and microscopic level [122], [123]. After all the obtained until now practical and theoretical experience makes us to believe that the complexity theory could be the prime motivator towards a global theoretical unification of our understanding of nature from the microscopic to the macroscopic level. However, for such a dream to come true, new physical and mathematical concepts must be used such as, cooperation of local, non–local and long–range correlations in complex systems. A new complementarity is developed fundamental local physical interactions and global ordering physical process including fractal space–time, or fuzzy space and non–communicative geometry or wild topologies [15], [36], [38], [46], [50], [84], [87]. In this direction and in order to remark the new state we can summarize by the following phrase of Castro: *It is reasonable to suggest that there must be a deeper organizing principle, from small to large scales, operating in Nature which might be based in the theories of complexity, non linear dynamics and information theory where dimensions, energy and information are intricately connected* [22].

According to the previous description, this study includes two distinct parts. In the first part (sections 2-4), we present the algorithm of chaotic analysis, as well as significant results by using the chaotic algorithm at experimental signals extracted by complex spatially extended systems. The physical presuppositions of the algorithm and significant applications are presented also. In the second part (sections 5-10), we introduce in a synthetic way significant theoretical concepts aiming at the unifying role of complexity theory from the microscopic to the macroscopic physical level.

2 Theoretical Presuppositions and New Tools for the Time Series Analysis

In general, the spatiotemporal dynamics of spatially extended physical systems is related to irreversible or non–equilibrium thermodynamics, as well as to non–equilibrium statistical physics [25], [58], [123]. According to Nicolis and Prigogine, [78]-[83], [110], [111] the existence of low dimensional order (periodic or chaotic) in an extended system is the central character of physical self organization and complexity theory. As we present here, different far from equilibrium spatially extended physical systems reveal self organized complexity and chaotic dynamics or other complex dynamics at the edge of chaos as well as non–equilibrium statistical profile according to Tsallis non–extensive statistical theory [123]. These significant characteristics of complexity theory have been revealed by the application of an extended algorithm of nonlinear time series analysis presented in the following.

The experimental study of spatiotemporal complexity includes the question: *"if you have a time signal, what is the kind of information that you hope to get out of it?"*. The physical systems, which we are interested in, are dissipative and spatially extended. The attempt to understand the complex deterministic motion of spatially extended nonlinear dissipative systems, so–called spatiotemporal chaos (STC), is at the forefront of research in nonlinear dynamics. In contrast to simple chaotic systems in the time domain, in which few degrees of freedom are nonlinearly coupled, spatially extended systems include an infinite number of spatially distributed degrees of freedom. For this reason tools and methods developed for low dimensional systems described by non-linear Ordinary Differential Equations (ODE) must be adapted to spatially distributed systems described by nonlinear Partial Differential Equations. The transition to chaos in spatially extended systems is widely investigated in many natural phenomena such as hydrodynamics or magnetohydrodynamics, chemical reactions, pattern formation in biology or brain activity. Spatiotemporal chaos involves intermediate situations between chaos and turbulence or to fully developed turbulence when the system is sufficiently confined. In these states it is possible to characterize the dynamics from a local time series alone estimating fractal dimensions, or Lyapunov exponents in the reconstructed phase space. When the physical extension of the system increases then quantities, measuring the amount of chaos, scale like the system size. When the system size is much larger than the correlation length then the system can be viewed as a collection of essentially independent sub–systems with a size of the order of correlation length, so that the amount of chaos should be proportional to the number of sub–systems. As long as $x_i > L$, where x_i is the correlation length and L the system size, we are dealing with a small system which may be chaotic in time and coherent in space. In the opposite limit $x_i << L$ the dynamical behavior is incoherent in space. This regime occurs for $L >> l_e - l_D$, where l_D is the dissipative length and l_e the excitation length, and it is the regime of spatial chaos or weak turbulence. Weak turbulence is characterized by the chaotic evolution of coherent structures roughly of the size of the correlation length ξ. The dimension of a local attractor can scale linearly with the system volume Ld, where d is the dimensionality of space while the attractor density D_H/L_d can be well defined. When the spatial structure plays an essential role then, according to Chate, Manneville and Wang [29], [30], [133] the spatiotemporal dynamics of the system can be very complex revealing chaotic synchronization, spatiotemporal intermittency or directed percolation, sporadic chaos, localized structures, and defect turbulence as well as phase transition processes. Moreover, defect turbulence and intermittent turbulence, self organized criticality (SOC), avalanche threshold dynamics, spinodal and nucleation phenomena and far from equilibrium phase transition, Tsallis entropies and non–Gaussian fluctuations as well as diffusion or Levy motion, are some of the different manifestations of spatiotemporal complexity and multiscale–multifractal phenomena that must be studied using nonlinear signal analysis [16], [63], [88], [122].

The chaotic algorithm that can be used to uncover the hidden nonlinear spatiotemporal dynamical characteristics underlying the experimental time series includes different group of tools summarized as follows:

i. Computation Phenomenological Characteristics
   (a) Autocorrelation Coefficient and Power Spectrum (Linear correlations, periodicities, scaling laws
   (b) (Mutual Information (Linear and Nonlinear Correlations)
   (c) Probability Distributions (Power Laws)
   (d) Hurst exponent (Persistence, anti-persistence, white noise)
   (e) Flatness Coefficient F (Intermittent turbulence)
   (f) Structure Functions (Turbulence, anomalous diffusion)
   (g) (Phase portrait (Low Dimensionality)
   (h) Entropy, energy, multifractal structures
   (i) Estimation of q-Tsallis Statistics
   (j) Wavelet analysis (Spatiotemporal structures)

ii. Computation of Geometrical characteristics in the reconstructed state space
   (a) Correlation Dimension (Degrees of freedom)
   (b) Generalized Dimension (Multifractals)
   (c) False Neighbors (Degrees of Freedom)
   (d) Singular values spectrum (SVD components, filtering)

iii. Computation of Dynamical Characteristics in the reconstructed state space
   (a) Maximum Lyapunov Exponent (Sensitivity in initial conditions)
   (b) Power Spectrum of Lyapunov Exponents (Sensitivity in initial conditions in all dimensions in space state)
   (c) Nonlinear modeling and nonlinear prediction algorithms

iv. Testing of Null Hypothesis in order to discriminate between low dimensional chaotic dynamics and linear high dimensional stochastic dynamics
   (a) Surrogate data
   (b) Discriminating statistics

v. Singular Value Analysis in order to
   (a) Estimate Degrees of Freedom
   (b) Filter signals from White or Colored Noise
   (c) Search for input-output dynamics

vi. Recently the above algorithm has been completed by significant new tools such as
   (a) Fuzzy analysis of time series
   (b) Cellular automata, genetic algorithms and neural network modeling (For spatiotemporal modeling and prediction of complexity)

3 Significant Applications of the Chaotic Algorithm

3.1 Solar activity

Solar activity is produced by the emergence of magnetic flux through the photosphere. The magnetic flux forms active regions which include sunspots and solar flares. The physical system underlying the solar activity and the solar cycle is the convection zone of the sun. Convection zone is a strongly turbulent region which occupies the one third of the solar interior. The generation of the magnetic field and its evolution inside the convection zone is one of the most challenging problems for the solar physics, related to the convection zone turbulence, the coronal heating, the solar flares, particle acceleration and transport. The random character of solar activity has been associated theoretically with chaotic behavior and a solar low dimensional strange attractor [106], [113]. For the first time [95] applied chaotic analysis at the sunspot index presenting some evidence for low dimensions solar chaos. Price *et al.* [108] have criticized the chaos hypothesis concerning the solar activity as they reported that applying the chaotic analysis to the wolf sunspot number time series no evidence was found for low dimensional deterministic nonlinear process. Oppositely, the self organized criticality (SOC) theory was introduced for the explanation of solar activity [132]. In a series of studies by Karakatsanis and Pavlos [60], Karakatsanis et al. [61] we have presented strong evidence for the coexistence of two clearly discriminated physical processes in the solar activity. The first process corresponds to the existence of self organized critical state according to the general profile of SOC theory process. The second process corresponds to low dimensional chaotic dynamics. These results were obtained after the nonlinear analysis of the Sunspot Index, according to which, the original signal reveals characteristics of a SOC process, that is, high dimensionality and zero value of the largest Lyapunov exponent. The low dimensional chaotic process was revealed after using high pass filtering of the original Sunspot Index and Solar Flare Index with the methods of first difference or singular value decomposition analysis. The dual character of the solar activity which is hidden in the observed Sunspot Index reveals a double input–output dynamics corresponding to the photospheric and sub-photospheric zones activity of the Solar system. Also, [60], [61] found strong evidence for intermittent solar turbulence as well as for non-extensive statistical processes, according to Tsallis q–statistics.

3.2 Chaos at the Earth and Jovian magnetospheres

The hypothesis of magnetospheric chaos was supported originally by Baker [12], Pavlos [90]-[94] and Vassiliadis [130]. In this direction Tom Chang [24] proposed for the far from equilibrium space plasmas dynamics the generalization of Wilson Renormalization Group theory predicting SOC or chaos states. Strong objection and criticism against the magnetospheric chaos was presented by Price and Prichard [107]-[109]. Pavlos [90], Pavlos et al. [95] paralelly to the nonlinear analysis of magnetospheric signals supported the hypothesis of magnetospheric self organization as basic nonlinear and holistic theory space plasmas, according to [45], [79]. Moreover, the hypothesis of SOC process was strongly supported as a physical explanation of the magnetospheric dynamics, by Klimas et al. [64], Consolini [31], Uritsky [128], Chapman [28] and others. After this in a series of papers we have shown the existence of two distinct magnetospheric dynamical components: one which is low dimensional and chaotic and a second which is high dimensional of SOC type [95]-[97], [102].

In particular we have shown:

- Change of the plasma sheet state from stochastic and high dimensional (SOC state) to low dimensional and chaotic (chaos state) during the development of a superstorm event.
- Strengthen of the intermittent character during the substorm period as well as development of global self-organization and long-range correlation at the plasma sheet.
- Low dimensional self-organization with long range, intermittent and correlated profile can be developed at regions near the magnetopause and the bow shock.
- High dimensional and stochastic processes and intermittent turbulence during quiet periods inside the plasma sheet
- q-Gaussian statistics and strong evidence for the application of non-extensive statistics, according to Tsallis theory to space plasmas

These results confirm the model of phase transition-like behavior of the magnetosphere during substorms introduced by Sitnov et al.[117]. According to this model the magnetospheric dynamics includes multiscale self organized criticality processes corresponding to second-order phase transition, as well as low dimensional and chaotic processes corresponding to first-order phase transition. The dual character of magnetospheric dynamics observed in situ by the space-craft GEOTAIL by Pavlos et al. [101], [104] is in agreement with our previous results [95], [97] and verify the concept of low dimensional chaos at the magnetospheric dynamics, as well as the intermittent turbulence and SOC in accordance with the general theory of Tom Chang [23], [27] concerning the far from equilibrium self-organization of the magnetospheric system and the far from equilibrium renormalization theory for critical dynamics.

3.3 Low dimensional chaotic seismogenesis

The existence of power law distributions led many scientists to explain earthquakes as a Self Organized Critical (SOC) process, according to Bak theory [10] as well as to consider seismicity as the turbulence of the solid earth crust [10], [11], [18], [40], [59]. These concepts showed that earthquakes can be understood via the general theory of statistical physics for dynamical processes of far from equilibrium phase transitions applied to distributed fault‘s systems. The SOC process has already been connected in the past to the unpredictability of earthquakes since the SOC dynamics is related to the edge of chaos phenomenon characterized by strong randomness and high-dimensionality [119] since SOC models have often been considered as alternatives to the low dimensional chaos interpretation of the seismic process [98], [100]. On the other hand, chaos includes low dimensional determinism, in contrast with complete unpredictability or randomness, which is lost over long enough time scales, but long-term, intermediate and short term prediction could be related to a chaotic seismic-cycle process [18], [40]. It was for the first time that Pavlos [96] constructed interevent seismic time series according to the dripping faucet dynamical system for testing the hypothesis of seismic chaos. The dripping faucet model is well known for illustrating the appearance of chaotic behavior in nonlinear systems, where time intervals between successive drop detachments are used to reconstruct the dynamics of the system [115]. At low dripping rates the system is periodic, while above a critical dripping rate the system exhibits chaotic behavior characterized by qualitatively different types of strange attractors. Using interevent time intervals between one drop to the next one, the reconstructed dynamics reveals low dimensional deterministic behavior in the reconstructed state space. According to Iliopoulos et al. [53] and Pavlos et al. [96] concerning a seismic process, the loading rate $m(t)$ of mass in the mechanistic dripping faucet model of Shaw corresponds to the transfer of stress in the fault system by the mantle and plate tectonic dynamics (the external driver of the system), while mass unloading corresponds to earthquakes, as releases of the elastic strain energy stored along a fault. The dripping faucet similarly to the earthquake process can be understood as a local, driven, threshold process. In a series of papers by Iliopoulos et al. [53]-[55] and Pavlos et al. [98] the seismic low dimensional chaos was faithfully supported for the case of the Hellenic seismogenesis. These studies showed clearly the existence of a global seismic strange attractor in the Hellenic region with low dimensionality and strong sensitivity to initial conditions concerning the spatiotemporal distribution of earthquakes. Moreover, the chaotic analysis of seismic time series revealed an independent high dimensional SOC dynamics concerning the energy release process. Moreover, [55] applied nonlinear analysis to various seismic time series indicating local low dimensional temporal chaotic character of the seismic process in the North Aegean area and high dimensional SOC process concerning the bursting seismic energy release. The non-extensive q-statistics of Tsallis was observed also by [55].

3.4 Brain activity during health and seizure state

The human brain can be modelled as a driven nonlinear threshold system including interacting spatial networks of statistically identical, nonlinear units or cells. Each cell fires or falls when the electrical potential or current reaches a threshold value. Numerical simulations of these systems reveal spatial and temporal patterns of firing while the dynamics may also be modified by the presence of noise. The spatiotemporal complexity of brain activity can reveal various dynamical states during health or seizure periods [11], [41], [52], [66], [69], [88], [98], [127], [134] applied chaotic analysis for EEG signals as the brain activity changes from health to epilepsy seizure. In this study, we have shown a phase transition process of brain activity from a high dimensional SOC state during the health period to a gradually developed low dimensional chaotic state. Tsoutsouras *et al.* [126] produced a cellular Automata (CA) model of healthy and epilepsy brain states. In this study, the chaotic clustering of neurons was indicated as a basic mechanism of the phase transition of the brain activity from high-dimensional stochastic dynamics to low-dimensional chaotic dynamics. The CA modeling of brain activity by [126] was found to simulate faithfully the brain dynamics underlying to real data. The brain phase transition process from high-dimensional stochastic to low-dimensional chaotic states supported by CA modeling and by real data chaotic analysis was found to belong to a universal complex spatiotemporal process observed at various physical systems such as the solar activity at the magnetospheric substorms, as well as at the seismic process of the earth lithospheric fault system [54], [60], [61], [101].

3.5 Self Organized Criticality and Chaos at the on-chip workload process

In recent years, the mobile technology was developed so dramatically that has opened new challenges in the embedded system design domain. Different, from the traditional desktop systems, embedded devices demand not only high processor performance but also low power consumption. Thus, the development of proper methods in power consumption management, which will extend the battery life, is without doubt, an imperative need. It was the first time that a SOC–Chaos phase transition process was observed during periods of on-chip workload by [136]. These are results of high significance concerning the design of Network-on-chip Architecture. Dynamic frequency scaling (DFS) is used to adjust the working frequency according to the system workload in order to save the power consumption without degrading the system performance significantly beyond the application tolerance. The main problem of DFS solutions is how to compute the system workload trend. Generally, the workload analysis is an open issue which has occupied the international literature by the beginning of the computer science technology. Recently, [136] used chaotic analysis of workload time series in order to develop novel power aware dynamic frequency scaling technique based on the workload treud of an embedded application. Particular chaotic analysis of workload signals showed the existence of workload critical dynamics and phase transition between states with distinct dynamical dimensionality. This gives the opportunity to handle dynamic data streams with complex behavior. The benefit of this approach based at the real time chaotic analysis of the workload signal is related to the management of the power consumption. The simulation results showed that the methodology based on chaotic analysis can achieve remarkable improvements at the final power consumption. The main idea of our approach is to adjust the processing frequency of our system by analysing the workload fluctuations without degrading the final performance or violating any deadline. The key of our methodology is that we use an abstract model of workload analysis that combines advanced mathematical tools from the Chaos Theory domain. This gives us the opportunity to handle dynamic data streams with complex behavior. The benefit of our approach is that it is a system-level platform independent technique, which permits us, through the analysis and the prediction of the workload trend, to manage the power consumption of an embedded system calibrating only the processing frequency. The simulation results showed that our methodology can achieve remarkable improvements at the final power consumption, which range between 17% and 38% depending on the restriction of the application deadlines. These results are complementary to previous analysis of system performance degray taking into account the interaction between workload fluctuations and the nonlinearity of the system. Moreover, the phase transition process described by Zompakis [136] can be used for proposing new adaptive techniques related with dynamics power management.

4 Theoretical Documentation of the Results Obtained by Chaotic Analysis

Until now, Chaos or SOC states, intermittent turbulence including multifractal and multiscale characteristics, Tsallis q-statistics, as well as out of equilibrium phase transition processes, were found to be of universal character at distinct physical or technological systems. Also, all the previous systems at which the chaotic analysis was applied belong to the general type of distributed and far from equilibrium input-output threshold nonlinear dynamics. For these systems the general theory of far from equilibrium non linear stochastic dynamics described by generalized Langeving equations can be applied [47]. According to Chang [23]-[25] the stochastic Lagrangian methodology for the solution of the stochastic Langevin equations and the far from equilibrium renormalization group theory can be used for the estimation of the fixed points of the dynamics. From this point of view, various macroscopic states and process can be related to different fixed points in the affine space of the stochastic Lagrangian dynamics. In general, far from equilibrium nonlinear stochastic dynamics can be described by a set of generalized Langevin equations [25]:

$$\frac{\partial \phi_i}{\partial t} = f_i(\phi, x, t) + n_i(x, t) \tag{1}$$

where $f_i(i = 1, 2, \ldots)$ are nonrandom forces corresponding to the functional derivative of the free energy functional, $x_\mu(\mu = 1, \ldots, d)$ are the spatial coordinates, t is the time and $\phi(x, t)$ represents the stochastic variables which describe the fault dynamics and $n_i(x, t)$ are random force fields or noises. According to Chang [24] the behavior of a nonlinear stochastic system far from equilibrium can be described by the density functional P, defined by path integral formulation:

$$P\phi(x, t) = \int D(x) \exp^{-i \int L(\dot{\phi}, \phi, x) dx dt} \tag{2}$$

where $L(\dot{\phi}, \phi, x)$ being the stochastic Lagrangian of the system, which describes the full dynamics of the stochastic system. Moreover, the far from equilibrium renormalization group theory applied to the stochastic Langrangian L gives the singular points (fixed points) in the affine space of the stochastic distributed system. At the fixed points the system reveals the character of criticality, as near criticality the correlations among the fluctuations of the random dynamic field are extremely long-ranged and there exist many correlation scales. Also, close to dynamic criticality certain linear combinations of the parameters, characterizing the stochastic Lagrangian of the system, correlate with each other in the form of power laws and the stochastic system can be described by a small number of relevant parameters characterizing the truncated system of equations with low or high dimensionality. According to these theoretical results, the stochastic system can exhibit low dimensional chaos or high dimensional SOC like behavior, including spatiotemporal fractal structures with power law profiles. The power laws are connected to the near criticality phase transition process which creates spatial and temporal correlations as well as strong or weak reduction (self-organization) of the infinite dimensionality corresponding to a spatially distributed system. According to Lyra and Tsallis [71], the power laws are not caused by the SOC process, but by the nonextensive statistics observed at far from equilibrium process with long range correlations. From this point of view, a SOC or low dimensional chaos interpretation depends upon the kind of the critical fixed (singular) point in the functional solution space of the system. When the stochastic system is externally driven or perturbed, it can be moved from a particular state of criticality to another characterized by a different fixed point and different dimensionality or scaling laws. Thus, SOC theory could be a special type of critical dynamics of an externally driven stochastic system [70]. Furthermore, according to Chang [26], Chang et al. [27] as well as Vieira [131], SOC and low dimensional chaos can coexist in the same dynamical system as a process manifested by different kinds of fixed (critical) points in its solution space. As the dynamical system evolves in time (autonomously or under external forcing), the state of the system described by the values of the dynamical parameters in the stochastic Lagrangian L, changes as well. The change of the critical state of the system can reveal different dynamical scenarios, as it evolves from one critical state to another, after external tuning. Also, it is possible to reveal local instabilities by creating metastable states which evolve to states of lower energy. This is a local symmetry breaking phenomenon and leads to a local phase transition process. Such local instabilities are connected to avalanche or nucleation dynamics, which can be present in systems that are at mean-field or near-mean-field state, with the possibility of spinodal decomposition process [63], [114]. Moreover, the theory of far from equilibrium critical dynamics can be related to the equilibrium phase transition theory, as both include local metastable states characterized by spinodal lines and spinodal phase transitions.

5 Is Complexity the Road for the Final Unification of the Physical Theory? New Concepts for an Old Problem.

The old problem in physics is the unification of mechanics and thermodynamics. For Albert Einstein, physical theory must correspond objectively to the physical reality. Thus, the hard core of a final physical theory must be the mathematical determinism. For Heisenberg, Bohr and Born, physical theory has to be related to the phenomenology of reality. Because of this, probabilism is the unavoidable hard core of every physical theory. On the other hand, the hard core of complexity is nonlinearity. Nonlinear dynamics includes determinism in the form of periodic attractors with integer dimension as well as probabilism in the form of chaotic fractal attractors. From this point of view we understand that there exists an internal connection between nonlinear dynamics and thermodynamics since entropy is a propabilistic property of a dynamical system. Nonlinear deterministic dynamics including strong irregularity of chaotic mixing or $f*$ exact type, includes thermodynamical characteristics [72]. Such a type of strongly nonlinear unstable determinism is equivalent also to stochastic or probabilistic dynamics [74], [83], [111].

Moreover determinism and probabilism can be interrelated also through Rochlin theorem [112] according to which every $f*$ exact transformation is the factor of a k automorphism. In this way Boltzmann‘s (stochastic) dynamics can be proved to be the trace of Liouville‘s (deterministic) dynamics or the Boltzmann‘s equation is the trace of Liouville equation [72]. In the same direction quantum probabilism can be related to classical determinism and dissipation according to [32], [49]. Another significant character of nonlinear deterministic dynamics is the bifurcation profile of solutions and the critical point dynamics (near or far from equilibrium) as the pattern formation or morphogenesis character [47]. According to Wilson [135] the critical physical theory related to the dynamics must take into account the entire spectrum of length scales from 10^{-8} cm to 10^{11} cm. However, more than this it seems that complexity and nonlinear dynamics includes the theoretical kernel for a global unification of physical theory, as we argue in a previous study by Pavlos *et al.* [101]. Critical dynamics showed the significance of renormalization group theory, the scale invariance principle related to fractal geometry, as well as the relation of dynamics with topological and dimensional characteristics of space. However, the deep question of complexity theory is how forms are generated in nature and what is the relationship between physical forces and the stable geometries of morphogenesis and pattern formation. According to Kovacs [67] somehow space-time symmetries create the fundamental laws of nature. Moreover, it could be possible to imagine some extended kind of geometrical invariance principle and novel topological characteristics to produce every kind of pattern, according to the dream of Einstein, concerning the geometrical unification of physical theory and physical dynamics. According to El Naschie [36], [39] and Ord [87] noncommutative geometries and fractal space-time can be used for the extension of complexity from the macroscopic to the microscopic level. Scale invariance principles also can be used for the unification of microscopic and macroscopic complexity. Some efficient evidence for such a dream we present indicatively in the following.

6 Complexity as a New Physical Theory

Is Complexity a major revolution in science such as Relativity and Quantum Theory? For many scientists attempts to explain complexity and self-organization by using the basic laws of physics have met with little success. Novel forms of self-organization are generally unexpected for the classic reductionistic point of view. However, while complexity is considered as a new and independent physical theory which was developed after the Relativity Theory and Quantum mechanics, it must be consistent with these theories. It is related to far from equilibrium dynamics and concerns the creation and destruction of spatiotemporal patterns, forms and structures. According to Balescu [8], Nicolis and Prigogine [78], [79], Nicolis [80], [81], and Prigogine [110], [111], complexity theory corresponds to the flow and development of space-time correlations instead of the fundamental local interactions. According to the classical point of view the physical phenomena (macroscopic or microscopic) must be reducible to a few fundamental interactions. However, according to Nicolis [82], since 1960 an increasing amount of experimental data challenging this idea has become available. This impose a new attitude concerning the description of Nature. Moreover, according to Sornette [118], systems with a large number of mutually interacting parts and open to their environment can self-organize their internal structure and their dynamics with novel and sometimes surprising macroscopic emergent properties. These general characteristics make the complexity theory, a fundamentally probabilistic theory of the non-equilibrium dynamics.

7 Complexity as a Form of Macroscopic Quanticity

The central point of complexity theory is the possibility for a physical system, which includes a great number of parts or elements, to develop internal long-range correlations leading to macroscopic ordering and coherent patterns. These long-range correlations can also appear at the quantum level. In particular, according to the general entanglement character of the Quantum theory, the quantum mechanical states of a system with two or more parts cannot be expressed as the conjunction of quantum states of the separate parts. This situation generally reflects the existence of non-local interactions and quantum correlations while the measurements bearing on either part correspond to random variables which are not independent and can be correlated independently with the spatial distance of the parts [116]. This means that the quantum density operator cannot be factored while the quantum state corresponds to the global and undivided system. The macroscopic manifestation of the quantum possibility for the development of long-range correlations is the spontaneous appearance of ordered behavior in a macroscopic system, examples of which are phenomena like superfluidity and superconductivity or lasers [68].

These quantum phenomena display coherent behavior involving the collective cooperation of a huge number of particles or simple elements and a vast number of degrees of freedom. They correspond also to equilibrium or nonequilibrium phase transition processes which constitute the meeting point of quantum theory and complexity. Here the development of quantum long-range correlations leads to a macroscopic phase transition process and macroscopic ordering. It is not out of logic or physical reality to extend the (unifying) possibility of quantum process to developed long-range correlations, according to the quantum entanglement character, into a macroscopic self-organizing factor causing also the far-from equilibrium symmetry breaking and macroscopic pattern formation. From this point of view we can characterize complexity as a form of a macroscopic quanticity [34], [135].

8 Quantum Theory as a Form of Microscopic Chaoticity and Complexity

Bohm and Hiley [21] imagined that the Quantum theory must be the manifestation of subquantum complex dynamics. During the last years, we observe the productive onset or the impetus invasion of chaos and complexity from macroscopic to the microscopic quantum level. In the following we present some novel concepts in this direction:

- Determinism at the Planck‘s scale [49]-[51].
- Order and deterministic chaos at the quantum level [32]
- Analytical continuation and fractal space-time can convert an ordinary diffusion equation into a Schrodinger‘s equation and a telegraph equation into a Dirac’s equation. From this point of view analytical continuation is a short cut of quantization [76], [77].
- Positive Lyapunov exponents of non-abelian gauge fields reveal the significance of chaos for the quantum field theory [20].
- Coupled map lattices with spatiotemporal chaotic profile can be used to simulate quantum field theories in an appropriate scaling limit [17], [22].
- Kaneko‘s coupled map lattices including chaotic strings provide the background for the Parisi-Wu stochastic quantization of ordinary string and quantum field theories [14], [15]. Chaotic strings can be used also to provide a theoretical argument why certain standard model parameters are realized in nature reproducing numerical values of the electroweak and strong coupling constants masses of the known quarks and leptons neutrino, W boson and Higg’s mass.
- Renormalization group (RG) flows on the superstring world sheet becomes chaotic and leads to non-Markovian Fokker-Planck equation with solutions describing the transition from order to chaos and revealing the Feigenbaum universal constant [65]. The appearance of this constant reveals the scaling of space-time curvatures at the fixed points of the RG flow which becomes chaotic near singularities where the curvature is very large [75], [105].
- The Parisi-Wu [89] stochastic quantization theory relates the quantum field theory in D-dimensions to a classical Langevin equation in $D{+}1$-dimensions where the Parisi-Wu fictitious time plays the role of an extra dimension. In this picture there exists a short of classical stochasticity and quantum theory duality [33]. The stochastic quantization can be transformed to chaotic quantization, similar to chaotic deterministic dynamical systems

which can generate Langevin dynamics in an appropriate scaling limit. In this approach, quantum field theories can be simulated by chaotic dynamics

- Non-extensive statistics [123], fractal string and branes, fractal statistics, fractons and anyons particles as well as chaotic M(atrix) theory indicate the establishment of chaos and complexity at the microscopic and the quantum level [22]. In this direction, Gerardus 't Hooft raised the conjecture that quantum theory can be derived as the low-energy limit of a microscopically deterministic but dissipative theory [50]. According to this concept classical Perron-Frobenius operators or deterministic automata can produce quantum states in Hilbert spaces as well as the Schrodinger equation [1], [9], [35], [42].

After all, we can imagine that quantum states are produced by a sub-quantum self-organized process. This sub-quantum self-organization must concern information process rather than simple material or energy self-organization.

9 Universality of Tsallis non-extensive statistical mechanics

According to Tsallis, Boltzmann-Gibbs statistical mechanics and standard thermodynamics do not seem to be universal. Tsallis extended the Boltzmann-Gibbs statistics and Boltzmann-Gibbs entropy to non-extensive statistical mechanics and non-extensive q-entropies. The classical Boltzmann-Gibbs extensive thermostatistics constitutes a powerful tool when microscopic interactions and memory are short ranged and space-time is a continuous and differentiable Euclidean manifold. However, far from equilibrium these characteristics are changed as multiscale coupling and non-locality characteristics can appears. In turbulence for example, the presence of long-range correlations imply non-local interactions between large and small scales as the relation between them is not local in space and time but functional. This indicates that small-scale fluctuations in each time space point depend on the large scale motions in the whole time-space domain and vice versa. Generally, the non-extensive statistical mechanics introduced by Tsallis rather than being just a theoretical construction it is relevant to many complex systems at the macroscopic or the microscopic level with long-range correlations-interactions or multifractal behavior. A crucial property of Tsallis entropy S_q is the pseudo-additivity for given subsystems A and B in the sense of factorizability of the microstate according to the relations $S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$ where $S_q = k_B \frac{(1-\sum p_i^q}{q-1}$ and k_B is Boltzmann‘s constant. The non-local coupling and long-range correlations of complex dynamics corresponds to the multiplicative tern of the previous relation. Also, the non-extensive behavior of macroscopic or microscopic complexity is related to the non-Euclidean and multi-fractal space time [22], [46], [58], the quantum gravity and quantum entanglement [122].

10 The road of Complexity for the Physical Theory Unification

According to previously presented concepts and descriptions, about the nonequilibrium statistical mechanics, the Feynman rules and diagrams become common tool from the estimation of probabilistic processes at the microscopic quantum level or the macroscopic level of continuous media as they are being described by the Ginzburg-Landau model [13], [19], [47]. In this direction, we could imagine Feynman rules and renormalization group theory as the universal characteristics of probabilistic processes at the microscopic and the macroscopic level. The renormalization group equations have many common features with non-linear dynamical systems, so that apart from existence of isolated fixed points, the coupling in a renormalizable field theory may flow also towards more general even fractal attractors. This could lead to Big Mess scenarios in application to multiphase systems, from spin-glasses and neural networks to fundamental string theory [75]. In this direction Cristopher Hill [46] introduced the fractal theory space where the key idea is that the Feynman path integral is invariant under a sequence of renormalization group transformations that map the kth lattice into the $k-1$ lattice. In the continuum limit these models produced quantum field theories in fractal dimensions D. These theories are connected to the scaling behavior of fractal strings (branes), while the couplings oscillate on a limit cycle. Moreover the concept of fractal space-time can be used for the foundation of an extended Einsteins Relativity Principle unifying the micro and macro levels [84], [85].

In this direction, [87] showed that fractal trajectories in space with Hansdorff dimension $D = 2$ exhibit both an uncertainty principle and a De Broglie wave - particle duality relations. Furthermore, Nottale [84], [85] introduced the principle of Scale Relativity according to which the laws of physics are scale invariant. This theory is related also to the concept of fractal space-time. According to Nottale, the consequence of scale invariance principle and space-time fractality opens the door for a grand unification of cosmos, from the microscopic quantum level to the macroscopic and cosmological level. The starting point of the theory is the refusing of the unjustified assumption of the differentiability of the space-time continuum. The non-differentiable space-time continuum is necessarily fractal. The development of the theory starts by making the various physical quantities explicitly dependent on the space-time scale while the fundamental laws become also scale dependent. In this frame of theory the non-differentiability of space-time implies the breaking of time reversibility, and the global unification of microscopical and macroscopical laws, [37], [56], [57], [86], [103].

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