Analysis of dynamical states of cosmogonical body formation based on the generalized nonlinear Schrödinger-like equation

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Abstract: This work investigates the different dynamical states of cosmogonical body formation using the generalized nonlinear Schrödinger-like equation. In particular, the cubic time-dependent Schrödinger-like equation describing of cosmogonical body forming in the state of soliton disturbances is derived. The soliton solution of the cubic generalized Schrödinger-like equation of a forming spheroidal body is considered. The reduced model of dynamical system into state-space of the cubic nonlinear Schrödinger equation is obtained.

Keywords: molecular clouds; initial oscillating interactions; spheroidal bodies; gravitational condensation; generalized nonlinear Schrödinger–like equation; cubic Schrödinger-like equation; soliton solution; reduced model.

1 Introduction

This work investigates different dynamical states of cosmogonical body formation using the generalized nonlinear Schrödinger-like equation obtained within the framework of statistical theory of gravitating spheroidal bodies. The statistical theory for a forming cosmogonical body (based on the model of so-called spheroidal body) has been proposed in our previous works [1-7]. As shown within the framework of this theory, interactions of oscillating particles inside a spheroidal cloud lead to a gravitational condensation increasing with the time. As a result, the generalized nonlinear time-dependent Schrödinger-like equation describing a gravitational formation of a spheroidal body has been derived [3, 4, 7]. As shown, this equation is found more general than analogous equations obtained in Nelson' stochastic mechanics [8] and Nottale's scale relativistic theory [9-11].

This work considers different dynamical states of a gravitating spheroidal body and respective forms of the generalized nonlinear time-dependent Schrödinger-like equation. In particular, the derived time-dependent generalized nonlinear Schrödinger-like equation describes not only the state of virial mechanical equilibrium and the quasi-equilibrium gravitational condensation state, but the

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initial equilibrium gravitational condensation state taking place in a forming gas-dust protoplanetary cloud as well as the soliton disturbance state and other gravitational instability states including the formation of the core of a cosmogonical body. Besides, the last case involves the avalanche gravitational compression increasing (when the parameter of gravitational condensation grows exponentially with the time), i.e. the case of unlimited gravitational compression leading to the collapse of a cosmogonical body.

In this paper, the cubic time-dependent Schrödinger-like equation describing formation of a cosmogonical body in the state of soliton disturbances has been derived. Then a reduced model into the state-space for the cubic Schrödinger-like equation is obtained. We show that the proposed model is represented by the system of four ordinary nonlinear differential equations with quadratic nonlinearity. We also note that the obtained attractor can demonstrate the complex dynamics into the state-space like the Lorenz one [12] or the similar attractor describing flows with the curvature of streamlines [13].

2 Some particular cases of the generalized nonlinear Schrödinger-like equation describing different dynamical states of a gravitating spheroidal body and the characterizing number K as a control parameter of these states

In the works [3, 4], the generalized nonlinear time-dependent Schrödinger-like equation describing a gravitational formation of a cosmogonical body has been derived:

$$\mathrm{i}\,2\mathrm{G}(t)\frac{\partial\Psi}{\partial t} = \left[-2\mathrm{G}^2(t)\nabla^2 + \varphi_g\right]\Psi + 2\frac{d\,\mathrm{G}(t)}{dt}\left[\ln|\Psi| + \arg\Psi\right]\Psi\,, \quad (1)$$

where Ψ is a wave function, φ_g is a gravitational field potential, $\mathrm{G}(t)$ is an antidiffusion function, i.e. the generalized *gravitational compression function* (GCF)

$$\mathrm{G}(t) = \frac{1}{2\alpha^2}\cdot\frac{d\alpha}{dt} \quad (2)$$

and $\alpha = \alpha(t)$ is a parameter of gravitational condensation of a spheroid-like gaseous cloud or, simply say, *spheroidal body* [1-6], $\mathrm{i} = \sqrt{-1}$.

Let us consider different dynamical states of a gravitating spheroidal body as well as the respective forms of the generalized nonlinear time-dependent Schrödinger-like equation (1). Indeed, this equation describes not only the state of virial *mechanical equilibrium* [1-4] when GCF $\mathrm{G}(t) = \mathrm{G}_s = \mathrm{const} \in \mathbf{R}$ (or $\mathrm{G}_s \in \mathbf{C}$) and $\Psi \in \mathbf{R}$ (or $\Psi \in \mathbf{C}$):

$$\mathrm{i}\,\mathrm{G}_s \frac{\partial \Psi}{\partial t} = \left[-\mathrm{G}_s{}^2 \nabla^2 + \frac{1}{2}\varphi_g \right] \Psi \tag{3}$$

and the *quasi-equilibrium* gravitational condensation state [3, 4] with a slowly (periodically) varying GCF increment when $\mathrm{G}(t) = \mathrm{G}_s[1 + \varepsilon \cos \omega t] \in \mathbf{R}$ (or $\mathrm{G}(t) \in \mathbf{C}$) and $\Psi \in \mathbf{C}$:

$$\mathrm{i}\,\mathrm{G}(t) \frac{\partial \Psi}{\partial t} = \left[-\mathrm{G}^2(t) \nabla^2 + \frac{1}{2}\varphi_g \right] \Psi + \frac{d\,\mathrm{G}(t)}{dt} \ln|\Psi| \cdot \Psi\,, \tag{4}$$

but also the *initial* equilibrium gravitational condensation state [1-4] occurring in a forming gas-dust protoplanetary cloud:

$$\mathrm{i} \frac{\partial \Psi}{\partial t} = -\mathrm{G}_s \nabla^2 \Psi \tag{5}$$

as well as the *soliton disturbances state* arising in a spheroidal body under formation [7]:

$$\mathrm{i} \frac{\partial \Psi}{\partial t} = \left[-\mathrm{G}(t) \nabla^2 + \frac{1}{2} \frac{d \ln \mathrm{G}(t)}{dt} |\Psi|^2 \right] \Psi \tag{6}$$

and the *gravitational instability states* [4,7] when GCF $\mathrm{G}(t) \in \mathbf{C}$ and $\Psi = |\Psi| e^{\mathrm{i} \arg \Psi} \in \mathbf{C}$:

$$\mathrm{i}\,\mathrm{G}(t) \frac{\partial \Psi}{\partial t} = \left[-\mathrm{G}^2(t) \nabla^2 + \frac{1}{2}\varphi_g \right] \Psi + \frac{d\,\mathrm{G}(t)}{dt} \left[\ln|\Psi| + \arg \Psi \right] \Psi\,, \tag{7}$$

including the increase of gravitational compression of spheroidal body providing the formation of a core of a cosmogonical body if $0 \le \arg \Psi < 2\pi$ (the case of unlimited gravitational compression leading to a collapse occurs when $\arg \Psi \to \arg \Psi \pm 2\pi n,\ n \in \mathbf{Z}$).

Let us note that the generalized nonlinear time-dependent Schrödinger-like equation (1) has been derived in [3, 4, 7] from the following equations:

$$\frac{\partial \vec{\mathrm{u}}}{\partial t} = -\mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\,\vec{\mathrm{v}}) - \mathrm{grad}(\vec{\mathrm{v}} \cdot \vec{\mathrm{u}}) + \frac{d \ln \mathrm{G}(t)}{dt} \vec{\mathrm{u}}\,; \tag{8a}$$

$$\frac{\partial \vec{\mathrm{v}}}{\partial t} = \vec{a} - (\vec{\mathrm{v}} \cdot \nabla)\,\vec{\mathrm{v}} + \mathrm{grad}(\vec{\mathrm{u}}^2/2) + \mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\,\vec{\mathrm{u}}) - \frac{d \ln \mathrm{G}(t)}{dt} \vec{\mathrm{u}}\,, \tag{8b}$$

where $\vec{u}$ is a the *antidiffusion velocity* (unlike of the ordinary hydrodynamic velocity $\vec{v}$) for a rotating spheroidal body [2, 4]:

$$\vec{u} = G(t)\frac{\nabla\rho}{\rho}, \tag{9}$$

where $\rho(r) = \rho_0 e^{-\alpha r^2/2}$ is a mass density, $\rho_0 = M(\alpha/2\pi)^{3/2}$, M is a mass of spheroidal body [1-5].

Since the generalized nonlinear Schrödinger-like equation (1) describes different dynamical states of a gravitating spheroidal body then we can carry out an analysis of dynamical states of a spheroidal body based on initial Eqs. (8a), (8b) introducing the scales of physical values T, L, V, U, F, G_s and the respective dimensionless variables $\tau, \vec{\xi}, \vec{v}, \vec{u}, \vec{f}, g$ as follows:

$$t = T\tau; \quad \vec{r} = L\vec{\xi}; \quad \vec{v} = V\vec{v}; \quad \vec{u} = U\vec{u}; \quad \vec{a} = F\vec{f}; \quad G(t) = G_s g(t). \tag{10}$$

By substituting Eqs. (10) in Eqs. (8a, b) we obtain:

$$\frac{U}{T}\frac{\partial\vec{u}}{\partial\tau} = -G_s g(\tau)\frac{V}{L^2}\,\mathrm{grad}(\mathrm{div}\vec{v}) - \frac{VU}{L}\,\mathrm{grad}(\vec{v}\vec{u}) + \frac{U}{T}\frac{d\ln g(\tau)}{d\tau}\vec{u}\,; \tag{11a}$$

$$\frac{V}{T}\frac{\partial\vec{v}}{\partial\tau} = F\vec{f} - \frac{V^2}{L}(\vec{v}\cdot\nabla)\vec{v} + \frac{U^2}{L}\,\mathrm{grad}(\vec{u}^2/2) + G_s g(\tau)\frac{U}{L^2}\,\mathrm{grad}(\mathrm{div}\vec{u}) - \frac{U}{T}\frac{d\ln G(\tau)}{d\tau}\vec{u} \tag{11b}$$

Similarly to [14], dividing Eq. (11a) by VU/L and Eq. (11b) by V^2/L we derive the following dimensionless equations [7]:

$$\mathrm{Sh}\,\frac{\partial\vec{u}}{\partial\tau} = -\frac{G_s}{\nu}\cdot\frac{g(\tau)}{K\cdot \mathrm{Re}}\,\mathrm{grad}(\mathrm{div}\vec{v}) - \mathrm{grad}(\vec{v}\vec{u}) + \mathrm{Sh}\,\frac{d\ln g(\tau)}{d\tau}\vec{u}\,; \tag{12a}$$

$$\mathrm{Sh}\,\frac{\partial\vec{v}}{\partial\tau} = \frac{1}{\mathrm{Fr}}\vec{f} - (\vec{v}\cdot\nabla)\vec{v} + K^2\mathrm{grad}(\vec{u}^2/2) + \frac{G_s g(\tau)}{\nu}\cdot\frac{K}{\mathrm{Re}}\,g(\tau)\,\mathrm{grad}(\mathrm{div}\vec{u}) - \mathrm{Sh}\cdot K\,\frac{d\ln g(\tau)}{d\tau}\vec{u} \tag{12b}$$

where $\mathrm{Sh} = L/VT$ is the Strouhal number, $\mathrm{Fr} = V^2/FL$ is the Froude number, $\mathrm{Re} = VL/\nu$ is the Reynolds number (ν is a kinematic coefficient of viscosity of flow of particles [14]), $K = U/V$ is a new number of similarity.

The new number of similarity is a measure of the values $|\vec{u}|$ versus $|\vec{v}|$ prevailing [7]:

$$K = \frac{|\vec{u}|}{|\vec{v}|}. \tag{13}$$

When this similarity number exceeds unity ($K >> 1$) then the antidiffusion condensation of a spheroidal body occurs exclusively, so that the value of hydrodynamic velocity is negligible $(|\vec{v}| \approx 0)$ because a gravitational field is

absent practically. If the similarity number becomes close to unity ($\mathrm{K} \approx 1$) then the hydrodynamic velocity $\vec{\mathrm{v}}$ of mass flow arises as a result of a gravitational contraction of a spheroidal body on the field stage of its evolution. As noted in [7], the value of antidiffusion velocity (9) becomes much less than the value of hydrodynamic velocity $|\vec{\mathrm{u}}| << |\vec{\mathrm{v}}|$ when $\mathrm{K} << 1$. This means that the growing magnitude of powerful gravitational field strength $\vec{a}$ induces the significant value of hydrodynamic velocity $\vec{\mathbf{v}}$ of mass flows moving into a spheroidal body. Thus, like the Mach number M [14] the new number of similarity K is a *control parameter* of dynamical states of a forming spheroidal body.
In particular, in the special case $\mathrm{K} >> 1$ corresponding to the generalized nonlinear Schrödinger-like equation for the initial equilibrium gravitational condensation state (5) two dimensionless Eqs. (12a, b) are reduced to one dimensionless equation of the kind:

$$\mathrm{K}\,\mathrm{grad}(\vec{u}^2/2) + \frac{\mathrm{G}_s}{\nu}\cdot\frac{1}{\mathrm{Re}}\,g(t)\,\mathrm{grad}(\mathrm{div}\vec{u}) = \mathrm{Sh}\cdot\frac{\partial\vec{u}}{\partial\tau}\,, \tag{14}$$

which corresponds the following equation [7]:

$$\mathrm{grad}(\vec{\mathrm{u}}^2/2) + \mathrm{G}(t)\,\mathrm{grad}(\mathrm{div}\vec{\mathrm{u}}) = \frac{\partial\,\vec{\mathrm{u}}}{\partial t}\,. \tag{15}$$

Except the antidiffusion solution, the equation (15) has a wave solution in the vicinity of equilibrium state when $\mathrm{G}_s = \mathrm{const}$ and $|\vec{\mathrm{u}}| < 1$:

$$\vec{\mathrm{u}} = \vec{\mathrm{u}}_0\, e^{\pm \mathrm{i}\vec{k}\vec{r} - \vec{k}^2 \mathrm{G}_s t}\,. \tag{16a}$$

Moreover, if we determine $\mathrm{G}(t) = \mathrm{i}\,\mathrm{G}_s$, solution (16a) becomes a *wave solution* of the kind :

$$\vec{\mathrm{u}} = \vec{\mathrm{u}}_0\, e^{\pm \mathrm{i}(\vec{k}\vec{r} - \vec{k}^2 \mathrm{G}_s t)}\,. \tag{16b}$$

In the initial equilibrium gravitational condensation state, the wave solutions (16a) and (16b) are generated respectively, moreover, they induce specific additional periodic forces and spatial oscillations (like the radial and the axial oscillations of Alfvén–Arrhenius [15, 16]) in the different domains of a forming spheroidal body.

3 The investigation of wave solutions of the generalized nonlinear Schrödinger-like equations of a forming cosmogonical body

Now let us consider some wave solutions of the generalized nonlinear Schrödinger-like equation taking into account its important particular cases (5) and (6).

The *initial equilibrium* gravitational condensation state is realized in a forming gas-dust protoplanetary cloud when the initial gravitational field φ_g is absent

($\varphi_g = 0$) and $\mathrm{G}(t) = \mathrm{G}_s = \mathrm{const}$, so that the generalized nonlinear Schrödinger-like equation (1) becomes the *linearized* Schrödinger equation (5). Like (16b) we are seeking a wave solution in the vicinity of the equilibrium state when $\mathrm{G}_s = \mathrm{const}$:

$$\Psi(\vec{r},t) = \Psi_0 e^{-\mathrm{i}(\omega_s t \pm \vec{k}\vec{r} + \varepsilon^0)},\ \mathrm{i} = \sqrt{-1}.$$

Indeed, let us calculate the derivatives of Ψ with respect to the spatial coordinate $\vec{r}$ and the time t:

$$\frac{\partial \Psi}{\partial \vec{r}} = -\mathrm{i}(\pm k)\Psi_0 e^{-\mathrm{i}(\omega_s t \pm \vec{k}\vec{r} + \varepsilon^0)} = \mp \mathrm{i}\, k\Psi; \qquad \frac{\partial^2 \Psi}{\partial \vec{r}^2} = -(\mp k)^2 \Psi_0 e^{-\mathrm{i}(\omega_s t \pm \vec{k}\vec{r} + \varepsilon^0)} = -k^2\Psi;$$

$$\frac{\partial \Psi}{\partial t} = -\mathrm{i}\,\omega_s \Psi_0 e^{-\mathrm{i}(\omega_s t \pm \vec{k}\vec{r} + \varepsilon^0)} = -\mathrm{i}\,\omega_s \Psi.$$

Comparing the two last relations we can see that

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = -\frac{\omega_s}{k^2}\frac{\partial^2 \Psi}{\partial \vec{r}^2}.$$

The obtained equation coincides with Eq. (5), therefore $\omega_s = \mathrm{G}_s k^2$. So, the wave solution of Eq. (5) can be rewritten in the following form:

$$\Psi(\vec{r},t) = \Psi_0 e^{-\mathrm{i}(\mathrm{G}_s k^2 t \pm \vec{k}\vec{r} + \varepsilon^0)}. \tag{17}$$

As mentioned relative to (16b), the analogous wave solution occurs for the antidiffusion velocity $\vec{\tilde{\mathrm{u}}} = \vec{\mathrm{u}}_0\, e^{-\mathrm{i}(\pm\vec{k}\vec{r} + \vec{k}^2 \mathrm{G}_s t + \varepsilon^0_{\mathrm{u}})}$.

Now we are going to investigate *nonlinear wave solutions* of the generalized nonlinear Schrödinger-like equation of a cosmogonical body formation. As noted in [4], as a result of the formation of a core of cosmogonical body (based on a model of a spheroidal body) from an initial weakly condensed molecular cloud, a *sharp increase* in the antidiffusion velocity of particles inside the cloud is highly likely leading to the gravitational field origin, subsequently. In this connection, we consider a possible scenario of transition from solutions in the form of plane waves (17) to *nonlinear wave solutions* of the generalized nonlinear Schrödinger-like equation (1) in the case $\mathrm{K} >> 1$.

In other words, let us pass from equation (5) to a more general equation with a time-varying function of gravitational compression $\mathrm{G}(t)$ under condition that the absolute value of the wave function is small, in other words, $|\Psi(t)| < 1$. To this end, we use the generalized nonlinear Schrödinger-like equation (1) of a spheroidal body formation from a molecular cloud in a state of gravitational instability but under the condition of smallness of the initial gravitational field φ_g and the absolute value $|\Psi(t)|$ respectively:

$$\mathrm{i}\,2\mathrm{G}(t)\frac{\partial \Psi}{\partial t} = \left[-2\mathrm{G}^2(t)\nabla^2 + \varphi_g\right]\Psi + \frac{d\,\mathrm{G}(t)}{dt}\left[\ln|\Psi|^2 + 2\arg\Psi\right]\Psi. \tag{18a}$$

In this case, the logarithmic function in the right-hand side of Eq. (18a) can be decomposed into a Taylor series and restricted to the first term of smallness:

$$\ln|\Psi|^2 = \ln(1+[|\Psi|^2 - 1]) = [|\Psi|^2 - 1] - [|\Psi|^2 - 1]^2/2 + \ldots \approx |\Psi|^2 - 1 . \quad (18b)$$

With regard to (18b), the equation (18a) takes the form:

$$\mathrm{i}\, 2\mathrm{G}(t)\frac{\partial \Psi}{\partial t} = -2\mathrm{G}^2(t)\nabla^2\,\Psi + \frac{d\,\mathrm{G}(t)}{dt}|\Psi|^2\,\Psi + \left[\varphi_g + \frac{d\,\mathrm{G}(t)}{dt}(2\arg\Psi - 1)\right]\Psi . \quad (19)$$

Dividing both sides of Eq.(19) by $2\mathrm{G}(t)$ we obtain:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = -\mathrm{G}(t)\nabla^2\,\Psi + \frac{\dot{\mathrm{G}}(t)}{2\mathrm{G}(t)}|\Psi|^2\,\Psi + \frac{1}{2\mathrm{G}(t)}\left[\varphi_g - \dot{\mathrm{G}}(t) + 2\dot{\mathrm{G}}(t)\arg\Psi\right]\Psi . \quad (20)$$

According to Eq. (26) from [3]: $\Psi = \sqrt{\Phi}\cdot \mathrm{e}^{\mathrm{i}\Im}$, the argument of the wave function is the statistical action $\Im$, so that the hydrodynamic velocity is to be its gradient:

$$\vec{\mathrm{v}} = 2\tilde{\mathrm{G}}(t)\,\mathrm{grad}\,\Im . \quad (21)$$

Since there is no practically hydrodynamic velocity for a motionless molecular cloud $(|\vec{\mathrm{v}}| \to 0)$, then the value of statistical action is also negligible:

$$\Im = \arg\Psi \to 0 . \quad (22)$$

Taking into account the condition (22), Eq. (20) goes to the following:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = -\mathrm{G}(t)\nabla^2\,\Psi + \frac{1}{2}\frac{d\ln\mathrm{G}(t)}{dt}|\Psi|^2\,\Psi + \frac{1}{2\mathrm{G}(t)}\left[\varphi_g - \dot{\mathrm{G}}(t)\right]\Psi . \quad (23)$$

When a spheroidal body is forming from an initial weakly condensed molecular cloud, its initial gravitational potential φ_g is proportional to $\dot{\mathrm{G}}(t)$, as noted in [7]. So, taking this circumstance into account, Eq. (23) is noticeably simplified:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = -\mathrm{G}(t)\nabla^2\,\Psi + \frac{1}{2}\frac{d\ln\mathrm{G}(t)}{dt}|\Psi|^2\,\Psi . \quad (24)$$

We can note that the obtained equation (24) fully corresponds to the announced nonlinear Schrödinger-like equation (6) of a spheroidal body forming in the state of soliton disturbances. Indeed, denoting in Eq. (24) by $\beta = \mathrm{G}(t)$, $\delta = \frac{1}{2}\frac{d\ln\mathrm{G}(t)}{dt}$, $A = \Psi$ we obtain the well-known nonlinear (cubic) Schrödinger equation (NSE) [17]:

$$\mathrm{i}\frac{\partial A}{\partial t} = -\beta\nabla^2 A + \delta|A|^2 A , \quad (25)$$

where $A = A(\vec{r}, t)$ is an amplitude of the envelope of wave packet and β, δ are some values.

NSE, a nonlinear second-order partial differential equation describing the wave packet envelope in a medium with dispersion and cubic nonlinearity, is one of the key equations playing an important role in the theory of nonlinear waves, in particular, in nonlinear optics and plasma physics [17, 18]. Using the Maxwell equations, as well as the equations of medium, in the case of a slowly varying amplitude $A = |\vec{\mathrm{E}}|$ of a linearly polarized wave

$$\vec{\mathrm{E}}(x, y, z, t) = \frac{1}{2} A(x, y, z, t) \exp[\mathrm{i}(kx - \omega_0 t)] \vec{\mathrm{e}}_x \tag{26}$$

in the reference system of a moving electromagnetic pulse (z, $t = t_{lab} - z / \upsilon_g(\omega_0)$ where $\upsilon_g(\omega_0) \equiv \partial\omega / \partial k|_{\omega_0}$ is the group velocity), a scalar equation of the NSE-type (25) can be obtained within the framework of the paraxial approximation [18]. In this case, the cubic term in the right-hand side of Eq.(25) describes the optical effect of Kerr, i.e. a change of the refractive index of optical material is proportional to the second power of strength of the acting electric field.

Since the NSE (25) completely corresponds to the cubic generalized Schrödinger-like equation (24) for the state of soliton perturbations, this means that, just as NSE (25) describes an evolution of the envelope of a wave packet of electromagnetic waves propagating in *nonlinear dispersible media*, the *cubic* nonlinear Schrödinger-like equation (24) describes an evolution of the envelope of a wave packet of *Jeans' substantial waves* that propagate in a nonlinear and dispersive medium of a forming cosmogonical body (in accordance with the theory of gravitational instability of Jeans [19]).

Under a suitable choice of parameters in Eq. (24) (or NSE (25)), we can write a *one-dimensional version* of the cubic generalized Schrödinger-like equation:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} + \frac{\partial^2 \Psi}{\partial x^2} + \kappa |\Psi|^2 \Psi = 0, \tag{27}$$

where, in the general case $\Psi(x, t)$ is a complex-valued function. Solution of Eq.(27) in the form of a traveling nonlinear wave satisfying the condition $\Psi \to 0$ at $|x| \to \infty$ is the following [17]:

$$\Psi(x, t) = a\sqrt{2\kappa}\, \frac{\exp\{-\mathrm{i}[2\upsilon x + 4(\upsilon^2 - a^2)\, t - \phi_0]\}}{\mathrm{ch}\; 2a(x + 4\upsilon t - x_0)}, \tag{28}$$

where a, υ and ϕ_0, x_0 are arbitrary constants. It is known [17, 18] that the envelopes of the NSE solution in the form of a traveling nonlinear wave (28) are also called *solitons* (see Fig. 1).

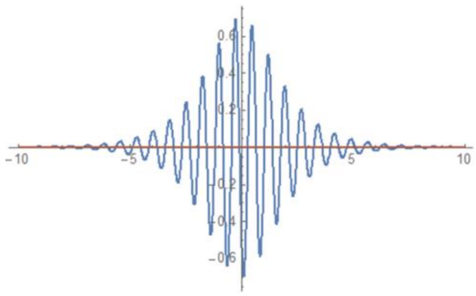

Figure 1. Soliton solution of a one-dimensional cubic generalized Schrödinger-like equation of a forming spheroidal body

Thus, this feature of the solution behaviour in Fig.1 has predetermined the title of equation (6).

4 Derivation of reduced model in the state-space of a nonlinear dynamical system describing behaviour of the cubic generalized Schrödinger-like equation

Considering the one-dimensional partial differential equation (27) we would like to obtain a system of ordinary differential equations (ODEs) in state-space like the well-known Lorenz system [12].
To this end, we intend to seek a solution (27) in the form:

$$\Psi(x,t) = \Psi_0(x,t)\cdot \mathrm{e}^{\mathrm{i}\Im(x,t)} \ , \tag{29}$$

where, according to the mentioned formula (26) in the paper [3], $\Psi_0(x,t) = \sqrt{\Phi(x,t)}$ and $\Phi(x,t)$ is an one-dimensional probability density function to locate a particle into a spheroidal body.
In this case, for the derivatives in Eq. (27), the following expressions are valid:

$$\frac{\partial \Psi}{\partial t} = \mathrm{i}\,\dot{\Psi}_0 \mathrm{e}^{\mathrm{i}\Im} + \mathrm{i}\,\Psi_0 \dot{\Im} \mathrm{e}^{\mathrm{i}\Im}, \quad \frac{\partial \Psi}{\partial x} = \Psi_0^{'} \mathrm{e}^{\mathrm{i}\Im} + \mathrm{i}\,\Psi_0 \Im^{'} \mathrm{e}^{\mathrm{i}\Im}, \tag{30a}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \Psi_0^{''} \mathrm{e}^{\mathrm{i}\Im} + 2\mathrm{i}\,\Psi_0^{'} \Im^{'} \mathrm{e}^{\mathrm{i}\Im} + \mathrm{i}\,\Psi_0 \Im^{''} \mathrm{e}^{\mathrm{i}\Im} - \Psi_0 \Im^{'2} \mathrm{e}^{\mathrm{i}\Im}, \tag{30b}$$

where the dot means differentiation with respect to the time while the dash is differentiation with respect to the coordinate in (30a,b) (the arguments of functions have been omitted for brevity).
Substitution of (30a) and (30b) into Eq. (27) leads to the following equation:

$$-\Psi_0 \dot{\Im} + \mathrm{i}\,\dot{\Psi}_0 = -\Psi_0^{''} - 2\mathrm{i}\,\Psi_0^{'} \Im^{'} - \mathrm{i}\,\Psi_0 \Im^{''} + \Psi_0 \Im^{'2} - \kappa \Psi_0^3, \tag{31}$$

so that after separation of the real and imaginary parts we obtain:

$$\begin{cases} -\Psi_0 \dot{\Im} = -\Psi_0^{''} + \Psi_0 \Im^{'2} - \kappa \Psi_0^3; \\ \quad \dot{\Psi}_0 = -2\Psi_0^{'} \Im^{'} - \Psi_0 \Im^{''}. \end{cases} \tag{32}$$

Let us represent the system of two equations (32) in the form:

$$\begin{cases} \dot{\Im} = \dfrac{\Psi_0^{''}}{\Psi_0} - \Im^{'2} + \kappa\Psi_0^2; \\ \dot{\Psi}_0 = -2\Psi_0^{'}\Im^{'} - \Psi_0\Im^{''}. \end{cases} \tag{33}$$

Relations (33) are a system of nonlinear equations leading to the *reduced model* like the Lorenz model allowing chaotic dynamics in state-space [12]. With a view to further transformation of the system (33) we assume that the amplitude $\Psi_0(x,t)$ depends on the coordinate rather weakly that initially takes place in the molecular cloud (when $\alpha \to 0$). This assumption permits us to neglect the term $\Psi_0^{''}/\Psi_0$ in the first equation of system (33). As a result, we obtain the following system of equations:

$$\begin{cases} \dot{\Im} = -\Im^{'2} + \kappa\Psi_0^2; \\ \dot{\Psi}_0 = -2\Psi_0^{'}\Im^{'} - \Psi_0\Im^{''}. \end{cases} \tag{34}$$

In (34), two variables x and t still appear in explicit form. To use only one variable (temporal) t we can apply the Galerkin's method known in aerohydrodynamics for flow stability problem solving [20]. According to this method we are going to look for the functions Ψ_0 and $\Im$ in the form of expansions in a set of orthogonal basic functions:

$$\begin{aligned} \Psi_0(x,t) &= \sum_n (A_n(t)\sin nkx + B_n(t)\cos nkx); \\ \Im(x,t) &= \sum_n (G_n(t)\sin nkx + H_n(t)\cos nkx). \end{aligned} \tag{35}$$

Choosing the concrete expansions (35), then substituting them into (34) and grouping the terms associated with the different components of these expansions we obtain various ODE systems of the kind:

$$\dot{q}_i = f(q_1, q_2, \ldots q_n), \tag{36}$$

where q_i are the amplitudes in the expansions (35), i.e. A_n, B_n etc., and the function $f(q_1, q_2, \ldots q_n)$ is a polynomial one in the case under consideration.

So, the nonlinearity in this reduced mathematical model is associated respectively with the nonlinear terms in equations of system (34), and it is clearly manifested when the multiplication of 2 trigonometric functions of the series (35) gives the 3rd one also presenting in the given decomposition. Later on we consider expansions involving *second order harmonics* only.

With a view to simplification, we can additionally assume a weak dependence of phase $\Im$ on the time leading to the condition $\dot{\Im} \approx 0$, that is consonant with the above mentioned condition (22). This means that we can pass from the system (34) to a single nonlinear equation relative to the function $\Psi_0(x,t)$. In this case, we obtain the expression for the coordinate derivative of Ψ_0 from the first equation of system (34):

$$\Im' = \sqrt{\kappa}\Psi_0, \tag{37}$$

which after substitution into the second equation of this system (34) leads to a simple nonlinear differential equation with respect to Ψ_0:

$$\dot{\Psi}_0 = -2\Psi_0'\sqrt{\kappa}\Psi_0 - \Psi_0\sqrt{\kappa}\Psi_0' = -3\sqrt{\kappa}\Psi_0\Psi_0'. \tag{38}$$

In order to eliminate the coordinate derivative from Eq. (38) and obtain the model in a reduced form (just as it was done in the works [12], [13], [21] concerning problems of Rayleigh–Benard (convection in the heated layer [20]), Couette–Taylor (flows between coaxial rotating cylinders [22], [23]), Görtler (flows past a concave wall [24])) we suppose that the function $\Psi_0(x,t)$ is periodic with respect to x, so that we can represent it in the form of a decomposition in a trigonometric series leaving the first and second harmonics:

$$\Psi_0(x,t) = A(t)\sin kx + B(t)\cos kx + C(t)\sin 2kx + D(t)\cos 2kx. \tag{39}$$

Then for the derivative with respect to the coordinate we get the expression:

$$\Psi_0'(x,t) = A(t)k\cos kx - B(t)k\sin kx + C(t)2k\cos 2kx - D(t)2k\sin 2kx. \tag{40}$$

After substituting (39) and (40) into Eq. (38) we have:

$$\begin{aligned}
&\dot{A}(t)\sin kx + \dot{B}(t)\cos kx + \dot{C}(t)\sin 2kx + \dot{D}(t)\cos 2kx = \\
&= -3\sqrt{\kappa}(A^2k\sin kx\cos kx - ABk\sin kx\sin kx + A2kC\sin kx\cos 2kx - A2kD\sin kx\sin 2kx + \\
&+ BAk\cos kx\cos kx - B^2k\cos kx\sin kx + B2kC\cos kx\cos 2kx - B2kD\cos kx\sin 2kx + \\
&+ CAk\sin 2kx\cos kx - CBk\sin 2kx\sin kx + C^2 2k\sin 2kx\cos 2kx - C2kD\sin 2kx\sin 2kx + \\
&+ DAk\cos 2kx\cos kx - DBk\cos 2kx\sin kx + D2kC\cos 2kx\cos 2kx - D^2 2k\cos 2kx\sin 2kx),
\end{aligned}$$

whence after separation of the terms associated with the various components of the decomposition (39) we obtain the following system of ODEs:

$$\begin{aligned}
\dot{A} &= \frac{3k\sqrt{\kappa}}{2}(AC + BD); \\
\dot{B} &= \frac{3k\sqrt{\kappa}}{2}(AD + BC); \\
\dot{C} &= \frac{3k\sqrt{\kappa}}{2}(B^2 - A^2); \\
\dot{D} &= -3k\sqrt{\kappa}AB.
\end{aligned} \tag{41}$$

Renaming the coefficients A, B, C, D with the preceding notation q_1, q_2, q_3, q_4 in Eq. (36) and introducing the control parameter $a = 3k\sqrt{\kappa}/2$ we obtain the following reduced model:

$$\begin{aligned}
\dot{q}_1 &= a(q_1q_3 + q_2q_4); \\
\dot{q}_2 &= a(q_1q_4 + q_2q_3); \\
\dot{q}_3 &= a(q_2^{\,2} - q_1^{\,2}); \\
\dot{q}_4 &= -2aq_1q_2.
\end{aligned} \tag{42}$$

The obtained system (42) is an ODE system with quadratic nonlinearity, so in this sense it is similar to the logistic parabola model [20] as well as the Lorenz model [12] and the model describing dynamical behaviour of flow with curvature of streamlines [13]:

$$\begin{aligned} \dot{q}_1 &= aq_2 - q_1 - q_2 q_3; \\ \dot{q}_2 &= q_1 q_3 + bq_1 + cq_2; \\ \dot{q}_3 &= q_1 q_2 + dq_3, \end{aligned} \tag{43}$$

however, unlike the last it contains 4 instead of 3 equations.
As seen from a comparison of Eq. (24) with Eq. (27), the value κ in Eq.(27) is proportional to $\dot{\mathrm{G}}(t)$ under consideration of one-dimensional version of the cubic generalized Schrödinger-like equation (24) of a spheroidal body forming in the state of soliton perturbations. This means that the control parameter of the reduced model (42) in the state-space of the nonlinear dynamical system (describing behaviour of the cubic generalized Schrödinger-like equation (24)) is determined by the value of $\sqrt{\dot{\mathrm{G}}(t)}$.

5 Conclusion

When a cosmogonical body (being in the perturbation state) is formed, linear and nonlinear waves of various types arise there including soliton-like waves. In the section 3, the soliton solution of the cubic generalized Schrödinger-like equation (24) of a forming spheroidal body is considered, i.e. the propagation of soliton waves of Schrödinger type during the formation of the core of a cosmogonical body is justified. In the section 4, the reduced model representing the system (42) of four ordinary nonlinear differential equations with quadratic nonlinearity for the cubic nonlinear Schrödinger-like equation is obtained.

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