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Experimental Detection of Wave Chaos in Quasi-Optical Microwave Cavity Resonator

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Abstract. Microwave resonant structures of spherical (cylindrical) geometry contained inhomogeneous inserts in the form of a metal sphere (disk) has been studied. The inner sphere (disk) is located either symmetrically or non-symmetrically with respect to the structure's side walls. For each of these states, the resonant frequency spectrum was measured in the 8-mm waveband. The correlation factors between the interline frequency intervals has been calculated. The symmetric spherical or cylindrical resonant structures with an inner sphere (disk) show correlation factors close to zero, whereas non-symmetric layered spherical structure where the inner sphere (disk) is placed asymmetrically has correlation factors C(1) > |0.2|. A transition between such states may occur within a narrow range of the structure's eccentricity. The probability distribution of the inter-line intervals has been calculated also. In the case of integrable systems these dependences are practically similar to the Poisson distribution, while the non-integrable system dependences tend to Wigner distribution which demonstrates the spectral line repulsion and can be a sigh of wave chaos in the given resonant structure.

Keywords: Quasi-optical microwave cavity resonator, correlation factors, wave chaos.

1 Introduction

Quasi-optical resonator millimeter and sub-millimeter ranges widely used in microwave technology, and the spectral properties of these devices are well studied [1,2]. At the same time, the influence on the spectrum of the resonator random inhomogeneities which may occur either during manufacture or artificially amended, much less explored fully. This issue is devoted to a few publications, among which can be specified in [3-5].

In [5] we have studied the spectral properties of the bulk quasi spherical optical resonators with inhomogeneities randomly distributed in the form of sapphire particles. Sapphire particles that have dimensions of the order of wavelength in the material, is a strong disturbance to the cavity and greatly influenced its spectrum. It was found that by filling the cavity of the degeneracy of the spectrum associated with the spherical shape of the device is fully removed, and the arrangement of resonance lines on a frequency scale acquired chaotic. In this case the resonance lines broadened considerably and the quality factor decreased, respectively. Sapphire particles having extremely small dielectric losses, almost did not make into the cavity resonator further dissipative losses.

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In [6] the statistical theory of quasi-optical microwave cavity resonator filled with randomly distributed bulk inhomogeneities. The action of irregularities influence on resonant lines due to inter-mode electromagnetic resonator scattering has been considered. It is found that the influence of the scattering on the offset and width of the resonance lines substantially dependent on the frequency interval between the lines. Strongly interact with each other due to irregularities only close in frequency modes. Since the intensity of the resonant line is proportional to resonator quality factor, under the influence of inhomogeneities the resonator spectrum has efficient “rarefaction”. A kind of "rarefaction" of the spectrum is that the intensity of broadened lines drops and lonely lines on which the quality factor of heterogeneity affect poorly maintained.

It is well known that the phase transition phenomenon is a familiar notion of physics to describe the variety of effects accompanying state alterations in solids. The changes are classified as either first or second order transitions. A first-order phase transition is a transfer of material sample from one phase to another which is accompanied by release or absorption of a certain amount of thermal energy. Such transitions are represented, for example, by melting or crystallization processes. At the point of transition the structure’s thermodynamic potential retains its continuity, while its first derivatives are discontinuous. We will be interested rather in the second-order transitions, characterized by changes in the symmetry of the structure’s state, hence appearance of a novel quality. As a representative example, one could mention the phase transition accompanying temperature variations in the crystalline barium titanate, $BaTiO_3$. At room temperature the crystal possesses a cubic lattice. If the temperature is decreased, the atoms get displaced, and it becomes energetically advantageous that the lattice should deform so as the cubic symmetry were reduced to the tetragonal.

This paper is based on the underlying considerations as follows. A second-order transition, as well as its analog at microwaves, can occur not only in a solid but also in a ‘resonant cavity+electromagnetic field’ system, should the latter be subject to a substantial change of symmetry. In case the symmetry has reduced to such an extent that the energy of the structure remains its sole integral of motion, the structure becomes non-integrable, and hence chaotic.

Accordingly, analysis of the second-order phase transition in a bulk microwave resonator, produced by a change in the structure’s symmetry, is related directly to the problem of wave (quantum) chaos (chaotic wave motion). This problem unites studies of the quantum systems whose classic analogs show chaotic performance. The problem emerged more than 50 years back and currently is a subject of intense work in many research institutions of the USA, the United Kingdom, and Germany. The research advanced considerably when it was realized that volumetric microwave resonators could be useful for experimental studies of quantum chaos. In view of the obvious similarity between the Schroedinger equation and the 2D Maxwell equation the results following from experiments with microwave resonators can be interpreted in quantum chaotic terms. In spite of the continued attention toward quantum chaos studies, a number of questions still remain poorly understood. This relates as well to the question of fundamental importance, namely that of line repulsion in the cavity spectrum, which effect is the principal indicator of quantum chaos.

The present paper is aimed at clarifying the nature of spectral line repulsion and understanding how this effect is related to non-integrability of a resonant structure and correlation between the interline frequency intervals. The question was first raised by Landau and Smorodinski [7] who wrote in their famous monograph: "It should be expected, from the considerations of a most general character, that the levels possessing equal spin values would be distributed so that the probability of small separations between them should be extremely low. The levels are kind of repulsed from one another. This is a question worth further investigation". Despite the fact that the monograph was published rather long ago, the question of line repulsion has not yet found its full explanation.

As has been noted, effects analogous to the second-order phase transition may occur in cavity quasi-optical resonators under the conditions of an essential change in their symmetry. A cavity resonator confined by symmetric side boundaries is an integrable system. Should the symmetry of the boundaries be violated, the resonator turns to a non-integrable system wherein the electromagnetic (EM) field is chaotic. The spectrum of such a system is characterized by line repulsion, which effect is the principal indicator of wave (quantum) chaos. The total energy of the 'resonator+EM field' system is conserved and serves as a sole integral of motion. Therefore, the system dynamics may be classified as such of non-integrable, or chaotic structures [8].

To study the spectral line repulsion experimentally, we have used the technique based on correlation factor measurements for the interline frequency intervals. The measuring facility involved a microwave (8 mm range) resonant cavity of spherical geometry containing a metal sphere inside. The whole structure is an integrable system as long as the internal sphere is disposed symmetrically with respect to the outer. Should the position of the internal sphere deviate from the symmetric, the whole structure becomes a non-integrable system. We assume that the reason for spectral line repulsion in a non-integrable system is the statistical dependence (correlation) between the lines, resulting from the occurrence of a phase transition. In this connection we have undertaken a study of the correlation properties demonstrated by the spectra of integrable and non-integrable resonant structures.

Should a resonant structure possess the right symmetry, such as to be an integrable system, the correlation factor of interline frequency intervals in its spectrum is equal to zero. In the case of a violated symmetry the correlation factor assumes a rather high magnitude and the 'resonator+sphere' system becomes non-integrable and chaotic [8]. It can be described in terms of the theory of random matrices that belong to a Gaussian orthogonal ensemble. According to that theory, the interline correlation factor of a dynamic system with a developed chaoticity may become comparatively high (in absolute value), reaching *e.g.*, $C(1) = -0.271$ [9].

This paper presents results of an experimental study of the integrable-to-non-integrable transition between the states of volumetric microwave resonators, which we regard as analogs to the second-order phase transition. The physical objects selected for the study of the chaotic spectral properties accompanying these transitions were quasi-optical microwave resonators of spherical and cylindrical geometries that contained metal inserts in the form of a sphere or a cylinder, respectively. By placing the inserts either symmetrically or non-symmetrically with respect to the general structure, it was possible to produce such conditions where the system was either integrable or non-integrable, thus altering its dynamics. With a violated symmetry of the structure the resonator's spectral lines are statistically coupled. The spectrum becomes chaotic, acquiring the properties pertaining to quantum chaos, specifically spectral line repulsion and its associated Wigner type distribution of the interline frequency intervals. So, if the state of a resonator can be changed between integrable and non-integrable, we are in a position to study the system in the course of a second-order phase transition.

2 Spherical resonator with inner sphere

First, we will consider the results concerning the 3*D* spherical resonator. The quasi-optical spherical resonant structure that was used in the experiment is shown schematically in Fig.1. It is a spherical cavity with copper walls, 270 mm in diameter, which can support electromagnetic oscillations belonging to the 8-mm range. The inner surface of the cavity has been manufactured to a high accuracy and polished so as to avoid additional losses in the resonator walls.

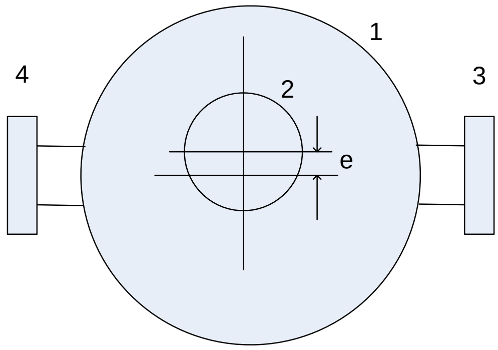


Fig**.** 1. Schematic of a 3*D*-resonator with an inner metal sphere: 1- spherical cavity; 2- inner metal sphere; 3 and 4- waveguide / dipole coupling elements for the resonator; *e* - eccentricity.

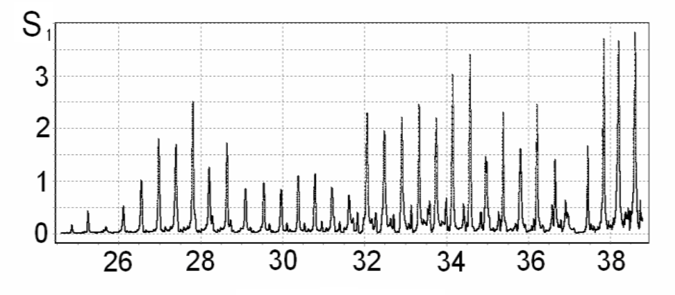
The electromagnetic oscillations could be excited in the resonator with a waveguide-based dipole antenna. The dipole was represented by a thin rod, 0.7 mm in diameter, driven from a standard waveguide. A short section of the antenna, of a smaller size than the operating wavelength, was placed inside the spherical resonator. The latter also contained an aluminum sphere with a polished surface, 100 mm in diameter that was suspended on a thin capron thread. Hooking the thread and manipulating the sphere position were possible from outside the resonator. The sphere could be placed at a fixed height inside the resonator, either at a symmetric or a non-symmetric position relative the side walls.

With a zero eccentricity of the inner sphere position in the resonator, $e = 0$, the resonant structure represents a fully symmetric, and hence an integrable system. To convert it into a non-integrable system, the inner sphere was displaced from the center, such that the spherical symmetry was violated.

The resonator spectrum is shown in Fig.2, and Fig.3. It involves more than 70 spectral lines over the frequency range 24…38 GHz. The resonator has been manufactured from a highly conducting material and the surfaces polished. Therefore, the Q-factor of the spectral lines is rather high, about (1or 2)$\times 10^3$. Note that the Q-factors of different lines may be sizably different. Admittedly, this is connected with the non-uniform field distribution in the cavity, which non-uniformity affects interaction of the cavity modes possessing different ohmic losses. The dissipative modes associated with the resonance lines tend to reduce the net Q-factor.

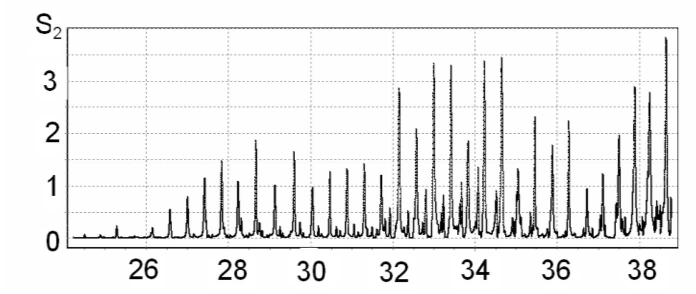
The spectra were measured and recorded with a КСВН Р2-65 panoramic meter, equipped with a special computer adapter enabling automatic measurements of the principal resonant line parameters, the proper frequencies and Q-factors. The time required for recording the entire spectrum, within the specified range of frequencies, was 40 s. The reasonably short recording time permitted us to avoid the negative effects associated with noise and interference in the transmission line connecting the source and the receiver. The level of microwave power at the resonator input never exceeded a few milliwatts.

The integrability-controlled dynamic properties of the resonator were studied through analysis of correlation characteristics of the spectrum. To do so, the data on the intervals separating neighboring spectral lines on the frequency scale were used to estimate the correlation factor $C(1)$ (in this notation the unity means that the value of C has been estimated for adjacent resonances).



Frequency, GHz

Fig. 2. The spectrum of a spherical resonant cavity with a symmetrically disposed inner sphere. S_1 is the spectral line intensity.



Frequency, GHz

Fig 3. The spectrum of a spherical resonant cavity with a non-symmetrically disposed metal sphere inside. S_1 is the spectral line intensity.

Despite the visible similarity of the spectra measured for the symmetric and the non-symmetric resonator, their dynamic properties are different. The correlation factor estimated for very small eccentricities is close to zero, *e.g.* $C(1) \cong 0.02$. Meanwhile, even slightly greater deviations in the position of the inner sphere from symmetry are sufficient for the correlation factor to sharply increase, reaching magnitudes that are characteristic of structures with a mature quantum chaos. The width of the eccentricity interval wherein the correlation grows as sharply is 0.1 (in terms of units reduced against the resonator radius). The relative width of the transition region corresponding to a two-fold increase in e is 0.05. Should the correlation factor be plotted as a function of the eccentricity, it would start from very low values near e=0 and reach a plateau $C(1) > 0.2$ (in absolute value) at greater values of e. This magnitude of $C(1)$ corresponds to a developed chaos [9] in the non-integrable system involving a resonator with an eccentrically disposed metal sphere inside. The growth in the magnitude of $C(1)$ at greater eccentricities e is evidence for a rearrangement in the positioning of the spectral lines. The probability of small interline intervals is reduced and the lines demonstrate kind of a mutual repulsion. In fact, this is the essence of the phase transition analog discussed in this paper. It can be stated that the transfer from an integrable to a non-integrable system is the reason for the spectral line repulsion characteristic of quantum chaotic states. The $C(1)$ vs e dependence as observed here might be characteristic of other effects accompanying second-order phase transitions with symmetry alterations.

Another manifestation of the phase transition is the changed distribution function of the interline frequency intervals, associated with the repulsion of the spectral lines. Evidence is given by the results of measurements presented in Figures 4 and 5.

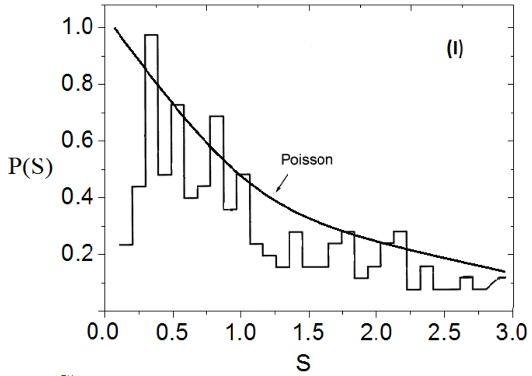


Fig 4. Histogram of interline frequency intervals in the spectrum of a spherical resonator with a symmetrically positioned metal inner sphere; *s* is the interline frequency separation, normalized to the mean value, and *P*(*s*) the probability for an interval to fall into a prescribed range.

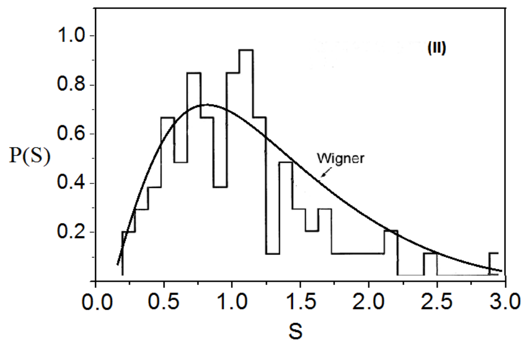


Fig 5. The histogram of interline frequency intervals in the spectrum of a spherical resonator with a non-symmetrically disposed metal inner sphere

It can be seen from Fig.5 that the distribution of interline intervals existing in the integrable system of a spherical resonator with a metal sphere is rather close to the Poisson distribution function, *i.e.* demonstrates an exponential fall-off of the probability in dependence on the mean interline separation, *s*.

3 Cylindrical resonator with inner disk

As soon as the inner sphere in the resonator occupies a non-symmetric position, bringing the structure to a non-integrable state, the distribution of the interline intervals undergoes an essential change. Specifically, the $P(s)$ curve acquires a maximum near s=1, suggesting a Wigner-type distribution. This is a situation where the resonator with the inner ball turns into a system with a fairly developed chaotic behavior.

In this connection the question arises, whether a similar phase transition can occur in a system of lower dimension, for instance, in a 2D system. To find an answer, we have experimented with a quasi-2D structure incorporating a quasi-optical volume resonator of cylindrical geometry. The resonator is shown schematically in Fig.6. It is a cylinder 120 mm in diameter and 16 mm in height, made of aluminum. The electromagnetic oscillations excited and measured in the structure belonged to the same frequency range as such in the spherical resonator. To excite the waves, a waveguide based diffraction antenna was used, specifically a 2.8 mm opening in the cap that closed the end face of a standard rectangular waveguide. The guide, in its turn, was soldered in the side wall of the cylindrical resonator. . To study the phase transition in the volume resonator of cylindrical geometry that could be transformed from an integrable to a non-integrable system, an aluminum disk of 60 mm in diameter was used, placed inside the resonant cavity. The position of the disk inside the cavity could be changed between the strictly symmetrical (when the structure represents an integrable system) and a fully non-symmetrical. In the latter case the structure manifests pronounced chaotic properties in its spectrum.

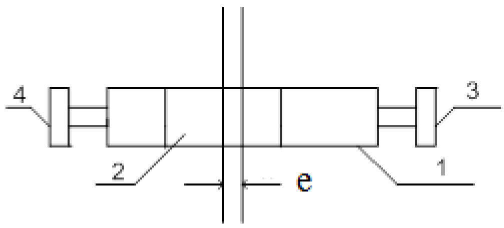


Fig. 6. Schematic of the 2*D* (cylindrical) volume resonator: 1- casing; 2-inner metal disk; 3 and 4-elements of resonator-to-load diffractional coupling. *e* is a measure of eccentricity in the position of the disk in the resonant cavity.

Note that the number of the resonance lines observed decreases at greater values of the eccentricity *e.* This effect was present in all the resonators with structural inhomogeneity that we have analyzed at the stage of altered symmetry, including the spherical one with an inner sphere and the cylindrical with a disk.

The experiments have shown that the effects demonstrated by the quasi-2D system of the cylindrical resonator are quite similar to those described above in connection with the spherical resonant cavity containing a metal ball. Specifically, the interline frequency intervals were distributed after Poisson's law if the disk is positioned symmetrically with respect to the main cavity, but according to a Wigner function if the disk occupied a non-symmetrical position. Similar as in the spherical resonator, a second order phase transition occurred as soon as the symmetrical position of the disk inside the cavity changed to become noticeably asymmetric.

4 Correlation factor

Accordingly, the correlation factor of interline frequency intervals increased sharply within a narrow range of eccentricities about zero. A second-order phase transition in the volume resonator (of either spherical or cylindrical geometry) suggests a qualitative change, associated with correlation properties of the structure's spectrum (see Fig.7). It is important that the volumetric resonator should possess a symmetry which could be violated essentially by introduction of disturbances into the cavity or deformation of the cavity's lateral surface.

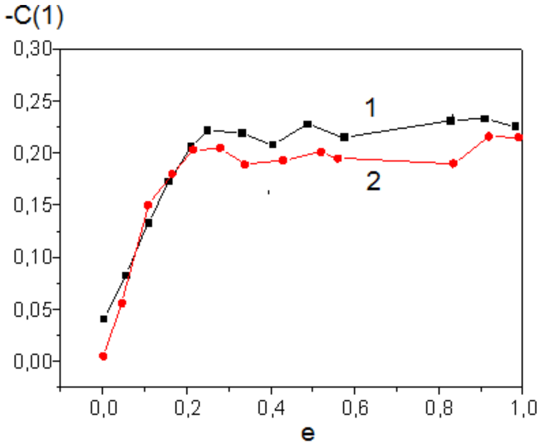


Fig. 7. Magnitude of the correlation factor $C(1)$ in the resonant cavity as a function of deviation e from the fully symmetric position of the inner spherical ball (curve 1) or disk (curve 2).

Along with the correlation factor, other parameters relating to quantum chaos manifestations may also be of interest. These include, in particular, the spectral rigidity which in the presence of chaos suggests a special form for the dependences of spectrum parameters upon the number of eigenfrequencies [10].

Conclusions

Analogs to the second order phase transition in volumetric resonators of spherical and cylindrical geometries have been discovered and studied. As has been found, an integrable spherical (cylindrical) resonant structure with an inner sphere (disk) in the cavity demonstrates practically no correlation of interline frequency intervals in the spectrum, whereas non-integrable systems where the inner sphere (disk) is disposed non-symmetrically a show correlation factor of magnitude $C(1) > 0.2$. Dependences of interline interval distribution functions upon the mean separation between the resonant lines have been determined for the spherical and the cylindrical resonator. In the case of an integrable system such a dependence is given by the Poisson law, whereas the non-integrable system demonstrates Wigner-type distribution which is characteristic of system states with repulsing resonance lines. Thus, it has been established that a change in the symmetry of a microwave resonant structure may result in effects analogous to a second order phase transition. At the transition the resonator becomes a non-integrable system demonstrating quantum chaotic behavior. The signs of quantum chaos are spectral line repulsion and non-zero correlation of interline frequency intervals.

As a result of the phase transition the spherical (cylindrical) resonator acquires a novel quality. Specifically, its eigen-frequency spectrum consisting of independent spectral lines, as long as the structure is an integrable system, gets converted to the spectrum of a non-integrable system wherein the lines are correlated. This effect can be useful for applications, in particular for creating an ultra-wideband noise generator at millimeter wavelengths.

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