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A Fractal Model of the Stimulus-Response Relationship

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Abstract. The models relating the intensity of perception to the stimulus are of great importance in sensorial analysis and practical applications. For example, in olfactometry they have been employed to determine: the odour threshold and suprathreshold, hedonic scale assessment and the degree of appreciation. The perceived intensity of a stimuli should be expressed in the form of function depending on a physico-chemical variable, eg concentration of the substance in fresh air or clean water. It must take into account a characteristic threshold of its perception, whereas at higher concentrations the perceived intensity should reach a plateau. Those requirements well satisfy the sigmoidal Gompertz function, which depends on the concentration variable and three parameters representing the initial perceived intensity, retardation constant and the initial increase rate constant. The application of the mapping procedure converts this function into power law formula $k(C)C^{n(C)}$ including the signal-dependent scaling factor k(C) and exponent n(C). Because the Gompertz model describes the self-similar and allometric processes of the fractal nature, the mapping procedure transfers those peculiar properties from the Gompertz function onto power law one. Hence, the phenomena described by the latter are endowed with the typical fractal properties; in particular n(C) is interpreted as C-dependent fractal dimension of the stimulus-response process. The formula obtained will be applied in modelling the odour intensity-concentration relationship for acetone, ethyl acrylate, pyridine, vanillin and then in calculating the scaling factor k(C) and exponent n(C), representing the C-dependent fractal dimension of the stimulus-response relationship.

Keywords: Sensorial analysis, Stimulus-response models, Fractality, Self-similar processes, Gompertz model, Mapping procedure

1 Introduction

The fractal characteristic of the objects and phenomena is usually determined in space-time with temporal or spatial fractal dimensions. Because time is a scalar variable it is tempting to extend the class of the fractal phenomena to include other scalar variables representing, for example, a magnitude of the physical stimulus – e.g. concentration of the substances used in the stimulus-response test analysis. In sensorial analysis the most important model relating the intensity of perception to the stimulus was introduced by Stevens [1] in the form of the power law

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$$y(C) = kC^n + y_0$$

in which C is the magnitude of the physical stimulus (e.g. concentration), y(C) is the psychophysical function describing the subjective magnitude of the sensation evoked by the stimulus, n is an exponent that depends on the type of stimulation and k is a scaling factor that depends on the type of stimulation and the units used; y0 is perceived intensity with pure medium as stimulus (e.g. clean air or pure solvent). Attempts to apply the Stevens' power law to fit the experimental data for a wide range of the physical stimulation revealed that it incorrectly reproduces them for strong stimulations. In this case, the intensity tends asymptotically to a maximum value and not to infinity as the Stevens' model predicts. To remove this anomaly, Easton [2] has suggested replacing the power law by the sigmoidal (S-shaped) function for quantitative description of the psychophysical laws for the magnitude estimation (saltiness). In particular he proved that the experimental data are better reproduced by the C-dependent (concentration dependent) sigmoidal Gompertz function [3]

$$G(C) = G_0 e^{\frac{b}{a}\left(1-e^{-aC}\right)}$$

then by the Stevens power one. The aim of this work is to modify the Stevens' formula in such a manner that its power form will be preserved but it will properly describe the experimental psychophysical data in an arbitrary range of the stimulus intensity. To this purpose we apply the mapping procedure to derive the analytical form of the C-dependent scaling factor k(C) and exponent n(C) indispensable to formulate the fractal stimulus-response relationship

$$y(C) = k(C)C^{n(C)}$$

This formula will be applied in modelling the odour intensity-concentration relationship for different chemical compounds employed in sensorial analysis and then in calculating the scaling factor k(C) and exponent n(C) characterizing substances under consideration.

2 The C-dependent mapping procedure

To formulate the fractal stimulus-response relationship, we convert the Gompertz function to the power form by making use of the mapping procedure, which permits interpolating any differentiable function by a family of power law curves

$$\left\{ y_i(C) = k(C_i)C^{n(C_i)} + y_0 \quad i = 1, 2, ...N \right\}$$

determined at the points {Ci, yi(Ci)}. Defining the sets of parameters k={k(Ci), i=1,2…}, n={n(Ci), i=1,2…}, one may derive the formula searched for by assuming that the Gompertz function is isovalued and isosloped with the power law function for the each value of the stimulus C. In such circumstances the equality of those functions as well as their first derivatives generate the set of nonlinear equations [4]

$$y_0 + kC^n = G(C) \qquad nkC^{n-1} = be^{-aC}G(C) \qquad y_0 = G_0$$

whose solutions provide the scaling factor k(C) and exponent n(C) in analytical forms

$$n(C) = bCe^{-aC}\frac{G(C)}{G(C)-G_0} \qquad k(C) = C^{-n(C)}\left[G(C)-G_0\right]$$

representing the functional version of the continuous sets n(C)={n(Ci), i=1,2…} and k(C)={k(Ci), i=1,2…}.

3 The C-dependent fractality

Because the phenomena described by the Gompertz model belong to the self-similar and allometric processes of the fractal nature [5], one may assume that the mapping procedure transfers those peculiar properties from the Gompertz function onto the fractal one. Hence, the phenomena described by it should be endowed with the typical fractal properties, whereas n(C) could be interpreted as the C-dependent fractal dimension of the stimulus-response process. To prove this thesis we employ the proof [5] that the Gompertz function is endowed with a self-similar characteristics by taking advantage the following relationships

$$G(C+s) = \alpha(s)G(C)^{\beta(s)} \qquad \beta(\mathrm{s}) = \exp(-as) \qquad \alpha(s) = G_\infty^{1-\beta(s)}$$

in which s denotes a change in the stimulus. Taking into account the equations specified above, one may prove, that proposed stimulus-response relationship is endowed with typical fractal self-similar characteristics

$$y(C) = k(C)C^{n(C)} \Rightarrow y(C+s) = \alpha(s)\left[y(C)\right]^{\beta(s)}$$

hence, the exponent n(C) can be interpreted as the C-dependent fractal dimension of the psychophysical process under consideration.

4 Applications

The formula obtained will be applied in modelling the odour intensity-concentration relationship for acetone, ethyl acrylate, pyridine, vanillin and then in calculating the scaling factor k(C) and exponent n(C). The experimental data have been taken from [6] (acetone, ethyl acrylate, pyridine) and [7] (vanillin).

Tab.1. The values of parameters fitted to the experimental data [6,7] by making use of the fractal formula $y(C)=k(C)C^{n(C)}$. R is the statistical indicator of the goodness of the fit.

Parameter	Acetone	Ethyl	Pyridine	Vanillin
a	0.0022(7)	4.53(1.97)	1.23(23)	0.0009(2)
b	0.0044(15)	6.01(2.96)	2.75(55)	0.0013(4)
G_0	7.68(1.97)	19.3(4.2)	7.38(1.16)	2.45(38)
R	0.9875	0.9507	0.9948	0.9810

The parameters evaluated are used for generating the analytical forms of n(C) and then their plots representing the C-dependent fractal dimension.

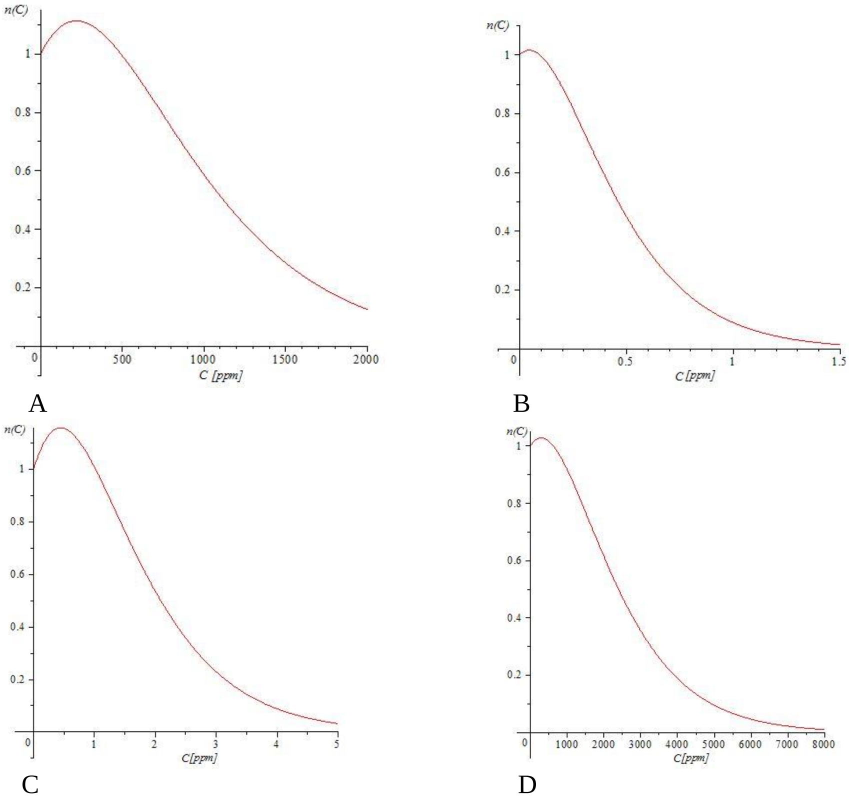

Fig. 1. Plots of the fractal dimension n(C) of the odour intensity-concentration relationship $y(C)=k(C)C^{n(C)}$ for acetone (A), ethyl acrylate (B), pyridine (C) and vanillin (D).

Their analysis reveals that the exponent n(C) increases from 1 for C=0 to the maximal value of 1.11 for C=218.8 [ppm] (acetone), 1.02 for C=0.0456 [ppm] (ethyl acrylate), 1.16 for C=0.4428 [ppm] (pyridine) and 1.03 for C=296,1 [ppm] for vanillin, and then decreases to zero. These results confirm King's [8] observation that the calculated exponent of the psychophysical power law depends on the stimulus-range and decreases as the range of the stimulus increases. The calculations performed reveal that the scaling parameter k(C) attains the maximum for C=1 [ppm], whereas minimum for identical concentration magnitudes as specified above(i.e. giving the maximal values of n(C)), and then it tends to the plateau as concentration increases to infinity. The results obtained indicate that the odour perception is governed by the same psychophysical power law formula $y(C)=k(C)C^{n(C)}$, characterized by the C-dependent fractal dimension n(C), whose mathematical characteristics is independent of the type of odourous compound and the range of its concentration. For example, in the case of acetone the experimental data range for C=<95.8,1040.4> [ppm], whereas for pyridine C=<0.025,1.997> [ppm] [6].

Conclusions

It has been demonstrated that the odour perception and the concentration of the stimulating compound are interrelated by the fractal relationship endowed with the self-similar and power law behavior. Deviation of exponent n(C) from unity indicates that derived relationship for the psychophysical phenomena, describes the self-similar and allometric processes of the fractal nature. Hence, the class of the objects and phenomena associated with temporal or spatial fractal characteristics can be extended to include the stimulus-response processes depending on the stimulus intensity e.g. concentration. Sensory intensity characteristic of odourous compounds are of great importance both in odour and flavor scientific research as well as in perfumery and flavoring industries. Because it should take into account both threshold of a perceived stimulus and plateau at higher stimulus intensity, the fractal power law can be applied in sensorial analysis independently of the range and the type of stimuli. It is well-known that the power law is a genuine property of the fractal phenomena and is symptomatic of self-similar patterns, for instance those appearing in physiology domain. The fractal power law successfully employed in olfactory analysis suggests that the fractality is an immanent property of the stimulus-response processes and their fractal characteristics is independent of the stimulus type and its intensity. Hence, the derived formula can be applied not only in olfactometry but also in sensorial analysis including, for example, saltiness, brightness and other area of perception [9].

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