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Construction of Dynamical Systems from Output Regular and Chaotic Signals

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Abstract: The problem of construction of the deterministic dynamical system from output signals (reconstruction) is very important. Two reconstruction methods have been used and compared. First one is the method of successive differentiation and the second is based on delay coordinates. It was firstly suggested to choose time delay parameter from the stable region of a divergence of the reconstructed system. Results show that both methods can capture regular and chaotic signals from reconstructed systems of the third order with nonlinear terms up to sixth order. Types of signals were examined with spectral methods, construction of phase portraits and Lyapunov exponents.
Keywords: Reconstruction, Dynamical system, Chaotic regime, Successive differentiation, Delay time.

1 Introduction

The problem of reconstruction of deterministic dynamical system from output signals is of great importance in studying of properties of experimental signals such as acoustic signals, ECG, EEG and so on. Reconstructed dynamical system may add a significant qualitative information to chaotic data analysis. Stability conditions, bifurcation curves, all types of steady – state regimes could be studied for solutions of a reconstructed system. Two reconstruction methods have been developed by Crutchfield and McNamara [1] and used for variety of signals later [2-4]. The first method is based on suggestion that the signal can be presented by a function that has at least three derivatives, so this is method of successive differentiation. Applying this method the dynamical system has a following form [1-4]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= F_3(x_1, x_2, x_3) \end{aligned}$$

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where $F_3(x_1, x_2, x_3)$ is a nonlinear function. The second method of reconstruction is based on delay coordinates. We need to reconstruct the dynamical system from the time series of some state variable $x(t)$ with the fixed sampling step dt. We have series of $s_k = x(kdt)$, , using value of time delay $\tau = ndt$ (which is chosen to yield optimal reconstruction [1]) we construct the dynamical system in the form [1-4]:

$$\dot{x}_1 = F_1(x_1, x_2, x_3)$$
$$\dot{x}_2 = F_2(x_1, x_2, x_3)$$
$$\dot{x}_3 = F_3(x_1, x_2, x_3)$$

where $x_1(t) = x(t)$; $x_2(t) = x(t+\tau)$; $x_3(t) = x(t+2\tau)$, $F_i(x_1, x_2, x_3)$ are nonlinear functions.

2 Construction of Dynamical Systems from Output Signals of Pendulum System

Reconstruction methods are applied to the signals of a deterministic dynamical system of pendulum oscillations which may have regular and chaotic regimes [5]**:**

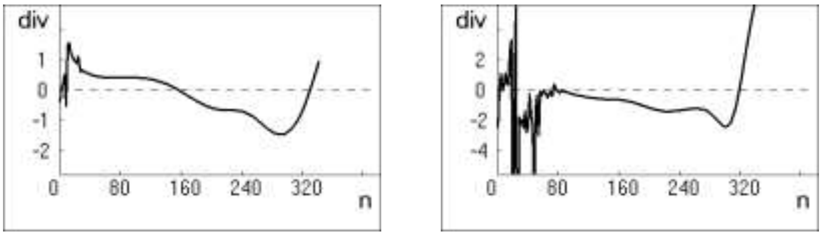
$$\dot{y}_1 = -0.1y_1 - y_2 y_3 - \frac{1}{8}(y_1^2 y_2 + y_2^3)$$
$$\dot{y}_2 = -0.1y_2 + y_1 y_3 - \frac{1}{8}(y_2^2 y_1 + y_1^3) + 1$$
$$\dot{y}_3 = -0.5y_2 - 0.61y_3 + F$$

Nonlinear functions $F_i(x_1, x_2, x_3)$ in the first and second systems have the following form:

$$F(x_1, x_2, x_3) = a + \sum_{i=1}^{3} a_i x_i + \sum_{i,j=1}^{3} a_{ji} x_j x_i + \ldots + \sum_{o,m,n,k,j,i=1}^{3} a_{omnkji} x_o x_m x_n x_k x_j x_i$$

with nonlinear terms up to third order for the regular signals and up to the six order for the chaotic.

The traditional way to obtain time delay parameter $\tau = ndt$ for the second method of reconstruction is to use time interval when the autocorrelation function is equal to zero [2-4]. For such chosen τ the divergence of a reconstructed system may not be negative. So that we introduce other way to choose τ. Real system is nonconservative and, the divergence of systems should be negative too. For example, for the original pendulum system div is equal to -0.81. In Figure 1 the dependence of reconstructed systems divergence on n in the steady – state regimes is shown. We choose n for time delay τ from the stable region of div.

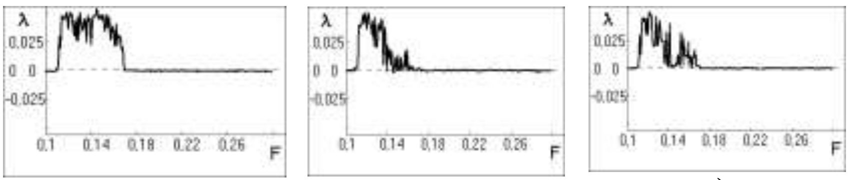


a) b)

Fig. 1. The dependence of reconstructed systems divergence on n for regular initial signal $F = 0.257$ (case a) and chaotic $F = 0.114$ (case b).

For every value of the bifurcation parameter F from the interval $0.1 \le F \le 0.3$ the reconstructed systems were built and the output signals were determined. And then the largest Lyapunov exponents [6] were calculated. For that purpose we use the fifth – order Runge – Kuttas method with the precision of $O(10^{-7})$. Initial conditions were selected in the vicinity of the original signal, and for the steady – state regime signals we choose $N = 2^{18}$, $dt = 0.004$.

The dependence of the largest Lyapunov exponent of the pendulum system on values of the bifurcation parameter F is shown in Figure 2.a. The dependences of the largest Lyapunov exponent on F for the first and the second reconstructed dynamical systems are shown in Figure 2.b – c correspondingly.



a) b) c)

Fig. 2.The largest Lyapunov exponent of the pendulum system (case a) and of the reconstructed systems (cases b and c).

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We may see similarity of both graphs to the dependence for the original system in Figure 2.a with the exception of the region $0.15 \le F \le 0.18$ where the transition to chaos occurs.

2 Construction Systems from Regular Output Signal

As was shown in the book [5] the solution of the pendulum system would be regular if bifurcation parameter is F=0.257. We used this value and solved the system in order to get the output signal. Then we reconstruct the system using the two methods.

For the second method we reconstruct the system using small initial value for the delay parameter and build the dependence of the divergence on value n and choose n from the stable interval of the delay parameter (Figure 1.a, n=240). As the result the system get the form with nonlinear terms only to the third order of nonlinearity.

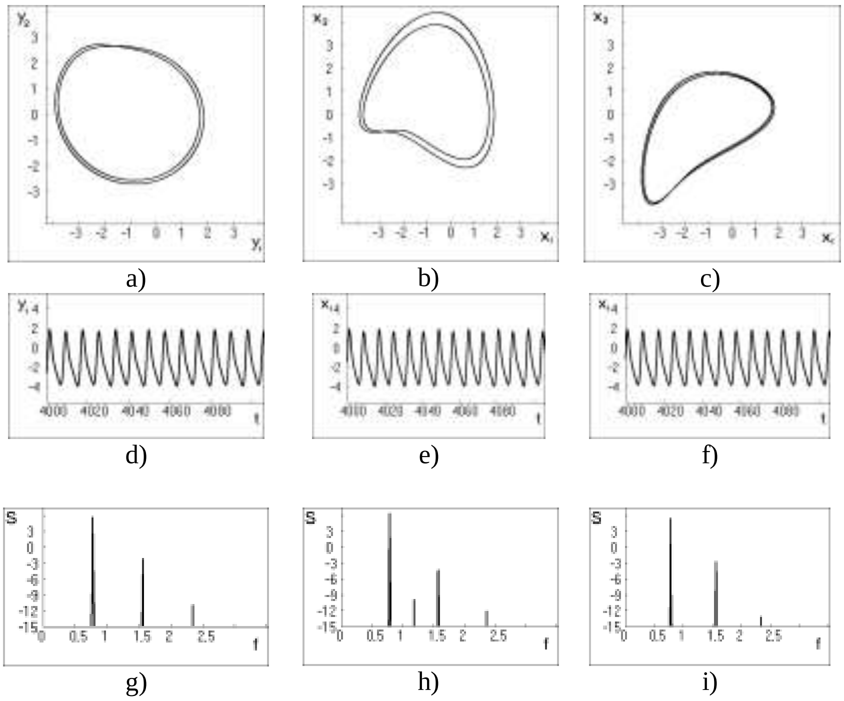


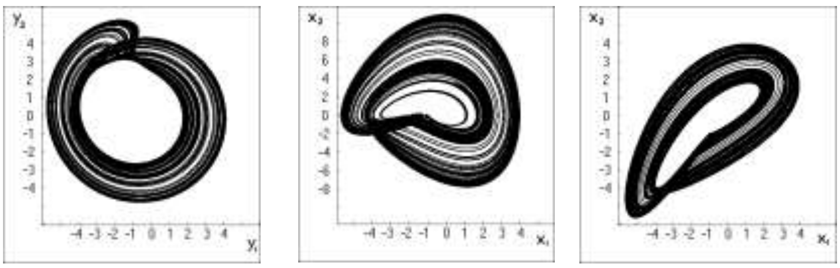
Fig. 3. The portrait of initial pendulum system (F=0.257), case a , the portraits of the reconstructed systems, cases b–c, their time realizations, cases d–f, and power spectrums, cases g–i.

Projections of the limit cycle with two loops on the plane are shown in Figure 3. a–c for the solution of the original system (Figure 3.a) and the reconstructed

first and second dynamical systems (Figure 3.b–c). Since for reconstruction we use only the first variable signal phase portrait projections on the plane with the second variable only qualitatively are look like the original limit cycle with two loops. Time realizations of the first variable and their power spectrums are presented in Figure 3.d–i. Figure 3.d and Figure 3.g describes the solution of the original system, and Figure 3.e–f and Figure 3.h– i gives the information about solutions of the reconstructed dynamical systems.
Since power spectrum indicates the power contained at each frequency, the peak heights corresponds to the squared wave amplitudes (i.e. the wave energy) at the corresponding frequencies. The first method of reconstruction gives the solution which the power spectrum for the regular signals coincides with the output signal power spectrum up to 96% for the first three peaks. The second method gives the precision up to 98%. Also the second method determines the maximum Lyapunov exponent more precisely for chaotic regimes (with a precision to $O(10^{-3})$) than the first method.

3. Construction Systems from Chaotic Output Signal

Now we use such parameter F for the pendulum original system when this system has the chaotic solution, namely F=0.114. Then we reconstruct the system using the two methods of reconstruction with nonlinear function $F_i(x_1, x_2, x_3)$ with nonlinear terms up to the sixth order. For the second method we reconstruct the system using small initial value for the delay parameter and build the dependence of the divergence on value n and choose n from the stable interval of the delay parameter (Figure 1.b, n=240).
Projections of the chaotic attractor of the initial system and of the reconstructed systems are shown in Figure 4.a–c. As could be seen from Figure 4 the both methods qualitatively good approximate chaotic attractor of the original system. Time realizations of the chaotic attractors after finished transient regimes are also similar and given in Figure 4.d–f. Power spectrums for the original signal and for the signals from the reconstructed systems are shown in Figure 4.g– i and may be approximated by the same decay function $S = -6.75 - 8.5f$.



a) b) c)

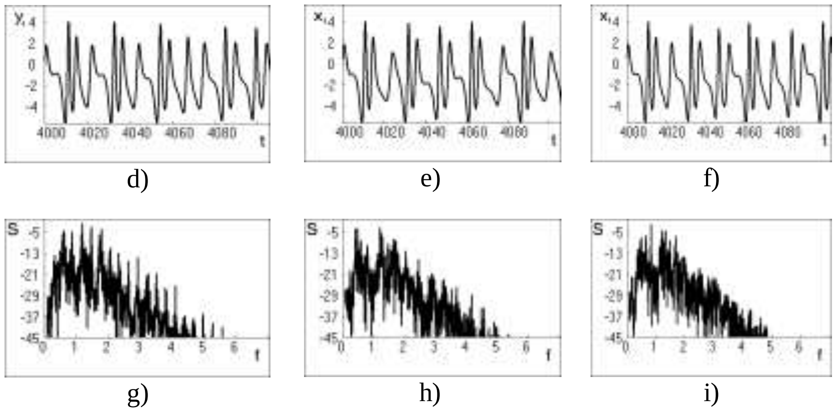


Fig. 4. The portrait of initial system (case a) (F=0.114), the portraits of the reconstructed systems (cases b –c), their time realizations (d –f) and power spectrums (g–i).

3 Construction System from Synthetic ECG Signal

As practical application of the considered methods the signal of a dynamical model for generating synthetic electrocardiogram signals [9] was used. This signal is regular and outwardly looks like the electrocardiogram of healthy man. Using the method of delay the system of eighth order was built. In Figure 5 temporal realization is represented by synthetic electrocardiogram. In Figure 6 temporal realization of the first coordinate of the solution of the reconstructed system is represented. As is obvious from graphs both signals are regular and have an identical period of oscillations.

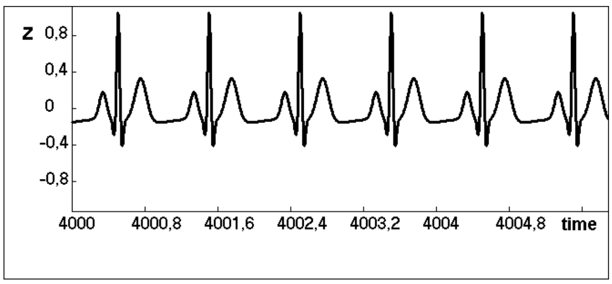


Fig. 5. Synthetic electrocardiogram signal.

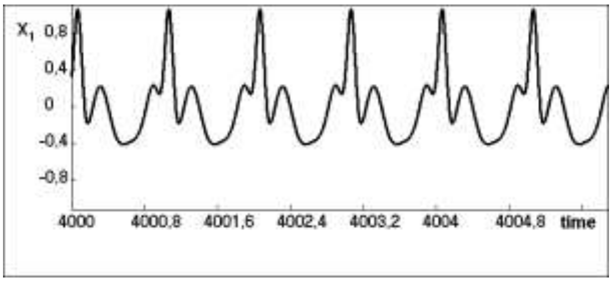


Fig. 6. Signal generated by reconstructed system.

4 Conclusions

Results show that both methods can capture regular and chaotic signals from reconstructed systems of the third order with nonlinear terms up to sixth order. Types of signals were examined with spectral methods, construction of phase portraits and Lyapunov exponents. The first method gives the solution which the power spectrum for the regular signals coincides with the output signal spectrum up to 96 % for the first three peaks. The second method gives a mistake around 2 %. And the second method determines the maximum Lyapunov exponent more precisely for chaotic regimes (with a precision to $O(10^{-3})$) than the first method.

Real systems are nonconservative and, a divergence of systems should be negative. It was suggested for the first time that the delay parameter for the second reconstruction method must be chosen from the stable region of the divergence behaviour of the reconstructed system.

The both methods qualitatively good approximate the phase portrait of chaotic attractor of the original system. Moreover, time realizations of the chaotic attractors after finished transient regimes are quiet similar. And what is more important, power spectrums for the original signal and for the signals from the reconstructed systems may be approximated by the same decay function $S = -6.75 - 8.5 f$. Calculations also show that more precisely the value of bifurcation parameter for chaotic regimes gives the second method of reconstruction.

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